GENERALLY COVARIANT UNIFIED FIELD THEORY

THE GEOMETRIZATION OF PHYSICS II

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I dedicate this book to
"Objective Natural Philosophy"

Preface

In this second volume, nineteen new papers on the Evans unified field theory are collected. These papers deal with different aspects of physics, and are intended to catalyze further research in each area by indicating the major points of progress in each case.

The first paper develops the objective laws of classical electrodynamics. By objective in this context is implied that the equations are those of general relativity, and are therefore generally covariant. They retain their form to any observer moving arbitrarily with respect to any other observer. The well known Maxwell-Heaviside (MH) equations of the standard model are covariant only under the Lorentz transformation of special relativity, in which one frame moves with respect to another at constant velocity. By making the MH equations objective equations the interaction of gravitation with electromagnetism can be analyzed objectively for the first time. This ability has major implications for the acquisition of electric power from Evans spacetime, and for the ability to control gravity with electromagnetism in the aerospace industries. Importantly for fundamental physics, the Evans spin field observed in the inverse Faraday effect is shown to be a direct consequence of general relativity.

Paper two develops the basics of the first and second order Aharonov Bohm (AB) effects in terms of the spinning spacetime of the Evans unified field theory. In this way the AB effects are explained straightforwardly without the need for multiply connected topology. The second order AB effect (the electromagnetic AB effect) is expected to be important in novel RADAR and stealth technologies.

Paper three develops the theory of the inverse Faraday effect and shows that the magnetization of matter by electromagnetism is due to the Evans spin field in general relativity (i.e. in objective physics). The quantized equivalent of the inverse Faraday effect is radiatively induced fermion resonance (RFR), which is expected to be important for MRI technology without permanent magnets, and in the development of portable MRI apparatus for clinics and hospitals.

Paper four discusses the origin of polarization and magnetization in the generally covariant electrodynamics of the Evans unified field theory. The visible part of a laser beam is shown to be due to the time and space variations of the potential four-vector, but this is always accompanied by an invisible region which is the swirling or spinning spacetime of general relativity defined by the spin connection, and which is the origin of polarization and magnetization, the inverse Faraday effect, and the electromagnetic AB effect.

Paper five is an analysis of the Eddington type of experiments in terms of refraction in the electromagnetic sector of the Evans field theory, thus showing that the Eddington experiments are not purely gravitational in nature. There is a number of electromagnetic effects expected from the unified field theory.

Paper six expresses the Coulomb and Ampère Maxwell laws in terms of the Schwarzschild metric as an illustration of the fact that the Evans unified field theory is a theory of both classical electromagnetism and classical gravitation. In the standard model this type of analysis does not exist, because the MH equations are developed in a Minkowski or flat spacetime in which gravitation is undefined. This paper in particular indicates the need to develop and apply the Evans unified field theory with the Schwarzschild and other known metrics, and with computational methods.

Paper seven develops a generally covariant or objective interpretation of the Heisenberg commutator equation for angular momentum and suggests why recent experiments show that the Heisenberg uncertainty principle is qualitatively incorrect (by many orders of magnitude in certain experimental configurations). These results are explained using angular momentum densities of general relativity defined by the torsion form of Cartan geometry.

Paper eight proves the tetrad postulate of Cartan geometry using seven independent methods. This is intended as a rigorous and self checking proof of the geometrical fundamentals of the Evans unified field theory. Paper nine continues the mathematical proof of Cartan geometry by deriving the Evans Lemma directly from the first Cartan structure equation of differential geometry and gives another proof of the tetrad postulate. The Evans Lemma is the subsidiary proposition of geometry that leads to the Evans wave equation of physics. Papers ten and eleven rigorously self check the proof of the Evans Lemma from the tetrad postulate and apply the Lemma to the generally covariant or objective Dirac equation.

Paper twelve applies the Evans unified field theory to the quark gluon theory of the standard model, and introduces considerations of gravitation into the quantum chromodynamics of special relativity. This paper is, as usual, intended as a sketch of what is possible in this area of physics.

Paper thirteen shows that the origin of intrinsic spin in physics is the basis set of elements in the tangent spacetime (Minkowski spacetime) to the base manifold (Evans spacetime) in Cartan's differential geometry. This finding is illustrated with electromagnetism, fermionic matter fields of the Dirac equation, strong fields (quarks) and the Majorana Weinberg spin equations. This paper shows with particular clarity that the Evans field theory is a true unified field theory in which the fundamental field is the tetrad. This is true for all radiated and matter fields.

Paper fourteen suggests that dark matter may be due to the effect of spacetime torsion on the well known Einstein Hilbert theory of gravitation of 1915. The gravitational sector of the Evans unified field theory is in general based on a Cartan geometry where torsion and curvature are both well defined. The neglect of torsion in the conventional general relativity of gravitation is adequate in earthbound experiments and in the solar system, but in general there is no reason to expect torsion to be absent. Thus there are well documented anomalies, data which indicate that the 1915 theory does not hold in general in cosmological contexts. Dark matter is a well known example.

Finally, paper fifteen develops a generally covariant quantum mechanics and defines the correctly objective conjugate variables needed in the Heisenberg equation. These are densities such as energy density and momentum density. The fundamental quantum of angular momentum density is shown to be the reduced Planck constant divided by the Evans rest volume of any particle or field.

The camera-ready form of this book we owe to the patient and meticulous labor of Linda Caravelli and Franklin Amador. The superb job they have done is herewith gratefully acknowledged.

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Contents

1 The Objective Laws Of Classical Electrodynamics: The Effect Of Gravitation On Electromagnetism **1**

1.1 Introduction 2
1.2 The Objective Laws of Classical Electrodynamics 3
1.3 Discussion 10

2 First And Second Order Aharonov Bohm Effect In The Evans Unified Field Theory **17**

2.1 Introduction 17
2.2 Argument Against The Standard Model 19
2.3 Explanation of the First Order AB Effect in Evans Theory . . . 20
2.4 Electromagnetic or Second Order Aharonov Bohm Effect 22
2.5 Discussion 23

3 The Spinning Of Spacetime As Seen In The Inverse Faraday Effect **27**

3.1 Introduction 27
3.2 Derivation of the Evans Spin Field from First Principles 28
3.3 The Inverse Faraday Effect in One Electron and Atomic and Molecular Material 31

4 On The Origin Of Magnetization And Polarization **35**

4.1 Introduction 35
4.2 Definition of Spacetime Magnetization and Polarization 37
4.3 Magnetization and Polarization in Field Matter Interaction . . . 39
4.4 Magnetization and Polarization in the Standard Model 40

5 Explanation Of The Eddington Experiment In The Evans Unified Field Theory **47**

5.1 Introduction 47
5.2 Classical Field Theory on the Eddington Effect 48
5.3 The Fundamental Vector Potential Magnitude $A^{(0)}$ 52

6 The Coulomb And Ampère-Maxwell Laws In The Schwarzschild Metric, A Classical Calculation Of The Eddington Effect From The Evans Unified Field Theory 57
6.1 Introduction . . . 58
6.2 The Coulomb and Ampère-Maxwell Laws From The Unified Field Theory . . . 58
6.3 Refraction Of Electromagnetic Radiation By Gravitation . . . 62

7 Generally Covariant Heisenberg Equation From The Evans Unified Field Theory 73
7.1 Introduction . . . 74
7.2 Angular Momentum Forms And Densities . . . 75
7.3 Discussion . . . 79

8 Metric Compatibility Condition And Tetrad Postulate 85
8.1 Introduction . . . 86
8.2 Seven Proofs Of The Tetrad Postulate . . . 87
8.3 Physical Meaning Of The Metric Compatibility Condition And The Tetrad Postulate . . . 92

9 Derivation Of The Evans Lemma And Wave Equation From The First Cartan Structure Relation 97
9.1 Introduction . . . 98
9.2 Proof Of The Theorem . . . 98
9.3 Application To The Class Of Aharonov Bohm Effects . . . 100

10 Proof Of The Evans Lemma From The Tetrad Postulate 105
10.1 Introduction . . . 105
10.2 Advantages Of Cartan Geometry . . . 106
10.3 First Proof Of The Evans Lemma From The Tetrad Postulate . . . 108
10.4 Second Proof Of The Evans Lemma From The Tetrad Postulate 110
10.5 Discussion . . . 112

11 Self Consistent Derivation Of The Evans Lemma And Application To The Generally Covariant Dirac Equation 117
11.1 Introduction . . . 117
11.2 Geometrical Self Consistency Of The Evans Lemma . . . 118
11.3 Structure Of The Dirac Equation . . . 124

12 Quark Gluon Model In The Evans Unified Field Theory 131
12.1 Introduction . . . 132
12.2 Perturbation Of Exact Flavor Symmetry By Momentum Exchange 134
12.3 Quark Color Symmetry . . . 137

13 The Origin of Intrinsic Spin and the Pauli Exclusion Principle in the Evans Unified Field Theory 141
13.1 Introduction 142
13.2 Electrodynamics 142
13.3 Fermionic Matter Field And The Dirac Equation 145
13.4 Strong Field Theory 146
13.5 Majorana And Weinberg Equations 147

14 On The Origin Of Dark Matter As Spacetime Torsion 153
14.1 Introduction 154
14.2 The Gravitational Sector Of The Evans Unified Field Theory . . 155
14.3 Approximations To The General Theory 155

15 Generally Covariant Quantum Mechanics 163
15.1 Introduction 164
15.2 A Simple Derivation Of The Heisenberg Equation Of Motion . . 165
15.3 The Generally Covariant Heisenberg Equation 166

16 General Covariance And Coordinate Transformation In Classical And Quantum Electrodynamics 173
16.1 Introduction 174
16.2 General Coordinate Transformation 174
16.3 Special Relativistic Limit 176

17 The Role Of Gravitational Torsion In General Relativity: The S Tensor 181
17.1 Introduction 182
17.2 Derivation Of The S Tensor 183
17.3 Field Equation For The S Tensor 185
17.4 Discussion 187

18 Explanation Of The Faraday Disc Generator In The Evans Unified Field Theory 193
18.1 Introduction 194
18.2 Rotational Dynamics and Cartan Geometry 196
18.3 Faraday Disk Generator 199
18.4 Induction By Solenoid 201

19 Experiments To Test The Evans Unified Field Theory And General Relativity In Classical Electrodynamics 207
19.1 Introduction 208
19.2 Faraday Rotating Disc Experiment 208
19.3 Suggested New Experiments 213
19.4 Discussion 216

Chapter 1

The Objective Laws Of Classical Electrodynamics: The Effect Of Gravitation On Electromagnetism

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Abstract

The four fundamental laws of classical electrodynamics are given in generallyco-variant form using the principles of differential geometry. In so doing it becomes possible to analyze in detail the effect of gravitation on electromagnetism. This developmentcompletes Einstein's generally covariant field theory of gravitation and shows that there is present in nature a source of electric power from the general four dimensional manifold. It is also shown that an electromagnetic field can influence gravitation, and there are majorimplications for power engineering and aerospace industries.

Key words: generally covariant classical electrodynamics; Evans field theory; electric powerfrom spacetime; the interaction of gravitation and electromagnetism.

1.1 Introduction

In order that physics be an objective subject it must be a theory of general relativity, in which ALL the equations of physics must be generally covariant. This is a well known and well accepted principle of natural philosophy first proposed by Einstein [1] who based his development on the philosophical ideas of Mach. Without this most fundamental principle there can be no objective knowledge (or science) of nature. However, the contemporary standard model does not conform correctly to this principle, because only gravitation is treated objectively. Classical electrodynamics in the standard model is a theory of special relativity, covariant only under the Lorentz transform [2]– [4] and unobjective under any other type of coordinate transformation. In other words electrodynamics in the standard model in general means different things to different observers. This is fundamentally unacceptable to natural philosophy and science, the objective observation of nature. In science, nature is objective to all observers, and if not we have no science (from the Latin word for knowledge). Furthermore the field theories of gravitation and electromagnetism in the standard model are conceptually different [4]. Gravitation is essentially a special case of Riemann geometry within Einsteins constant k, electromagnetism is a distinct, abstract, entity superimposed on the Minkowski (flat) spacetime. It is well known that the origins of contemporary classical electrodynamics go back to the eighteenth century, to an era when space and time were also considered as distinct philosophical entities, not yet unified into spacetime. The contemporary standard model is still based on this mixture of concepts and is the result of history rather than reason.

In order to unify electromagnetism and gravitation in a correctly objective manner, it has been shown recently [5]– [36] that physics must be developed in a four dimensional manifold defined by the well known principles of differential geometry, notably the two Maurer Cartan structure equations, the two Bianchi identities, and the tetrad postulate [3]. The Einstein field theory of gravitation is essentially a special case of differential geometry, and electromagnetism is described by the first Bianchi identity within a fundamental voltage. Gravitation and electromagnetism are unified naturally by the structure of differential geometry itself. This means that one type of field can influence the other, leading to the possibility of new technology as well as being a major philosophical advance. In the last analysis gravitation and electromagnetism are different manifestations of the same thing, geometry. This is hardly a new idea in physics, but the Evans theory [5]– [36] is the first correct unified field theory to be based on well accepted Einsteinian principles.

In this paper the four laws of classical electrodynamics are developed in correctly covariant form from the first Bianchi identity of differential geometry. In the standard model these four laws together constitute the Maxwell Heaviside theory of the electromagnetic sector. The standard model is fundamentally or qualitatively unable to analyze the important effects of gravitation on electrodynamics because the two fields are treated differently as described already. String theory makes matters worse by the introduction of adjustable mathematical parameters known optimistically as dimensions. These have no physical significance and this basic and irremediable flaw in string theory originated a few years after the Einstein theory of 1916 in the fundamentally incorrect introduction of an unphysical fifth dimension in an attempt to unify gravitation with electromagnetism. The Evans field theory [5]– [35] achieves this aim by correctly using only the four physical dimensions of relativity, the four dimensions of spacetime. String theory should therefore be abandoned in favor of the simpler and much more powerful Evans field theory, which is the direct and logical outcome of Einsteins own work.

In Section 1.2 the correctly objective laws of classical electrodynamics are developed straightforwardly from the first Bianchi identity. The objective form of the Gauss law applied to magnetism and of the Faraday law of induction is obtained from the Bianchi identity itself, and the objective form of the Coulomb law and Ampere Maxwell law is obtained from the appropriate Hodge duals used in the Bianchi identity. Therefore all four laws become a direct consequence of the first Bianchi identity of differential geometry. Within a scalar $A^{(0)}$ with the units of volt s/m the electromagnetic field is the torsion form and the electromagnetic potential is the tetrad form. In Section 1.3 a discussion is given of some of the major consequences of these objective laws of classical electrodynamics.

1.2 The Objective Laws of Classical Electrodynamics

The first Bianchi identity of differential geometry is well known to be [3]:

$$D \wedge T^a = R^a{}_b \wedge q^b \tag{1.1}$$

Here $D\wedge$ denotes the covariant exterior derivative, $d\wedge$ is the exterior derivative, T^a is the torsion form and $R^a{}_b$ is the curvature form, also known as the Riemann form. The covariant exterior derivative is defined [3] as:

$$D \wedge T^a = d \wedge T^a + \omega^a{}_b \wedge T^b, \tag{1.2}$$

where $\omega^a{}_b$ is the spin connection of differential geometry. As is customary in differential geometry [3] the indices of the base manifold are suppressed (not written out), because they are always the same on both sides of any equation of differential geometry. Therefore only the indices of the tangent bundle are given in Eq.(1.1). The first Bianchi identity is therefore:

$$d \wedge T^a = -\left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b\right) \tag{1.3}$$

which implies the existence of the base manifold indices as follows:

$$d \wedge T^a{}_{\mu\nu} = -\left(q^b \wedge R^a{}_{b\mu\nu} + \omega^a{}_b \wedge T^b{}_{\mu\nu}\right) \tag{1.4}$$

The basic axiom of differential geometry is that in a given base manifold there is a tangent bundle to that base manifold at a given point [3]. The tangent bundle was not used by Einstein in his field theory of gravitation, Einstein considered and needed only the restricted base manifold geometry defined by the Christoffel symbol and metric compatibility condition [3]. These considerations of Einstein were sufficient to describe gravitation, but not to unify gravitation with electromagnetism. No one knew this better than Einstein himself, who spent thirty years (1925 - 1955) in attempting objective field unification in various ways.

The Bianchi identity (1.3) becomes the equations of electrodynamics using the following fundamental rules or laws:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu \tag{1.5}$$

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu} \tag{1.6}$$

defining the electromagnetic potential $A^a{}_\mu$, and the electromagnetic field $F^a{}_{\mu\nu}$. These appellations are used only out of habit, because both $A^a{}_\mu$ and $F^a{}_{\mu\nu}$ have now become parts of the unified field, i.e. of electromagnetism influenced by gravitation (or vice versa). The homogeneous field equation of the Evans field theory (HE equation) is therefore:

$$d \wedge F^a = -A^{(0)} \left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b \right) \tag{1.7}$$

and the homogeneous current of the HE is:

$$j^a = -\frac{A^{(0)}}{\mu_0} \left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b \right) \tag{1.8}$$

When this current vanishes the HE becomes:

$$d \wedge F^a = 0 \tag{1.9}$$

and is for each index a the homogeneous field equation of the Maxwell Heaviside theory:

$$d \wedge F = 0. \tag{1.10}$$

Equation (1.10) is a combination in differential form notation [3] of the Gauss law applied to magnetism:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = \mathbf{0} \tag{1.11}$$

and of the Faraday law of induction:

$$\boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial \mathbf{t}} = \mathbf{0}. \tag{1.12}$$

These two laws are well tested experimentally so the homogeneous current must be very small or zero within contemporary instrumental precision. These experimental considerations define the free space condition:

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b. \tag{1.13}$$

In general relativity however the homogeneous current may be different from zero, and so general relativity means that the Gauss law and Faraday induction law are special cases of a more general theory. This is the objective theory given by the HE. Similarly it will be shown that the Coulomb and Ampère Maxwell laws are special cases of the inhomogeneous equation (IE) of the Evans field theory. The IE is deduced from the HE using the appropriate Hodge duals, those of F^a and $R^a{}_b$. Therefore objective classical electrodynamics is deduced entirely from the Bianchi identity (1.1) using the rules (1.5) and (1.6). Within contemporary instrumental precision the HE can therefore be written in differential form notation as Eq. (1.9).

In tensor notation Eq. (1.7) becomes:

$$\partial_\mu F^a{}_{\nu\rho} + \partial_\rho F^a{}_{\mu\nu} + \partial_\nu F^a{}_{\rho\mu} = \mu_0 \left(j^a{}_{\mu\nu\rho} + j^a{}_{\rho\mu\nu} + j^a{}_{\nu\rho\mu} \right) \tag{1.14}$$

and this is the same equation as:

$$\partial_\mu \widetilde{F}^{a\mu\nu} = \mu_0 \widetilde{j}^{a\nu} \tag{1.15}$$

where:

$$\widetilde{F}^{a\mu\nu} = \frac{1}{2} |g|^{1/2} \epsilon^{\mu\nu\rho\sigma} F^a{}_{\rho\sigma} \tag{1.16}$$

is the Hodge dual tensor defined [3] by:

$$|g| = |g_{\mu\nu}| \tag{1.17}$$

In Eq.(1.17) $|g|^{1/2}$ is the positive square root of the metric determinant [3]. However the Hodge dual of the current on the right hand side is defined by:

$$\widetilde{j}^{a\sigma} := 3 \left(\frac{1}{6} |g|^{1/2} \epsilon^{\mu\nu\rho\sigma} j^a{}_{\mu\nu\rho} \right) \tag{1.18}$$

and so $|g|^{1/2}$ cancels out on either side of Eq.(1.15). The simplest form of the homogeneous field equation is therefore:

$$\partial_\mu \widetilde{F}^{a\mu\nu} \sim 0 \tag{1.19}$$

and in vector notation this becomes:

$$\boxed{\boldsymbol{\nabla} \cdot \mathbf{B}^a \sim 0} \tag{1.20}$$

and

$$\boxed{\frac{\partial \mathbf{B}^a}{\partial t} + \boldsymbol{\nabla} \times \mathbf{E}^a \sim \mathbf{0}}. \tag{1.21}$$

Experimentally it is found that the homogeneous current $\widetilde{j}^a$ is very tiny. Geometrically this implies that:

$$R^a{}_b \wedge q^b = -\frac{1}{2} \kappa \omega^a{}_b \wedge T^b. \tag{1.22}$$

Using the Maurer Cartan structure relations

$$T^a = D \wedge q^a \tag{1.23}$$

$$R^a{}_b = D \wedge \omega^a{}_b \tag{1.24}$$

eq.(1.22) becomes:

$$(D \wedge \omega^a{}_b) \wedge q^b = \omega^a{}_b \wedge \left(D \wedge q^b\right). \tag{1.25}$$

A possible solution of this equation is:

$$\omega^a{}_b = -\frac{1}{2}\kappa \epsilon^a{}_{bc} q^c \tag{1.26}$$

in which the Levi-Civita symbol is defined by:

$$\epsilon^a{}_{bc} = g^{ad} \epsilon_{dbc}. \tag{1.27}$$

Here g^{ad} is the metric in the orthonormal tangent bundle spacetime. This defines the geometry of the Gauss Law and of the Faraday Law of induction in the Evans unified field theory.

In order to deduce the inhomogeneous field equations of the unified field theory it is first noted that the inhomogeneous field equations of the Maxwell-Heaviside field theory are encapsulated in the well known:

$$d \wedge F = 0 \tag{1.28}$$

$$d \wedge \widetilde{F} = \mu_0 J \tag{1.29}$$

where $\widetilde{F}$ is the Hodge dual of F and where J is the charge current density three-form. The appropriate Hodge dual of Eq.(1.7) is:

$$d \wedge \widetilde{F}^a = -A^{(0)} \left(q^b \wedge \widetilde{R}^a{}_b + \omega^a{}_b \wedge \widetilde{T}^b\right) \tag{1.30}$$

and this is the inhomogeneous field equation of the Evans unified field theory (denoted IE). When there is no field matter interaction the appropriate inhomogeneous field equation is:

$$d \wedge \widetilde{F}^a = 0. \tag{1.31}$$

Therefore for electromagnetic radiation in free space:

$$\left(q^b \wedge \widetilde{R}^a{}_b + \omega^a{}_b \wedge \widetilde{T}^b\right)_{e/m} = 0. \tag{1.32}$$

For centrally directed gravitation (Einstein / Newton gravitation):

$$T^a = 0 \tag{1.33}$$

$$R^a{}_b \wedge q^b = 0 \tag{1.34}$$

$$\widetilde{R}^a{}_b \wedge q^b \neq 0. \tag{1.35}$$

If it is assumed that Eq.(1.32) continues to be true approximately in the presence of field matter interaction then the only term that contributes to the right hand side of the IE is that from the gravitational Eq.(1.35). (This approximation is analogous to the well known minimal prescription:

$$p_\mu \longrightarrow p_\mu + eA_\mu \tag{1.36}$$

in which it is seen that the electromagnetic property (A_μ) is unchanged by the field matter interaction.) Therefore the IE becomes:

$$d \wedge \widetilde{F}^a = -A^{(0)} \left(\widetilde{R}^a{}_b \wedge q^b \right)_{grav}. \tag{1.37}$$

This is the inhomogeneous field equation linking electromagnetism to gravitation. Any type of electromagnetic field matter interaction is described by Eq.(1.37) provided Eq.(1.32) remains true for the electromagnetic field when the latter interacts with matter.

In tensor notation Eq.(1.37) is

$$\partial_\mu \widetilde{F}^a{}_{\nu\rho} + \partial_\rho \widetilde{F}^a{}_{\mu\nu} + \partial_\nu \widetilde{F}^a{}_{\rho\mu} = -A^{(0)} \left(q^b{}_\mu \widetilde{R}^a{}_{b\nu\rho} + q^b{}_\rho \widetilde{R}^a{}_{b\mu\nu} + q^b{}_\nu \widetilde{R}^a{}_{b\rho\mu} \right) \tag{1.38}$$

which is the same equation as:

$$\partial_\mu F^{a\mu\nu} = -A^{(0)} R^a{}_\mu{}^{\mu\nu} \tag{1.39}$$

where we have used:

$$R^a{}_{\lambda\nu\mu} = q^b{}_\lambda R^a{}_{b\nu\mu}. \tag{1.40}$$

Eq.(1.39) is the simplest tensor formulation of the IE.

In vector notation Eq.(1.39) becomes the Coulomb law of the Evans unified field theory and the Ampère Maxwell law of the Evans unified field theory. The Coulomb law is derived using

$$\nu = 0, \mu = 1, 2, 3 \tag{1.41}$$

to give:

$$\partial_1 F^{a10} + \partial_2 F^{a20} + \partial_3 F^{a30} = -A^{(0)} R^a{}_i{}^{i0} \tag{1.42}$$

where summation over repeated indices i is implied. Now denote the fundamental voltage

$$\phi^{(0)} = cA^{(0)} \tag{1.43}$$

to obtain:

$$\boxed{\boldsymbol{\nabla} \cdot \mathbf{E}^a = -\phi^{(0)} R^a{}_i{}^{i0}} \tag{1.44}$$

with charge density:

$$\rho^a = -\epsilon_0 \phi^{(0)} R^a{}_i{}^{i0}. \tag{1.45}$$

Eq.(1.44) is the Coulomb law unified with the Newton inverse square law in the Evans unified field theory.

The units on both sides of Eq.(1.44) are volt m^{-2} and it is seen in Eq.(1.45) that charge density originates in ${R^a}_i{}^{i0}$, the sum of three Riemann curvature elements. These elements are calculated from the Einstein field theory of gravitation in our approximation (1.32). In the weak field limit it is well known that the Einstein field equation reduces to the Newton inverse square law and so Eq.(1.44) unifies the Newton and Coulomb laws in the weak field limit. Given the existence of $\phi^{(0)}$ it is seen from Eq.(1.44) and (1.45) that an electric field can be generated from gravitation.

Similarly the Ampère-Maxwell law in the Evans unified field theory is:

$$\boxed{\boldsymbol{\nabla} \times \mathbf{B}^a = \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} + \mu_0 \mathbf{J}^a} \tag{1.46}$$

where

$$\mathbf{J}^a = J_x^a \mathbf{i} + J_y^a \mathbf{j} + J_z^a \mathbf{k} \tag{1.47}$$

and where:

$$J_x^a = -\frac{A^{(0)}}{\mu_0}\left({R^a}_0{}^{10} + {R^a}_2{}^{12} + {R^a}_3{}^{13}\right) \tag{1.48}$$

$$J_y^a = -\frac{A^{(0)}}{\mu_0}\left({R^a}_0{}^{20} + {R^a}_1{}^{21} + {R^a}_3{}^{23}\right) \tag{1.49}$$

$$J_z^a = -\frac{A^{(0)}}{\mu_0}\left({R^a}_0{}^{30} + {R^a}_1{}^{31} + {R^a}_2{}^{32}\right) \tag{1.50}$$

From Eqs.(1.47) to (1.50) it is seen that current density also originates in sums over different Riemann tensor elements. This finding has the important consequence that electric current can be generated by spacetime curvature, the relevant Riemann tensor elements are again calculated from the Einstein theory of gravitation. The unified Coulomb/Newton law (1.44) can be further simplified to:

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = -\phi^{(0)} R^a \tag{1.51}$$

where

$$R^a = {R^a}_1{}^{10} + {R^a}_2{}^{20} + {R^a}_3{}^{30}. \tag{1.52}$$

Here the units of $\phi^{(0)}$ are volts and the units of R^a are inverse square meters. The index a denotes a state of polarization and originates in the index of the tangent bundle spacetime. For example if $\mathbf{E}^a$ is in the Z axis and if we use the complex circular basis $a = (1), (2), (3)$ then:

$$E_z = E^{(3)} \tag{1.53}$$

and we obtain:

$$\begin{aligned} \frac{\partial E_z}{\partial z} &= -\phi^{(0)} R_z \\ &= -\phi^{(0)} \left({R^{(3)}}_1{}^{10} + {R^{(3)}}_2{}^{20} + {R^{(3)}}_3{}^{30}\right) \end{aligned} \tag{1.54}$$

Using the antisymmetry properties of the Riemann tensor gives the simple equation:

$$\frac{\partial E_z}{\partial z} = -\phi^{(0)} R_z \tag{1.55}$$

where

$$R_z = R^{(3)}{}_1{}^{10} + R^{(3)}{}_2{}^{20}. \tag{1.56}$$

Eq.(1.55) is therefore the law that governs the interaction between two charges placed on two masses, and this law shows in the simplest way that an electric field is always generated by the scalar curvature R multiplied by the fundamental voltage $\phi^{(0)}$. This product defines the charge density as:

$$\rho = -\epsilon_0 \phi^{(0)} R_z. \tag{1.57}$$

The minus sign in Eq.(1.55) indicates that charge density is a compression of spacetime. The Einstein field equation indicates that:

$$R_z = -k_1 T_z \tag{1.58}$$

where k_1 is a constant proportional to the Newton constant G and where T_Z is an index contracted (i.e. scalar) canonical energy-momentum tensor. (The constant k_1 In Eq.(1.58) I snot the same numerically as the Einstein constant k because R_Z in Eq.(1.58) is not defined in the same way as the original R of the Einstein field theory.) Therefore Eq.(1.55) can be written as:

$$\frac{\partial E_z}{\partial z} = \phi_1^{(0)} G T_z \tag{1.59}$$

where $\phi_1^{(0)}$ is a fundamental voltage numerically different from $\phi^{(0)}$. Eq.(1.59) shows that the Coulomb law derives in the last analysis from spacetime energy momentum denoted T, and T can be transferred to an electric circuit. The curvature R is greatest near an electron and for a point electron R becomes infinite. There are however no infinities in nature so point electrons are idealizations of the traditional theory of electrodynamics. The Evans field theory removes this infinity and also removes the need for renormalization and Feynman calculus in quantum electrodynamics.

If we repeat our consideration of the $a = (3)$ index in the Ampère-Maxwell law it is seen that current density is generated in the simplest way by a time varying electric field:

$$J_z = -\epsilon_0 \frac{\partial E_z}{\partial z} = -\frac{A^{(0)}}{\mu_0} R_z \tag{1.60}$$

because for $a = (3)$:

$$\boldsymbol{\nabla} \times \mathbf{B}_z = \mathbf{0}. \tag{1.61}$$

It is seen that current density is generated by different curvature components from those that generate charge density in the Coulomb/Newton law (1.55) of the Evans unified field theory. More generally the transverse $a = (1)$ and (2) components are given by:

$$\nabla \times \mathbf{B}^{(1)} = \frac{1}{c^2}\frac{\partial \mathbf{E}^{(1)}}{\partial t} + \mu_0 \mathbf{J}^{(1)} \tag{1.62}$$

$$\nabla \times \mathbf{B}^{(2)} = \frac{1}{c^2}\frac{\partial \mathbf{E}^{(2)}}{\partial t} + \mu_0 \mathbf{J}^{(2)}. \tag{1.63}$$

In free space there is no current density due to mass and so Eqs.(1.62) and (1.63) reduce to:

$$\nabla \times \mathbf{B}^{(1)} = \frac{1}{c^2}\frac{\partial \mathbf{E}^{(1)}}{\partial t} \tag{1.64}$$

$$\nabla \times \mathbf{B}^{(2)} = \frac{1}{c^2}\frac{\partial \mathbf{E}^{(2)}}{\partial t} \tag{1.65}$$

1.3 Discussion

It has been shown that classical electromagnetism and classical gravitation can be unified with the well known methods of differential geometry, notably the Maurer-Cartan structure relations and the Bianchi identities. The end result produces in this paper the four classical laws of electrodynamics unified with the classical laws of gravitation as given in the Einstein field theory. Charge density in the Coulomb law and current density in the Ampère-Maxwell law have been shown to originate in sums over scalar components of the Riemann tensor of the Einstein gravitational theory. This result shows, for example, that the curl of a magnetic field and the time derivative of an electric field can produce gravitation through the current density of the Ampère-Maxwell law. The gradient of an electric field produces gravitation through the charge density through the Coulomb law. Therefore an electromagnetic device can counter gravitation, and this is of clear importance to the aerospace industry. Conversely the curvature of spacetime as embodied in the Riemann tensor of gravitation can produce electromagnetic current density through the Ampère-Maxwell law and charge density through the Coulomb law, and so can produce electric power. This is of clear interest to the electric power industry, because circuits can be designed in principle to produce electric power from gravitation in an original way. The vector formulation produced in this paper of the four laws of classical electrodynamics shows this clearly.

In deriving the simplest possible expressions of the unified field theory as given in this paper a type of minimal prescription has been used where it has been assumed that the electromagnetic field is unchanged in the interaction with matter. This produces a simple and clear result as described already. If this minimal prescription is not used the unified field equations become more complicated but still soluble numerically given the initial and boundary conditions.

In deriving the unified field equations in this paper no account has been taken of polarization and magnetization. The development of these properties in the unified field theory will be the subject of future work. Essentially both polarization and magnetization become spacetime properties, and the electromagnetic field interacts with matter through molecular property tensors which are also spacetime properties and which in the unified field theory, incorporate the effects of gravitation. This is the subject of generally covariant non-linear optics.

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Chapter 2

First And Second Order Aharonov Bohm Effect In The Evans Unified Field Theory

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Abstract

The first and second order Aharonov Bohm effects are explained straightforwardly in the Evans unified field theory using the spin connection generated by electromagnetism as spinning spacetime.

Key words: Evans unified field theory; first and second order Aharonov Bohm effects.

2.1 Introduction

The class of first order Aharonov Bohm (AB) effects [1] (those due to a static magnetic field) can be defined as AB effects in which the wavenumber (κ) of a matter beam such as an electron beam is shifted by the electromagnetic potential A acting at first order in the minimal prescription:

$$\kappa \longrightarrow \kappa + \frac{e}{\hbar} A. \tag{2.1}$$

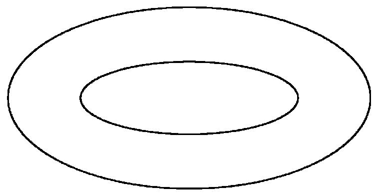

Figure 2.1: Area Overlapping

Here $-e$ is the charge on the electron and $\hbar$ the reduced Planck constant. Experiments on the AB effect can be summarized schematically with reference to Fig. 2.1, which defines one area within another as follows: In the well known Chambers experiment [2] for example the outer area is that enclosed by the interacting electron beams in a Young diffraction set up, and the inner area is that enclosed by an iron whisker within which is trapped a static magnetic flux density **B**. In the standard model, electromagnetism always is a theory of special relativity and:

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} \tag{2.2}$$

where **A** is the vector potential. The AB effect is observed in the Chambers experiment as a shift in the diffraction pattern of the electron beams, a shift that is proportional to:

$$\Phi = \int_s d \wedge A \quad (\texttt{outer}) \tag{2.3}$$

in which the surface integral is around the OUTER boundary defined by the paths of the two electron beams. This is despite the fact that **B** and therefore $\boldsymbol{\nabla} \times \mathbf{A}$ are confined to the INNER boundary [2]– [3] defined by the circumference of the iron whisker. The latter is placed between the openings of the Young interferometer. In the standard model, if B vanishes then so does $d \wedge A$. This is clearly stated in a standard textbook such as ref. [2]. Therefore in the standard model there cannot be regions in which $d \wedge A$ exists and in which B does not exist. Despite this simple inference it is often claimed confusingly that the first order AB effect is due to the effect of non-zero $d \wedge A$ where B is zero or that the AB effect is a pure quantum effect with no classical counterpart. Other attempts [2] at explaining the first order AB effect in the standard model rely on the classical concept of gauge transforming A. This confusion shows that the standard model does not explain the first order AB effect satisfactorily, or at all. This much is evidenced by over fifty years of theoretical controversy, all caused by the use of special relativity where general relativity is needed. The Evans field theory [3]– [6] is the first successful unified field theory that develops electromagnetism unified with gravitation as a correctly objective field theory of general relativity.

In Section 2.2 it is argued that the gauge transform theory of the standard model violates Stokes' Theorem in non-simply connected regions, and so is erroneous and unable to explain correctly the first order AB effect. In Section 2.3, the first order AB effect is explained correctly and straightforwardly using the spin connection of the Evans field theory. The latter is therefore preferred experimentally and mathematically to the standard model. Finally in Section 2.4 the second order or electromagnetic Aharonov Bohm effect is explained through the conjugate product of potentials in the Evans field theory, a conjugate product that defines the well known Evans spin field and which is observed in the inverse Faraday effect IFE [7]. The IFE is explained from the first principles of general relativity in the Evans unified field theory [3]– [6] but cannot be explained in the standard model without the empirical or ad hoc introduction of the conjugate product [8] in non-linear optics. Similarly for the second order AB effect which is implied by the well observed IFE.

2.2 Argument Against The Standard Model

Adopting the well known [9] notation of differential geometry the following three equations summarize the attempted description of the first order Aharonov Bohm effect in the standard model:

$$F = d \wedge A \tag{2.4}$$

$$d \wedge F = 0 \tag{2.5}$$

$$\kappa \longrightarrow \kappa + \tfrac{e}{\hbar} A. \tag{2.6}$$

Experimentally the observed Aharonov Bohm effect in an experiment such as that of Chambers is proportional to the magnetic flux (in weber) within the outer boundary of Fig 2.1 (the boundary defined by the paths of the electron beams):

$$\Phi = \int_S d \wedge A = \int_S F = \oint A \quad (\texttt{outer boundary}) \tag{2.7}$$

However, the magnetic flux density of the iron whisker is at the same time confined within the inner boundary

$$\Phi = \int_S d \wedge A = \int_S F = \oint A \quad (\texttt{inner boundary}) \tag{2.8}$$

and $d \wedge A$ is also confined within the inner boundary in the standard model. There is a contradiction between Eqs.(2.7) and (2.8) because the experimentally measured flux is given by Eq.(2.7) but the physical magnetic flux is given by Eq.(2.8). In a standard model textbook such as ref. [2], pp. 101 ff. an attempt is made to explain this contradiction in the first order Aharonov Bohm effect using the gauge transformation:

$$A \longrightarrow A + d\chi. \tag{2.9}$$

The standard model uses the Stokes Theorem to argue that:

$$\oint d\chi \neq 0 \quad ? \tag{2.10}$$

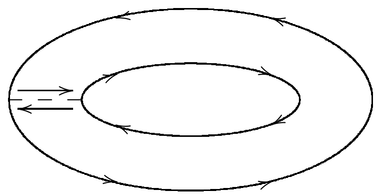

Figure 2.2: Integration Over Circumferences

in the region between the inner and outer boundary of Fig 2.1 and that the Aharonov Bohm effect is due to the integral over $d\chi$ in Eq.(2.10). However, the basis of electromagnetic gauge theory in the standard model is the Poincaré Lemma:

$$d \wedge (d\chi) := 0 \tag{2.11}$$

which is true for simply AND multiply connected spaces. The integrated form of the Poincaré Lemma is the Stokes Theorem:

$$\Phi = \int_S F = \int_S d \wedge (d\chi) = \oint d\chi := 0 \tag{2.12}$$

which is also true for multiply connected spaces [10]. The standard model [2] attempts to explain the first order AB effect by asserting INCORRECTLY that:

$$\Phi = \int_S F = \int_S d \wedge (d\chi) = \oint d\chi \neq 0. \tag{2.13}$$

In order to apply the Stokes Theorem to Fig 2.1 for example, a cut [10] is made to join the outer and inner boundaries as follows: and contour integration proceeds in one direction around the inner boundary, across the cut, in the opposite direction around the outer boundary, and back across the cut. Examples of such procedures are to be found in a standard textbook on vector algebra [10], in problems on the application of the Stokes Theorem.

We must look to general relativity and the Evans unified field theory for first correct explanation of the first order Aharonov Bohm effect.

2.3 Explanation of the First Order AB Effect in Evans Theory

In the correctly objective description of the first order AB effect [11] the electromagnetic field is defined by the first Maurer Cartan structure equation:

$$F^a = D \wedge A^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{2.14}$$

where $D\wedge$ is the covariant exterior derivative, $d\wedge$ is the exterior derivative, $\omega^a{}_b$ is the spin connection in the well known Palatini formulation of general relativity [12,13] in which the tetrad $q^a{}_\mu$ is the fundamental field. (In the original Einstein Hilbert formulation of general relativity the metric is the fundamental field.) The electromagnetic potential field is the fundamental tetrad field within a primordial or universal scalar $A^{(0)}$, where $cA^{(0)}$ has the units of volts, and where c is the speed of light in vacuo:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu. \tag{2.15}$$

It is seen that this gives a natural field unification scheme, because the metric used by Einstein and Hilbert is well known [9, 14] to be the dot product of two tetrads:

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{ab} \tag{2.16}$$

where η_{ab} is the Minkowski metric of the tangent bundle whose index is a. The latter becomes essentially a polarization index [3]– [6] in the Evans field theory. For example:

$$a = (1),\ (2),\ (3) \tag{2.17}$$

describes circular polarization where $((1), (2), (3))$ is the well known [15] complex circular basis. Again, it is well known [16] that the tetrad is developed into the spin 3/2 gravitino in supersymmetry theory, and that the Einstein Hilbert and Palatini variations of general relativity are inter-related by the tetrad postulate [9, 16]:

$$D_\nu q^a{}_\mu = 0. \tag{2.18}$$

One of the major inferences of the Evans field theory is that the tetrad field is the fundamental entity of objective (i.e. generally covariant) unified field theory, a unified field theory which satisfies the fundamental requirements of objectivity and general covariance in physics, the principles of general relativity. Electromagnetism in the standard model is a theory of special relativity, and is Lorentz covariant only. So the standard model is not a correctly objective theory of physics. This is the fundamental reason why it cannot describe the first order Aharonov Bohm effect, and gauge theory in special relativity [2] suffers from the same fundamental defect.

From Eq.(2.14) the magnetic flux in weber from the Evans field theory is defined as:

$$\Phi^a = \int_S F^a = \oint A^a + \int_S \omega^a{}_b \wedge A^b \tag{2.19}$$

and is in general the sum of two terms, one involving the spin connection $\omega^a{}_b$ of general relativity. It is $\omega^a{}_b$ that gives rise to the first (and second) order Aharonov Bohm effects. The fundamental reason is that the second term on the right hand side of Eq.(2.19) exists in the outer region of Fig 2.1 even though the magnetic flux density F^a is confined to the inner region and so is zero in the region between the inner and outer boundaries. The second term on the right hand side of Eq.(2.19) does not vanish, and gives rise to the AB effects. In the Chambers experiment, for example, the observed shift in the electron diffraction pattern is:

$$\delta = x \int_S \omega^a{}_b \wedge A^b \tag{2.20}$$

where x is a proportionality constant. The integration in Eq.(2.20) is around the outer boundary as required experimentally, the boundary defined by the diffracting electron beams in the Young interferometer of the Chambers experiment. The latter therefore observes the spin connection of the Evans theory directly. The spin connection is not present in the standard model, which has no explanation (Section 2.2) for the AB effects. The spin connection is a direct consequence of the major discovery of the Evans field theory that electromagnetism is the spinning of spacetime [3, 6, 17] - the spinning spacetime gives rise directly to the spin connection in the Palatini variation of general relativity.

The homogeneous field equation of the Evans field theory is [3]– [6]:

$$d \wedge F^a = 0 \tag{2.21}$$

implying that:

$$\omega^a{}_b = -\frac{1}{2}\kappa\epsilon^a{}_{bc}q^c. \tag{2.22}$$

In the complex circular basis:

$$\Phi^{(3)*} = \int_S F^{(3)*} = \oint A^{(3)*} - i\frac{e}{\hbar}\int_S A^{(1)} \wedge A^{(2)}. \tag{2.23}$$

From Eq.(2.23) it is seen that $F^{(3)*}$ and $A^{(3)*}$ are confined to the iron whisker (being in the Z axis of the iron whisker perpendicular to the plane of the paper), but $A^{(1)}$ and $A^{(2)}$ exist outside the iron whisker (i.e. in the plane of the paper) and interact with the electron beams. The observed fringe shift is proportional to $\Phi^{(3)*}$. The electron wavenumber is shifted by:

$$\kappa \longrightarrow \kappa + \frac{e}{\hbar}A^{(0)} \tag{2.24}$$

where

$$A^{(0)} = -i\left(A^{(1)} \wedge A^{(2)}\right)^{1/2}. \tag{2.25}$$

Here

$$eA^{(0)} = \hbar\kappa. \tag{2.26}$$

2.4 Electromagnetic or Second Order Aharonov Bohm Effect

The existence of the reproducible and repeatable inverse Faraday effect [3]– [6] implies that there is an electromagnetic or second order Aharonov Bohm effect. This is not a shift in the electron wave function but is due to magnetization by the Evans spin field $B^{(3)}$ [3]– [6]:

$$B^{(3)*} = -igA^{(1)} \wedge A^{(2)}. \tag{2.27}$$

In generally covariant unified field theory [3]– [6] the $B^{(3)}$ field is a fundamental manifestation of the fact that electromagnetism is spinning spacetime.

The latter gives rise to the spin connection in Eq.(2.14), and the second term in this equation gives the $B^{(3)}$ spin field using Eqs.(2.22) and (2.27). The magnetization of the inverse Faraday effect is to second order in the potential, so gives rise to a second order electromagnetic Aharonov Bohm effect (EAB) [3]– [6]. In a Chambers type experiment the EAB would be due to a circularly polarized electromagnetic beam directed between the interfering electron beams but isolated from the electron beams. The resulting fringe shift would be proportional [3]– [6] to the magnetic flux:

$$\Phi^{(3)*} = \mu_0 \int_S M^{(3)*} \quad (\texttt{outer}) \tag{2.28}$$

where integration again occurs around the outer boundary in Fig 2.1. The EAB has important consequences for RADAR and stealth technology because objects can be detected outside the width of the RADAR beam using the EAB.

2.5 Discussion

The explanation of the EAB is simply that the second term on the right hand side of Eq.(2.14) exists when it is arranged experimentally that the first term, the exterior derivative of the potential, is zero. This explanation means that an electromagnetic beam of given diameter will interact with an electron placed outside the electron beam. If so, the diameter of the electromagnetic beam must be defined. In the standard model there is no answer to this question because the standard explanation of the AB effects violates the Poincaré Lemma. The latter is identically zerofor any function in both simply and non-simply connected spaces because:

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} := 0 \tag{2.29}$$

is an identity independent of the function or topology. A laser beam at visible frequencies has a definite diameter and color, it can be focused or expanded, reflected and so on. However, the term $\omega^a{}_b \wedge A^b$ is invisible, for the first time in physics it is seen that there is something more to an electromagnetic beam than $d \wedge A^a$. Similarly the Chambers experiment shows that there is something more to a magnetic field than the curl of a vector potential. Again, this is the $\omega^a{}_b \wedge A^b$ of general relativity, caused by the spinning of spacetime itself.

The beam diameter must therefore have been defined by the way that the beam was originally created, or radiated by the source charge-current density J^a in the inhomogeneous Evans field equation (IE):

$$d \wedge \widetilde{F}^a = \mu_0 J^a. \tag{2.30}$$

For example if the beam is radiated by a non-relativistic electron in a circular orbit, the diameter of the beam is the diameter of the orbit. A laser is more complicated than this but this illustration gives the principle. The circling electron causes a spacetime spinning. The radiated magnetic and electric components of the laser beam are confined within the laser beam diameter by Eq.(2.30) and its Hodge dual in free space, the homogeneous Evans field equation (HE):

$$d \wedge F^a = 0. \tag{2.31}$$

The invisible term $\omega^a{}_b \wedge A^b$ exists both inside and outside the laser beam and outside the laser beam interacts with the electron in the Aharonov Bohm effect.

So what we see as the visible laser beam is defined by the exterior DERIVATIVES of F^a, its Hodge dual $\widetilde{F}^a$, and the potential A^a. The exterior derivative summarizes the time and space variation of these entities within the diameter defined by the circling electron in the source term J^a of Eq.(2.30). Outside of this diameter there is no visible radiation. However, the spacetime spinning indicated by $\omega^a{}_b \wedge A^b$ exists outside the visible laser beam because spacetime itself exists outside the laser beam. The spatial and temporal variations of A^a are also confined within the beam diameter. Only $\omega^a{}_b \wedge A^b$ exists outside the beam, and this contains no spatial or temporal variations of A^a. In the Chambers experiment the latter interacts with an electron at first order, and the second order AB is indicated by the existence of the inverse Faraday effect (IFE). This is the first correct explanation of the Aharonov Bohm effects, the important new principle at work is that the diameter of a beam of electromagnetic radiation is always defined by spatial and temporal variations of the potential, electric and magnetic fields. Similarly a static magnetic field is defined by the curl of a magnetic potential inside the iron whisker or solenoid, but spacetime outside the iron whisker is spun by the magnetic field. A useful analogy is to think of the electromagnetic beam or iron whisker as a stirring rod and the spinning spacetime as the whirlpool set up by the rod at its center.

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Chapter 3

The Spinning Of Spacetime As Seen In The Inverse Faraday Effect

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Abstract

The inverse Faraday effect is the observation of spinning spacetime in general relativity. The spinning gives rise to a spin connection in the Palatini variation of general relativity and in the Evans unified field theory. For the free electromagnetic field the spin connection is the dual of the tetrad field in the tangent spacetime, and from this inference the Evans spin field $\mathbf{B}^{(3)}$ is deduced straightforwardly. The spin field is a magnetic flux density which is observed in the reproducible and repeatable inverse Faraday effect as a magnetization occurring in all materials, in the simplest instance one electron.

Key words: Inverse Faraday effect; Evans field theory, general relativity, Evans spin field, spin connection, tetrad field, Palatini variation.

3.1 Introduction

In the Evans unified field theory [1]– [5] the electromagnetic field is spinning spacetime in general relativity and the gravitational field is curving spacetime in general relativity. The two fields are unified and inter-related by standard differential geometry as first developed by Cartan and others. The fundamental field is the tetrad form of the standard Palatini variation [6]– [9] of general relativity. In this paper the concept of spinning spacetime is established experimentally by reference to the reproducible and repeatable inverse Faraday effect [10]– [12], the magnetization of matter by the circularly polarized component of electromagnetic radiation at any frequency. In the standard model the electromagnetic sector is Lorentz covariant only, and is not therefore objective to all observers, or generally covariant. In consequence, the effect of gravitation on electromagnetism and vice versa cannot be analyzed with the standard model. In the latter, electromagnetism is a spinning and propagating entity superimposed on the Minkowski spacetime of special relativity. The Minkowski spacetime itself does not spin, and in consequence there is no spin connection in the standard models description of electromagnetism. The result is that there is no Evans spin field $\mathbf{B}^{(3)}$ [1]– [5] in the standard model, and no generally covariant or objective explanation in the standard model for the inverse Faraday effect. Available explanations [13, 14] of the inverse Faraday effect in the standard model rely on the conjugate product of transverse potentials or transverse electric or magnetic fields. The conjugate product is introduced empirically [15] and in the standard model it is not realized that the conjugate product defines the Evans spin field. For this, general relativity is needed as explained straightforwardly and simply in this paper.

In Section 3.2 the spin field is defined for the free electromagnetic field using the first Maurer Cartan structure equation of standard differential geometry. In Section 3.3 the inverse Faraday effect is explained from first principles of Evans unified field theory for one electron, and suggestions made for further work on atomic and molecular materials, where the inverse Faraday effect is mediated by a hyperpolarizability which is also a property of differential geometry in the Evans field theory. In this way a self consistent unified field theory of non-linear optics in general can be built up from differential geometry.

This paper therefore emphasizes a key difference between Evans field theory and the obsolete standard model, the spin field exists in the former but not in the latter. The spin field is observed experimentally, so the Evans field theory is preferred because it explains the spin field in a generally covariant manner as required of any valid theory of physics. To be valid, a theory must be objective to all observers, the principle of general relativity.

3.2 Derivation of the Evans Spin Field from First Principles

The starting point is the first Maurer Cartan structure equation of standard differential geometry [1]– [5], which defines the torsion form (T^a) as the covariant exterior derivative ($D\wedge$) of the tetrad form (q^a). In the standard Palatini variation of general relativity the tetrad form becomes the fundamental field. (In the Einstein Hilbert variation of general relativity the symmetric metric becomes the fundamental field.) In the standard notation [16] of differential geometry the first Maurer Cartan structure equation is:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{3.1}$$

where $\omega^a{}_b$ is the spin connection one-form and where $d\wedge$ denotes the exterior derivative of Cartan. The indices in Eq.(3.1) are those of the tangent spacetime. The indices of the base manifold in differential geometry are always the same on both sides of any equation of differential geometry, so are customarily omitted [16]. Reinstating the base manifold indices we obtain:

$$T^a{}_{\mu\nu} = (d \wedge q^a)_{\mu\nu} + \omega^a{}_{\mu b} \wedge q^b{}_\nu. \tag{3.2}$$

Therefore differential geometry consists of equations of the tangent bundle valid for each and every index such as μ and ν of the base manifold. This inference provides us with one of the fundamental principles of Evans field theory: equations of physics such as the Dirac equation can be written in the tangent bundle and the effect of gravitation on these equations is measured by the way in which the tangent bundle and base manifold are related geometrically. The fundamental field is the tetrad because the latter is defined by:

$$V^a = q^a{}_\mu V^\mu \tag{3.3}$$

where V^a is any vector or spinor in the tangent bundle and where V^μ is the corresponding vector or spinor in the base manifold. The latter is defined by Evans spacetime [1]– [5] and the tangent bundle by the Minkowski spacetime. So the tetrad inter-relates the tangent bundle and base manifold and gives the information required to measure the effect of a curving base manifold: gravitation; or the effect of a spinning base manifold: electromagnetism.

In electromagnetism and electrodynamics the Evans Ansatz [1]– [5] defines the potential field ($A^a{}_\mu$) as the tetrad field within a scalar-valued factor $A^{(0)}$ with the units of volt s/m. Thus $cA^{(0)}$ has the units of volts, and $cA^{(0)}$ is a primordial quantity (analogous to Feynman′s well known [17] description of electromagnetic potential in special relativity as the universal influence). Thus:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu \tag{3.4}$$

from which the anti-symmetric electromagnetic field tensor is defined as:

$$\begin{aligned} F^a{}_{\mu\nu} &= (d \wedge A^a)_{\mu\nu} + \omega^a{}_{\mu b} \wedge A^b{}_\nu \\ &= \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + \omega^a{}_{\mu b} A^b{}_\nu - \omega^a{}_{\nu b} A^b{}_\mu. \end{aligned} \tag{3.5}$$

In the standard model the Lorentz covariant equivalent of Eq.(3.5) is:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3.6}$$

In the correctly objective or generally covariant description of electrodynamics (Eq.(3.5)) there appears an extra magnetic flux density for the free field:

$$B^a{}_{\mu\nu} = \omega^a{}_{\mu b} A^b{}_\nu - \omega^a{}_{\nu b} A^b{}_\mu. \tag{3.7}$$

When the magnetic flux density (3.7) interacts with matter it produces the magnetization of the inverse Faraday effect [1]– [5]. Evidently, the standard model's Eq.(3.6) does not produce an inverse Faraday effect. The latter is due to the Evans spin field (3.7) and to the spin connection $\omega^a{}_{\mu b}$ set up by the spinning spacetime that we know as electromagnetism.

By free electromagnetic field we mean the propagating field in the absence of mass (material matter). The free field is defined by the homogeneous Evans field equation, (HE), which is simply Eq.(3.5) developed into the Bianchi identity [1]–[5]:

$$\begin{aligned} d \wedge F^a &= \mu_0 j^a \\ &= A^{(0)} \left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \right). \end{aligned} \tag{3.8}$$

The HE is a combination [18] of the Gauss Law applied to magnetism and the Faraday law of induction. Both laws are well known to hold to high precision in the laboratory, from which it is deduced that the homogeneous current j^a is zero within contemporary experimental precision in the laboratory. (In cosmological contexts in contrast j^a may be measurable experimentally.) Therefore we may write:

$$j^a \sim 0. \tag{3.9}$$

Eq.(3.9) in geometrical terms is:

$$\left(D \wedge \omega^a{}_b \right) \wedge q^b = \omega^a{}_b \wedge \left(D \wedge q^b \right) \tag{3.10}$$

and a solution of Eq.(3.10) is [1]– [5]:

$$\omega^a{}_{\mu b} = -\frac{\kappa}{2} \epsilon^a{}_{bc} q^c{}_\mu \tag{3.11}$$

where κ is the free space wavenumber of the electromagnetic radiation. It follows [18] from Eq.(3.11) that for the free field the Evans spin field is:

$$\mathbf{B}^{(3)*} = -i \frac{\kappa}{A^{(0)}} \mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{3.12}$$

in vector notation and in the complex circular basis [1]– [5]. In Eq.(3.12) the vector cross product on the right hand side is the well known conjugate product of non-linear optics [19]. The conjugate product has therefore been derived from the first principles of general relativity and defines the Evans spin field $\mathbf{B}^{(3)}$. In the standard model the conjugate product is introduced empirically and cannot be related to the Evans spin field because in special relativity the spin field does not exist (Eq.(3.6)). It does not exist because the Minkowski spacetime of special relativity does not spin.

If we accept general relativity we must accept the Evans spin field and objectivity in physics. If we reject the Evans spin field we must reject general relativity and objectivity in physics.

3.3 The Inverse Faraday Effect in One Electron and Atomic and Molecular Material

The interaction of the spin field with one electron produces the observable magnetization of the inverse Faraday effect as follows:

$$\mathbf{M}^{(3)*} = -\frac{i}{\mu_0}\frac{\kappa'}{A^{(0)}}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = \frac{1}{\mu_0}\frac{\kappa'}{\kappa}\mathbf{B}^{(3)*} \tag{3.13}$$

where [15]:

$$\frac{\kappa'}{\kappa} = \frac{N}{V}\left(\frac{\mu_0 e^2 c^2}{2m\omega^2}\right). \tag{3.14}$$

Here N is the number of electrons in a sample of volume $V, -e$ is the charge on the electron, m is the mass of the electron, and ω is the angular frequency $2\pi f$ of the electromagnetic radiation where f is its frequency in hertz. (Here ω should not be confused with the spin connection $\omega^a{}_{\mu b}$.) In order to calculate Eq.(3.14) in the Evans field theory a minimal prescription method may be used as follows:

$$p^a{}_\mu = p^a{}_\mu + eA^a{}_\mu \tag{3.15}$$

and the angular momentum imposed to the electron by the circularly polarized electromagnetic field calculated in the non-relativistic limit [15]. If we wish to include relativistic effects the Hamilton-Jacobi method may be used [15].

The key point is that the observable magnetization of the one electron inverse Faraday effect directly observes the Evans spin field from Eq.(3.13) within the factor κ'/κ.

It is observed from Eq.(3.11) that the inverse Faraday effect in samples of many electrons, such as atomic and molecular samples, arises from the particular form taken by the three index spin connection in the atom or molecule. Only in free space is the spin connection dual to the tetrad through Eq.(3.11). In the interaction of a circularly polarized field with one electron Eq.(3.11) becomes:

$$\omega^a{}_{\mu b} = -\frac{1}{2}\kappa' \epsilon^a{}_{bc} q^c{}_\mu \tag{3.16}$$

but in more complicated samples the simple Eq.(3.16) no longer applies, and the inverse Faraday effect is defined by hyperpolarizabilities constructed from the spin connection. In other words hyperpolarizabilities are properties of differential geometry. This deduction is generalized finally to the basic principle of Evans unified field theory: physics is differential geometry.

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Chapter 4

On The Origin Of Magnetization And Polarization

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Abstract

The origin of magnetization and polarization in the Evans field theory is traced to the spin connection term in the first Maurer Cartan structure equation of differential geometry. The spin connection term originates in the spinning or swirling of spacetime, which sets up a spacetime magnetization around for example a visible laser beam. Spacetime magnetization is not visible but is detectable experimentally in the electromagnetic (and other) Aharonov Bohm effects and in non-local phenomena in general. Magnetization and polarization in material matter is then a change in the spacetime magnetization, i.e a change in the spin connection of the Evans unified field theory.

Key words: Evans unified field theory, magnetization, polarization, spacetime magnetization, spin connection, field matter interaction, magnetization and polarization of matter.

4.1 Introduction

Recently the Evans field theory has provided a workable framework upon which to unify the theories of gravitation and electromagnetism [1]– [6]. In this paper the theory is used to trace the origin of magnetization and polarization in the spin connection term of the definition of the electromagnetic field tensor. The inference is made of a novel spacetime magnetization M^a, a magnetization caused by the swirling or spinning of spacetime around a propagating electromagnetic field. If the latter is a visible laser for example, M^a is an invisible region of magnetization around the visible electromagnetic field, the latter being always defined by the spatial or temporal derivatives of the electromagnetic potential A^a. In the Evans field theory A^a is the fundamental tetrad field [7]– [10] within a primordial or universal factor $A^{(0)} = \phi^{(0)}/c$ where $\phi^{(0)}$ has the units of volts and where c is the speed of light. The presence of M^a is detected experimentally by a novel electromagnetic or second order Aharonov Bohm effect (EAB) [7,8]. If the laser is replaced by a static magnetic field the spacetime magnetization around the magnetic field is detected with the original first order (magnetic) Aharonov Bohm effect first demonstrated by Chambers [9]. Similarly there is a spacetime polarization P^a around a static electric field (a polarization detectable in the Aharonov Bohm effect due to a static electric field), and a spacetime polarization also accompanies the EAB. Magnetization and polarization in matter is then a change in the spin connection defining the fundamental spacetime magnetization and polarization. This is a correctly objective or generally covariant description as required by the principle of general relativity and as such is a major advance on the standard model. In the latter, electromagnetism is a theory of special relativity and is Lorentz covariant only, and not generally covariant as required of any objective theory of physics. In consequence the effect of gravitation (curved spacetime) on electromagnetism (flat spacetime in the standard model) cannot be analyzed in the standard model, and the concept of spacetime magnetization and polarization is missing. The standard model attempts [10] to explain the class of AB effects with non-simply connected regions of space in special relativity, but it has been shown recently [11] that such an explanation must always violate the fundamental Poincaré Lemma for any type of space.

In Section 4.2 spacetime magnetization and polarization is defined from the first Maurer Cartan structure equation of differential geometry, which in the Evans field theory becomes [1]– [6] the fundamental definition of the antisymmetric field tensor. In section 4.3 it is argued that spacetime magnetization and polarization becomes the magnetization and polarization of matter through a change in the spin connection of the Evans field theory. This traces the origin of magnetization and polarization to differential geometry and general relativity. Finally in Section 4.4 a discussion is given of the description of magnetization and polarization in the standard model, whereupon it becomes clear that the Evans field theory has several theoretical and experimental advantages over the standard model.

4.2 Definition of Spacetime Magnetization and Polarization

In the correctly objective (i.e. generally covariant) unified field theory of Evans [1]– [6] the anti-symmetric electromagnetic field tensor is defined by the following vector valued two-form:

$$F^a = D \wedge A^a = d \wedge A^a + \omega^a{}_b \wedge A^b. \tag{4.1}$$

Here $D\wedge$ denotes the covariant exterior derivative, $d\wedge$ the exterior derivative, A^a the vector valued potential one-form and $\omega^a{}_b$ is the spin connection of the Palatini variation of general relativity [12]– [14]. The homogeneous Evans field equation (HE) is the correctly objective form [1]– [6] of the standard model's [10] homogeneous Maxwell-Heaviside field equation (HME). The HE is:

$$d \wedge F^a = -A^{(0)} \left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b \right). \tag{4.2}$$

Here $R^a{}_b$ is the tensor valued Riemann or curvature two-form [14], T^a is the vector valued torsion two-form, and q^a is the vector valued tetrad one-form. In the Palatini variation the tetrad is the fundamental field of general relativity [12]– [14]. In the original Einstein - Hilbert variation [14] the symmetric metric $g_{\mu\nu}$ is the fundamental field. The symmetric metric is factorized [14] into the dot product of two tetrads as follows:

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{ab} \tag{4.3}$$

where η_{ab} is the Minkowski metric of the tangent bundle spacetime at any point P in the base manifold (Evans spacetime).

The Evans Ansatz [1]– [6] is as follows:

$$A^a = A^{(0)} q^a. \tag{4.4}$$

From Eq.(4.4):

$$F^a = A^{(0)} T^a \tag{4.5}$$

Eqs. (4.4) and (4.5) show that electromagnetism in the Evans unified field theory is differential geometry within a factor $A^{(0)}$, a fundamental, C negative, universal and primordial influence, the vector potential magnitude. Within this factor $A^{(0)}$ the field tensor (4.5) is the first Maurer Cartan structure equation [14] of standard differential geometry:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{4.6}$$

and within the factor $A^{(0)}$, the HE is the first Bianchi identity of standard differential geometry:

$$D \wedge T^a = R^a{}_b \wedge q^b. \tag{4.7}$$

Eq.(4.7) can be rewritten as:

$$d \wedge T^a = -\left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b \right) \tag{4.8}$$

The homogeneous electromagnetic current of the unified field theory is defined as:

$$j^a = -\frac{A^{(0)}}{\mu_0}\left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b\right) \tag{4.9}$$

where μ_0 is the vacuum permeability in S.I. units. Under laboratory conditions:

$$j^a \sim 0 \tag{4.10}$$

because Eq.(4.2) is the correctly objective combined expression of the Faraday Law of induction and the Gauss Law of magnetism [1]– [6]. Both these laws appear to hold within contemporary experimental precision under laboratory conditions. In a cosmological context however, for example the deflection of light grazing an intensely gravitating object in an Eddington experiment, j^a is already known to be observable, because it is the electromagnetic current responsible for this deflection of light. In other words j^a is a measurable influence of intense gravitation on electromagnetism propagating in free space. It is well known that Einstein predicted the deflection of light by intense gravitation using a purely gravitational theory, the photon being assumed implicitly to be a mass that is attracted gravitationally by the mass of the sun. In the Evans unified field theory this SAME influence or mutual interaction appears classically through j^a. Therefore the Evans field theory correctly predicts the results of the Eddington experiment, (deflection of light by the sun), and in addition, the Evans theory shows that the Faraday Law of induction and Gauss Law of magnetism no longer hold in the presence of intense gravitation. In order to realize this a UNIFIED field theory is evidently needed, a purely gravitational theory such as that used originally by Einstein, is not enough to give us the homogeneous current j^a. The fact that the well known Gauss and Faraday laws appear always to hold in the laboratory is due to the fact that the gravitation of the Earth is too small to detect j^a. Therefore in the unified field theory there are experimentally verifiable effects such as the Eddington experiment which do not exist in the standard model. Another example is the well known relativistic pulsar radiation [15], and this will be the subject of future work to analyze the effect of intense gravitation on synchrotron radiation. A pulsar is essentially a synchrotron located on a rotating and very intensely gravitating neutron star.

A third example (out of many) is the subject of this paper and is defined in this section - spacetime magnetization and polarization.

In the unified field theory a visible frequency laser beam, for example, is defined on the classical level by:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{4.11}$$

$$d \wedge F^a = \mu_0 j^a \sim 0. \tag{4.12}$$

The first term in Eq.(4.11) describes the visible part of the beam, and the invisible second term in Eq.(4.11) describes spacetime swirling around the beam in analogy to a whirlpool set up by a stirring rod. The latter is the analogy for the beam, and the water of the whirlpool is the analogy for swirling or spinning spacetime ITSELF. This produces SPACETIME MAGNETIZATION:

$$M^a = \frac{1}{\mu_0} \omega^a{}_b \wedge A^b. \tag{4.13}$$

It is concluded that: 1) The visible light of a laser is defined by $d \wedge A^a$, i.e. by the spatial and temporal DERIVATIVES of the potential or tetrad field A^a. These derivatives define the electric and magnetic fields of the laser beam. 2) The spacetime magnetization M^a surrounding the laser beam is invisible but gives rise to an Aharonov Bohm effect at second order - the electromagnetic Aharonov Bohm effect (EAB) [7, 8].

The EAB occurs in regions where:

$$d \wedge A^a = 0 \tag{4.14}$$

but where:

$$M^a \neq 0. \tag{4.15}$$

Similarly in the well known Chambers experiment [9] the magnetic field inside the iron whisker is defined by Eq.(4.11) and the well known Chambers effect or magnetic Aharonov Bohm effect is due to the spacetime magnetization set up by the static magnetic field rather than by the laser (electromagnetic field). 3) It is seen that the origin of magnetization (and also polarization) is differential geometry, the existence of spinning or swirling spacetime.

Spacetime magnetization is defined by the Evans spin field [1]– [6]:

$$M^a = \frac{1}{\mu_0} B^a \tag{4.16}$$

and the B^a Field is observed directly in the inverse Faraday effect [16]. Both M^a and B^a are due to the spinning of spacetime in general relativity. In the standard model both M^a and B^a are undefined from first principles because electromagnetism in the standard model is a theory of special relativity in which the Minkowski spacetime is flat and static. Since M^a and B^a are both experimental observables, the Evans theory is preferred experimentally to the standard model. The Evans theory is also preferred philosophically and theoretically because it is correctly objective and generally covariant and because it is a unified field theory of all radiated and matter fields.

4.3 Magnetization and Polarization in Field Matter Interaction

When there is field matter interaction, magnetization and polarization are defined by the following changes in the spin connection and field tensor:

$$\omega^a{}_b \longrightarrow \Omega^a{}_b \tag{4.17}$$

$$F^a \longrightarrow G^a, \tag{4.18}$$

so Eq.(4.1) becomes:

$$G^a = d \wedge A^a + \Omega^a{}_b \wedge A^b \tag{4.19}$$

and $\Omega^a{}_b$ is seen to be the origin of magnetization and polarization of matter by an electromagnetic field. The interaction process is described by the inhomogeneous Evans field equation (IE) [1]– [6]:

$$d \wedge \widetilde{G}^a = \mu_0 J^a \tag{4.20}$$

in which the inhomogeneous current is defined by:

$$J^a = -\frac{A^{(0)}}{\mu_0}\left(q^b \wedge \widetilde{R}^a{}_b + \Omega^a{}_b \wedge \widetilde{T}^b\right). \tag{4.21}$$

Here $\widetilde{R}^a{}_b$ is the Hodge dual of $R^a{}_b$ and $\widetilde{T}^b$ is the Hodge dual of T^b. In the absence of magnetization and polarization of matter, the spin connection $\Omega^a{}_b$ reverts to $\omega^a{}_b$ and eqn.(4.20) becomes:

$$d \wedge \widetilde{F}^a = \mu_0 J_0^a \tag{4.22}$$

where the inhomogeneous current is now defined by:

$$J_0^a = -\frac{A^{(0)}}{\mu_0}\left(q^b \wedge \widetilde{R}^a{}_b + \omega^a{}_b \wedge \widetilde{T}^b\right). \tag{4.23}$$

Eqn.(4.21) therefore describes an idealized or mathematical type of field matter interaction which does not produce magnetization or polarization. In situations of interest to physics however, magnetization and polarization are always produced in matter by an electromagnetic field, even on the one electron level [17]. The HE (Eq.(4.2)) describes the propagation of electromagnetic radiation in free space.

It is seen that a concise and rigorously objective description of magnetization and polarization is given by differential geometry in the Evans field theory.

4.4 Magnetization and Polarization in the Standard Model

In the standard model classical electromagnetism is a theory of special relativity and the electromagnetic field is a mathematical or abstract entity superimposed on a flat and static Minkowski spacetime. Magnetization (**M** in Am^{-1}) and polarization (**P** in Cm^{-2}) are introduced phenomenologically (i.e. from the existence of the phenomenon and not by reasoning or deduction from a first principle) in the constitutive equations that define the magnetic field strength **H** (in Am^{-1}) and the electric displacement **D** (in Cm^{-2}):

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \tag{4.24}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}. \tag{4.25}$$

Here $\mathbf{E}$ is the free space electric field strength (in volt m^{-1}) and $\mathbf{B}$ is the free space magnetic flux density (in tesla). In Eqs.(4.24) and (4.25) the permittivity in vacuo (ε_0) and permeability in vacuo (μ_0) have the S.I. values:

$$\epsilon_0 = 8.854188 \times 10^{-12} J^{-1} C^{-2} m^{-1} \tag{4.26}$$

$$\mu_0 = 4\pi \times 10^{-7} J s^2 C^{-2} m^{-1}. \tag{4.27}$$

The equivalents of Eq.(4.2) in the standard model are:

$$\nabla \cdot \mathbf{B} = 0 \tag{4.28}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}, \tag{4.29}$$

and the equivalents of Eq.(4.20) in the standard model are:

$$\nabla \cdot \mathbf{D} = \rho \tag{4.30}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}. \tag{4.31}$$

Here ρ is charge density (in Cm^{-3}) and J is current density (in Am^{-2}). Eq(4.28) is the Gauss law applied to magnetism; Eq.(4.29) is the Faraday law of induction; Eq.(4.29) is the Coulomb law and Eq. (4.31) is the Ampère Maxwell law.

The standard model's description of electromagnetism is not generally covariant and in consequence cannot analyze the effect of gravitation on electromagnetism or vice versa. Evidently the notion of spacetime curvature and torsion is absent from the standard model entirely, whereas they are present in the Evans theory through $R^a{}_b$ and T^a. The spin connection $\omega^a{}_b$ is missing from the standard model because the latter describes electromagnetism with a static Minkowski spacetime. In consequence the Evans spin field B^a and the spacetime magnetization M^a are missing from the standard model. This means in turn that the standard model is not able to describe the inverse Faraday effect or the Aharonov Bohm effects from the first principle of objective physics, the principle of general relativity. The standard model's description of electromagnetism is not able, again, to describe the Eddington experiment and gravitational lensing: the deflection of light by gravitation. In the standard model a beam of light in vacuo (the "source free" region of the standard model) is described by Eqs.(4.28) and (4.29), the Gauss and Faraday laws, and if this beam of light grazes an intensely gravitating object such as the sun no effect is expected in the standard models Eqs.(4.28) and (4.29). Yet it is observed that the light is deflected by the mass of the sun during an eclipse (the original Eddington experiment). Einstein's well known explanation of this phenomenon uses or implies the concept of particulate photon mass but the very concept of the photon as quantum of electromagnetic energy is missing entirely from Eqs.(4.28) to (4.31).

In the Evans field theory the Eddington experiment can be described on a classical level from Eq.(4.2), the HE. This result can be seen qualitatively as follows. Einstein's essentially quantum and particulate explanation of the Eddington experiment is based on Riemann geometry with a symmetric or Christoffel or Levi-Civita connection. This geometry is summarized succinctly by:

$$q^b \wedge R^a{}_b = 0. \tag{4.32}$$

$$\omega^a{}_b \wedge T^b = 0. \tag{4.33}$$

Eq.(4.32) is the Bianchi identity used by Einstein [14], and Eq.(4.33) follows from the fact that in Einsteins 1916 theory of general relativity the torsion form is zero:

$$T^b = 0. \tag{4.34}$$

It follows from Eqs.(4.32) and (4.33) that:

$$d \wedge F^a = \mu_0 j^a = 0. \tag{4.35}$$

This result is due to the fact that in the Einstein theory gravitation is not unified with electromagnetism on the classical level and there can never be any mutual influence of one field on another if we use the geometry defined by Eqs. (4.32) to (4.34). Eq.(4.35) means that the beam of light is not deflected. Electromagnetism uninfluenced by gravitation is defined [1]– [6] in the Evans field theory by the free space condition:

$$q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b = 0 \tag{4.36}$$

and Eq.(4.36) again leads to Eq. (4.35). Again there is no deflection of the beam of light by the sun. This result is again due to the fact that gravitation and electromagnetism are not mutually influential.

In order for the beam of light to be deflected by the sun in an Eddington experiment, the homogeneous current j^a must be non-zero. Conversely the experimental observation of deflection of light by the sun means that the Evans field theory is verified on a classical level, and the standard model is invalidated on a classical level. The geometry needed for this deflection is, from Eq.(4.2), defined by:

$$j^a = -\frac{A^{(0)}}{\mu_0}\left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b\right) \neq 0. \tag{4.37}$$

The presence of the sun therefore has the effect of changing:

$$d \wedge F^a = 0 \tag{4.38}$$

to

$$d \wedge F^a = \mu_0 j^a \neq 0. \tag{4.39}$$

This means that the beam is refracted by mass, i.e. its path is deflected. Similarly a beam of light is refracted when it interacts with matter (e.g. water).

The refraction by mass is described by the homogeneous current j^a, the refraction by water is described by the inhomogeneous current J^a. The refraction is accompanied in general by a change in polarization from the circular polarization of Eq.(4.38) to a modified polarization in Eq.(4.39). To calculate these changes quantitatively needs a computer in general, but the results can be seen qualitatively as described without any further calculation.

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Chapter 5

Explanation Of The Eddington Experiment In The Evans Unified Field Theory

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Abstract

The Evans unified field theory offers a classical explanation of the refraction of electromagnetic radiation by gravitation (the Eddington effect or gravitational lensing). In so doing a number of other electromagnetic effects of gravitation are predicted by the theory.

Key words: Evans unified field theory, Eddington experiment, gravitational lensing, electromagnetic effects of gravitation.

5.1 Introduction

The Eddington experiment (1919 - 1922) [1] observed refraction of light by starlight in regions near the sun during an eclipse. The effect was a confirmation of the Einstein/Hilbert theory of general relativity [2] because the orbit of a photon around the sun is Einsteinian rather than Newtonian. Although based on the classical field theory of generally covariant gravitation, the Einstein/Hilbert explanation of the Eddington effect does not involve classical electrodynamics. In this paper a straightforward classical explanation of the Eddington effect (gravitational lensing) is offered using the Evans unified field theory [3]– [10]. In Section 5.2 the equations needed to explain the Eddington effect are written out in terms of standard differential geometry. These are the homogeneous Evans field equation (HE) and the inhomogeneous Evans field equation (IE). In general the electromagnetic effects of gravitation are found by solving these two equations simultaneously with initial and boundary conditions. The equations show that in general, gravitation will cause all the kinematic and electrodynamic effects familiar from the phenomena observed in the interaction of electromagnetic radiation with a dielectric [11]. These include refraction, the deflection of light (visible frequency electromagnetic radiation) by gravitation, the Eddington effect.

In Section 5.3 a short discussion is given of the origin of the fundamental vector potential magnitude $A^{(0)}$ of the Evans field theory.

5.2 Classical Field Theory on the Eddington Effect

The classical explanation of the Evans unified field theory is based in general on the solution of the HE and IE simultaneously, given initial and boundary conditions. The explanation is summarized as follows:

$$d \wedge F^a = 0 \quad \longrightarrow \quad d \wedge F^a = \mu_0 j^a \tag{5.1}$$

$$d \wedge \widetilde{F}^a = 0 \quad \longrightarrow \quad d \wedge \widetilde{F}^a = \mu_0 J^a . \tag{5.2}$$

Here F^a is the differential two-form [12] representing the electromagnetic field tensor and $\widetilde{F}^a$ is its Hodge dual. The symbol $d\wedge$ denotes the exterior derivative of differential geometry. The effect of gravitation on light grazing the sun is analyzed by the homogeneous current:

$$j^a = -A^{(0)} \left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b \right) \tag{5.3}$$

and the inhomogeneous current:

$$J^a = -A^{(0)} \left(q^b \wedge \widetilde{R}^a{}_b + \omega^a{}_b \wedge \widetilde{T}^b \right) \tag{5.4}$$

of the Evans field theory [3]– [10]. Here μ_0 is the S.I. permeability in vacuo, T^a is the torsion form of differential geometry [12], $R^a{}_b$ is the Riemann form of differential geometry, $\omega^a{}_b$ is the spin connection of differential geometry and q^a is the tetrad form of differential geometry. The scalar valued $A^{(0)}$ is a primordial vector potential magnitude with the units of volt s/m. Its origin and meaning is discussed further in Section 5.3.

In the absence of gravitation (in regions far from the sun) the currents j^a and J^a are vanishingly small, but for light grazing the sun the currents become finite and cause the Eddington effect. The origin of the currents is spacetime itself and this inference means that spacetime itself can act as a source for an electromagnetic field given the primordial vector potential magnitude $A^{(0)}$. The existence of the latter is also indicated by the Eddington effect on the classical level. The very fact that a light beam is refracted (i.e. deflected) by mass (the sun) proves the Evans unified field theory qualitatively on the classical level. Einsteins famous explanation of the Eddington effect is implicitly quantum in nature because the explanation is based on the gravitational attraction of the particulate photon by the sun. The light beam is made up of an ensemble of photons. This explanation does not use classical electrodynamics and kinematics because the explanation is based on a theory of gravitation only, and not on a unified field theory as required for a fuller understanding of the phenomenon. In the Maxwell-Heaviside (MH) theory of the contemporary standard model refraction by mass does not occur at all, because mass and gravitation do not occur in classical MH electrodynamics, in which the source of electromagnetism is accelerated charge. The charge is considered in MH theory as a point charge without mass and without volume. The latter is introduced only through the charge density. Similarly the current in MH theory is the motion of point charges, and volume is introduced only through current density. These nineteenth century concepts predate general relativity and are not compatible with general covariance or objectivity in physics. In consequence the MH equations are Lorentz covariant equations of special relativity but not generally covariant as required by general relativity. The standard model is therefor flawed fundamentally in several ways [3]– [10]. The inability of classical electrodynamics to explain the Eddington effect is a clear indication of these flaws.

In the Evans field theory j^a and J^a are properties of spacetime with both curvature and torsion, so gravitation can cause the refraction of light. There are also other effects predicted by the Evans unified field theory, effects such as absorption and dispersion due to gravitation, and in general any classical electrodynamical effect of a "dielectric". The "dielectric" in this case is spacetime ITSELF, specifically the Evans spacetime [3]– [10] defined by the presence of both curvature and torsion.

The simplest approximation to Eqs.(5.2) and (5.2) is:

$$j^a = 0 \tag{5.5}$$

$$J^a = -A^{(0)} q^b \wedge \widetilde{R}^a{}_b \tag{5.6}$$

i.e.:

$$d \wedge F^a = 0 \tag{5.7}$$

$$d \wedge \widetilde{F}^a = \mu_0 J^a = -A^{(0)} q^b \wedge \widetilde{R}^a{}_b \tag{5.8}$$

In this approximation

$$q^b \wedge R^a{}_b = 0 \tag{5.9}$$

$$\omega^a{}_b \wedge T^a = 0 \tag{5.10}$$

for gravitation and

$$q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^a = 0 \tag{5.11}$$

for electromagnetism. Eqs.(5.9) and (5.10) define the differential geometry appropriate to the Einstein field theory of gravitation [3]– [10], and Eq.(5.11) defines the differential geometry of electromagnetism in free space [3]– [10].

In this simplest approximation the Eddington effect is caused by Eqs.(5.7) and (5.8), which must be solved simultaneously with given initial and boundary conditions. Written out in vector notation [13] these equations are as follows:

$$\nabla \cdot \mathbf{B}^a = \mathbf{0} \tag{5.12}$$

$$\nabla \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mathbf{0} \tag{5.13}$$

$$\nabla \cdot \mathbf{E}^a = -cA^{(0)} \left(R^a{}_1{}^{10} + R^a{}_2{}^{20} + R^a{}_3{}^{30} \right) \tag{5.14}$$

$$\begin{aligned} \nabla \times \mathbf{B}^a = \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} - \frac{A^{(0)}}{\mu_0} \left(\left(R^a{}_0{}^{10} + R^a{}_2{}^{12} + R^a{}_3{}^{13} \right) \mathbf{i} \right. \\ \left. + \left(R^a{}_0{}^{20} + R^a{}_1{}^{21} + R^a{}_3{}^{23} \right) \mathbf{j} + \left(R^a{}_0{}^{30} + R^a{}_1{}^{31} + R^a{}_2{}^{32} \right) \mathbf{k} \right) \end{aligned} \tag{5.15}$$

A quantitative explanation of the Eddington effect in the Evans field theory therefore requires a knowledge of the scalar-valued Riemann components in Eqs.(5.14) to (5.15), and a knowledge of $A^{(0)}$. The Riemann scalar elements can be calculated from the Einstein field theory for a given metric, notably the Schwarzschild metric [12] for the sun. All the kinematic and electrodynamic effects normally associated with a dielectric are also expected from Eqs.(5.12) to (5.15) and this inference illustrates the predictive power of the Evans field theory. The standard model is unable to make these predictions.

It is helpful to summarize the above explanation in a barebones notation which suppresses all indices to leave the basic structure of the equations.

The Eddington effect therefore is the refraction of light by Evans spacetime near the sun. The spacetime is considered to be a dielectric defined by two differential equations:

$$d \wedge F = \mu_0 j \tag{5.16}$$

$$d \wedge \widetilde{F} = \mu_0 J. \tag{5.17}$$

In the simplest approximation we assume that the interaction of electromagnetism with gravitation does not change the free space fields **E** and **B**, respectively the electric field strength and the magnetic flux density of the electromagnetic field. This is a standard approximation used also in MH theory, in which the homogeneous equations are written in terms of **E** and **B** and the inhomogeneous equations in terms of **D** and **H**, respectively electric displacement and magnetic field strength. This approximation, when used in the Evans field theory, means that we assume:

$$q \wedge R + \omega \wedge T = 0 \tag{5.18}$$

in the HE and we assume:

$$q \wedge \widetilde{R} + \omega \wedge \widetilde{T} = 0 \tag{5.19}$$

in the IE. This approximation is equivalent to a standard minimal prescription [13] and simplifies Eq.(5.17) to:

$$d \wedge \widetilde{F} = \mu_0 J = -A^{(0)} q \wedge \widetilde{R} \tag{5.20}$$

where $q \wedge \widetilde{R}$ in Eq.(5.20) indicates the gravitational geometry of the Einstein field theory. The latter is defined geometrically [2]– [10] by:

$$q \wedge R = 0 \tag{5.21}$$

$$\omega \wedge T = 0 \tag{5.22}$$

but:

$$q \wedge \widetilde{R} \neq 0. \tag{5.23}$$

In general J of Eq.(5.20) is the sum of two terms:

$$J = J_c + J_p \tag{5.24}$$

where J_c is due to free charges and J_p is due to polarization and magnetization in the dielectric (i.e. Evans spacetime). This deduction can be seen from the structure of the MH inhomogeneous equations [14]:

$$\nabla \cdot \mathbf{D} = \rho \tag{5.25}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \tag{5.26}$$

where

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \tag{5.27}$$

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}. \tag{5.28}$$

Here ϵ_0 is the S.I. vacuum permittivity, μ_0 is the S.I. vacuum permeability, ρ is charge density, $\mathbf{J}$ is current density, $\mathbf{P}$ is polarization and $\mathbf{M}$ is magnetization.

Eq.(5.26) can be rewritten in terms of the free fields $\mathbf{E}$ and $\mathbf{B}$ as:

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \left(\mathbf{J} + \mathbf{J}_p \right) \tag{5.29}$$

where:

$$\mathbf{J}_p = \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M}. \tag{5.30}$$

In the absence of free charges (i.e. in a dielectric such as glass):

$$\mathbf{J} = \mathbf{0} \tag{5.31}$$

and

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}_p. \tag{5.32}$$

This equation may be generalized to:

$$d \wedge \widetilde{F} = \mu_0 J_p \tag{5.33}$$

Finally we assume that:

$$J_p = -\frac{A_p^{(0)}}{\mu_0} q \wedge \widetilde{R} \tag{5.34}$$

where $A_p^{(0)}$ is the equivalent of $A^{(0)}$ for an uncharged dielectric.

The Eddington effect is described by Eqs. (5.33) and (5.34) as the refraction of light in a dielectric, a well known problem solved in many standard texts [15].

The mass of the sun creates $q \wedge \widetilde{R}$ in the regions of Evans spacetime where starlight grazes the sun in an eclipse. Conceptually the Eddington effect becomes the familiar refraction seen for example in a prism or lens. As for any dielectric such as glass or water the refraction is accompanied by reflection, dispersion, and frequency shifts of the radiation. The frequency shifts in the Evans field theory are frequency shifts of light caused by gravitation. There are also polarization changes of electromagnetic radiation due to gravitation expected in general. The Evans spacetime is characterized by a refractive index, as for any dielectric such as glass or water. The permittivity ϵ and permeability μ of the dielectric (i.e. the Evans spacetime with curvature and torsion) are different from ϵ_0 and μ_0 defined in S.I. units by:

$$\epsilon_0 \mu_0 = \frac{1}{c^2}. \tag{5.35}$$

These differences are again due to gravitation. None of these effects occur in the MH theory, and none occur in the Einstein field theory of gravitation. They occur only in a unified field theory, and clearly illustrate the predictive power of the Evans field theory. The Eddington effect then follows from the standard textbook theory of refraction, i.e. from the fact that ϵ and μ are different from ϵ_0 and μ_0 in regions close to the sun in a solar eclipse. Thus the terminology gravitational lensing - the Evans spacetime around the sun is a giant lens through which starlight passes before reaching the observer.

5.3 The Fundamental Vector Potential Magnitude $A^{(0)}$

A classical expression can be derived for $A^{(0)}$ starting from the standard definition [15] of total electromagnetic field energy En within the volume of radiation V:

$$En = \frac{1}{\mu_0} \int B^2 dV \tag{5.36}$$

a definition which can be found in the standard texts [15] of classical electrodynamics. Now use dimensionality to find that:

$$B = \kappa A = \frac{\omega}{c} A \tag{5.37}$$

where κ has the dimensions of wavenumber (inverse meters). From Eqs.(5.36) and (5.37)

$$A^2 = \frac{\mu_0}{\kappa^2} \frac{\partial En}{\partial V} \tag{5.38}$$

and it is possible to define the root mean square:

$$A^{(0)} = \frac{c}{\omega} \mu_0^{1/2} \left\langle \left(\frac{\partial En}{\partial V} \right)^{1/2} \right\rangle . \tag{5.39}$$

Therefore $A^{(0)}$ is seen to originate in the root mean square of the derivative of En with respect to V, a pure classical definition.

The quantity En is defined in terms of the electromagnetic energy density U:

$$En = \int U dV \tag{5.40}$$

where

$$U = \frac{\kappa^2}{\mu_0} A^2 . \tag{5.41}$$

Therefore

$$A^{(0)} = \left\langle A^2 \right\rangle^{1/2} = \mu_0^{1/2} \frac{c}{\omega} U^{1/2} \tag{5.42}$$

and it is seen that $A^{(0)}$ can be defined as being proportional to the square root of the electromagnetic energy density U. The latter can be related to the power density I of the electromagnetic field:

$$I = cU \tag{5.43}$$

so:

$$A^{(0)} = \mu_0^{1/2} c^{1/2} \left(\frac{I^{1/2}}{\omega} \right) \tag{5.44}$$

where I is measured in watts per square meter. Therefore $A^{(0)}$ is proportional in free space to the square root of I and inversely proportional to the angular frequency ω of the beam in radians per second.

A quantum classical equivalence can be forged for an electromagnetic beam consisting of one photon occupying a volume V. In this case the total electromagnetic field energy is that of the photon, i.e. $\hbar\omega$, and so:

$$\hbar\omega = \frac{1}{c} \int I dV \tag{5.45}$$

and $A^{(0)}$ can be expressed in terms of the energy of one photon in a volume of radiation V. In classical electrodynamics **A** and **B** are entities superimposed on a Minkowski spacetime, but in the Evans field theory **A** and **B** are properties of spinning spacetime [3]– [10]. The electromagnetic field in the Evans theory is generally covariant and the electromagnetic field is spinning spacetime itself. An index contracted canonical energy - momentum density T can be defined for the Evans unified field, and is proportional to scalar curvature:

$$R = -kT. \tag{5.46}$$

Here k is the Einstein constant. It is deduced that $A^{(0)}$ originates classically in R [3]– [10]. It is seen from Eq.(5.42) that electromagnetic energy density U is proportional to $A^{(0)2}$. This equation indicates that there are two signs of $A^{(0)}$ for one sign of energy density. $A^{(0)}$ may be positive (positive charge) or negative (negative charge), and this is the origin of the fact that there are two signs of charge in nature. The fundamental reason is that energy density U is quadratic in $A^{(0)}$, and therefore R is also quadratic in $A^{(0)}$ in the Evans unified field theory.

The Eddington effect shows that gravitation can deflect light on the classical level. This phenomenon can be understood in terms of J^a, a charge current density three-form, so the phenomenon is clear and important proof that electric power can be generated by Evans spacetime acting as J^a, the source of the power. This has very important technological consequences which must be worked out by computer simulation, i.e. by solving the HE and IE simultaneously in a circuit designed to amplify the available power to practical levels.

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Chapter 6

The Coulomb And Ampère-Maxwell Laws In The Schwarzschild Metric, A Classical Calculation Of The Eddington Effect From The Evans Unified Field Theory

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Abstract

The Schwarzschild metric is used in the Evans unified field theory to calculate the slowing of the speed of light and angle of deflection of light by a gravitating object such as the sun. Thus, Einsteins well known gravitational explanation of this effect in terms of the photon mass is completed within the context of a unified field theory in which electromagnetism is considered to be at once particulate and undulatory.

Key words: Evans unified field theory; gravitational lensing; diffraction of light

by gravitation; Eddington experiment; Schwarzschild metric.

6.1 Introduction

The diffraction of electromagnetism by gravitation appears to be precisely explained for the sun and some stars by the Einstein/Hilbert gravitational theory of general relativity [1]. The theory is however incomplete, as Einstein pointed out, because it does not involve classical electromagnetism. In consequence there are anomalous gravitational shifts which cannot be explained by the Einstein Hilbert theory [2]. The latter explanation is based on the gravitational attraction of the photon by the sun, and was originally verified by Eddington et al. [3]. In this paper the diffraction of electromagnetic waves by mass is calculated both in classical electrodynamics and with the Einstein/Hilbert theory using the Evans unified field theory [4]– [10]. The structure of the latter theory is based solidly on universally accepted Cartan differential geometry [11], the homogeneous and inhomogeneous equations of classical electrodynamics being derived from the first Bianchi identity. The relation between the potential and the field is derived from the first Cartan structure equation, and all the wave equations of physics are derived from Cartans tetrad postulate, which links Cartan differential geometry to Riemann geometry and which is a universally accepted fact of geometry like the Pythagoras Theorem.

The Coulomb and Ampère-Maxwell laws from the Evans unified field theory are set up in Section 6.2, the charge current density being calculated using the Schwarzschild metric for the sake of illustration. Other metrics can be used if preferred. In Section 3 the slowing of the speed of light by the suns gravitational attraction is calculated straightforwardly with the Einstein/Hilbert field theory directly from the Schwarzschild metric and the result is compared with that of Newtonian dynamics. This is part of the Evans unified field theory, which reduces to the Einstein/Hilbert field theory when the torsion form of Cartan vanishes [4]– [10]. Another part of the Evans field theory is then used to calculate straightforwardly the refraction expected in classical electrodynamics by the slowing of the speed of light. This is a first qualitative calculation using the suns equatorial radius, a calculation which may refined and extended within the context of the unified field theory.

6.2 The Coulomb and Ampère-Maxwell Laws From The Unified Field Theory

The Coulomb and Ampère-Maxwell laws in the unified field theory are derived from the first structure equation of Cartan:

$$T^a = D \wedge q^a \tag{6.1}$$

and the first Bianchi identity:

$$D \wedge T^a = R^a{}_b \wedge q^b. \tag{6.2}$$

Here $D\wedge$ denotes the covariant exterior derivative:

$$D\wedge = d\wedge + \omega^a{}_b\wedge \tag{6.3}$$

where $d\wedge$ is the exterior derivative of Cartan. In Eq.(6.3) $\omega^a{}_b$ is the spin connection, T^a is the torsion form, and $R^a{}_b$ is the Riemann form. Eqs.(6.1) and (6.2) become the homogeneous field equations using the basic ansatz:

$$A^a = A^{(0)} q^a \tag{6.4}$$

where $A^{(0)}$ is a primordial quantity with the units of volt sm^{-1}, and A^a is the vector potential of the unified field theory, a vector valued one-form.

The antisymmetric field tensor of the unified field theory is a vector-valued two form which is derived from the potential using the Cartan structure equation:

$$\begin{aligned} d\wedge F^a = \mu_0 j^a = -A^{(0)}\left(q^b\wedge R^a{}_b + \omega^a{}_b\wedge T^b\right), \\ F^a = d\wedge A^a + \omega^a{}_b\wedge A^b. \end{aligned} \tag{6.5}$$

Therefore electrodynamics becomes a geometrically based theory as required by the basic philosophy of general relativity. Finally the temporal and spatial dependence of the electromagnetic field tensor is determined by the first Bianchi identity of differential geometry:

$$\begin{gathered} D\wedge T^a = R^a{}_b\wedge q^b \\ \downarrow \\ D\wedge F^a = R^a{}_b\wedge A^b \end{gathered} \tag{6.6}$$

This equation is the homogeneous field equation (HE) of the Evans unified field theory and the current j^a is the homogeneous current. The HE is a condensed version of the Gauss law applied to magnetism and of the Faraday law of induction, which are known to hold to high precision in the laboratory. The experimental data in the laboratory therefore [4]– [10] imply that there is no interaction between gravitation and electromagnetism measurable by the Gauss and Faraday laws in the laboratory, i.e. the homogeneous current vanishes within instrumental precision in the laboratory:

$$j^a \sim 0. \tag{6.7}$$

However in a cosmological context gravitational fields may become very intense, and the homogeneous current is non-zero in general.

The inhomogeneous field equation (IE) of the Evans unified field theory is obtained by Hodge dual transformation [4]– [10] of the HE, giving:

$$d\wedge \widetilde{F}^a = \mu_0 J^a = -A^{(0)}\left(q^b\wedge \widetilde{R}^a{}_b + \omega^a{}_b\wedge \widetilde{T}^b\right) \tag{6.8}$$

The IE is a condensed summary of the properly covariant inhomogeneous laws of electrodynamics, the Coulomb and Ampère-Maxwell laws. In general the inhomogeneous current J^a is made up of both spacetime curvature and torsion, reflecting the interaction of electromagnetism and gravitation. In order to describe gravitational lensing in general, the HE and IE must be solved simultaneously with given initial and boundary conditions. This is in general a problem for the computer.

However, if a minimal prescription is used, i.e. if it is assumed [4]– [10] that the electromagnetic geometry of the IE is described by:

$$\left(q^b \wedge \widetilde{R}^a{}_b + \omega^a{}_b \wedge \widetilde{T}^b\right)_{e/m} = 0 \tag{6.9}$$

then the IE reduces to:

$$d \wedge \widetilde{F}^a = -A^{(0)} \left(q^b \wedge \widetilde{R}^a{}_b\right)_{grav}. \tag{6.10}$$

Eq.(6.10) is a combination of the Coulomb Law:

$$\nabla \cdot \mathbf{E}^0 = -\phi^{(0)} \left(R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30}\right) \tag{6.11}$$

where

$$\mathbf{E}^0 = E^0_x \mathbf{i} + E^0_y \mathbf{j} + E^0_z \mathbf{k} \tag{6.12}$$

and the Ampère-Maxwell law

$$\nabla \times \mathbf{B}^a = \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} + \mu_0 \mathbf{J}^a \tag{6.13}$$

where the components of the current term are given by:

$$J^1_x = -\frac{A^{(0)}}{\mu_0}\left(R^1{}_0{}^{10} + R^1{}_2{}^{12} + R^1{}_3{}^{13}\right) \tag{6.14}$$

$$J^2_y = -\frac{A^{(0)}}{\mu_0}\left(R^2{}_0{}^{20} + R^2{}_1{}^{21} + R^2{}_3{}^{23}\right) \tag{6.15}$$

$$J^3_z = -\frac{A^{(0)}}{\mu_0}\left(R^3{}_0{}^{30} + R^3{}_1{}^{31} + R^3{}_2{}^{32}\right). \tag{6.16}$$

The electric and magnetic fields appearing in the Ampère-Maxwell law are:

$$\mathbf{E}^a = E^1_x \mathbf{i} + E^2_y \mathbf{j} + E^3_z \mathbf{k}, \tag{6.17}$$

$$\mathbf{B}^a = B^1_x \mathbf{i} + B^2_y \mathbf{j} + B^3_z \mathbf{k}. \tag{6.18}$$

In order to proceed in this minimal approximation the Riemann elements appearing in equations (6.11) to (6.18) must be calculated for a given metric such as the Schwarzschild metric [12] (SM). The SM is well known to be the first solution discovered of the Einstein/Hilbert field equation of 1915 - the spherically symmetric solution to:

$$G_{\mu\nu} = 0 \tag{6.19}$$

and known as the vacuum solution. Here $G_{\mu\nu}$ is the Einstein field tensor. The SM therefore describes the spacetime around a gravitating mass of any kind. In the Eddington experiment this is the sun of mass M and in general relativity M is a parameter of curved spacetime. The SM is a metric solution corresponding to the Riemann geometry used by Einstein and Hilbert. In the notation of differential geometry this is:

$$R^a{}_b \wedge q^b = 0 \tag{6.20}$$

$$T^a = 0 \tag{6.21}$$

$$\widetilde{R}^a{}_b \wedge q^b \neq 0 \tag{6.22}$$

Eq.(6.20) is the first Bianchi identity of Riemann geometry used by Einstein and Hilbert. The identity is true if and only if the connection is symmetric in its lower two indices: if and only if the torsion tensor vanishes. It is this geometry that was used by Einstein in his explanation of the Eddington experiment [13]. The explanation by Einstein [13] was based on the gravitational attraction of a photon of mass m a distance r from the sun of mass M. This is a purely dynamical explanation without reference to classical electromagnetism. In this paper we give a fuller explanation than Einstein/Hilbert in terms of generally covariant unified field theory [4]– [10].

The SM is given in spherical polar coordinates by [11, 12]:

$$\begin{aligned} ds^2 = \left(1 - 2\frac{GM}{rc^2}\right)(cdt)^2 - \left(1 - 2\frac{GM}{rc^2}\right)^{-1} dr^2 \\ -r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \end{aligned} \tag{6.23}$$

where G is Newtons universal gravitational constant and where c is the speed of light, a universal constant. The velocity v_0 of an object in orbit around the sun is given in the frame of the observer by:

$$v_0 = \frac{dr}{dt} = \frac{\left(1 - 2\frac{GM}{rc^2}\right)^{1/2}}{\left(1 - 2\frac{GM}{rc^2}\right)^{-1/2}} \frac{dr'}{dt'} = \left(1 - 2\frac{GM}{rc^2}\right)_{v'}. \tag{6.24}$$

For a photon:

$$v_0 = \left(1 - 2\frac{GM}{rc^2}\right) c. \tag{6.25}$$

Therefore the photon appears to be slowed by:

$$\frac{v_0}{c} = 1 - 2\frac{GM}{rc^2}. \tag{6.26}$$

From this result the observed diffraction of light by the sun can be calculated with the general relativistic theory of gravitation of Einstein and Hilbert. The Evans theory reduces to this theory when:

$$q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b \longrightarrow 0 \tag{6.27}$$

$$T^a \longrightarrow 0, \tag{6.28}$$

so the Einstein/Hilbert field theory is a well defined limit of the Evans unified field theory [4]– [10], a limit in which classical electrodynamics is not considered at all.

Einsteiń explanation of the Eddington experiment is accepted because it is repeatable for some stars to within 0.02% uncertainty. The corresponding Newtonian result is calculated from the Newton inverse square law:

$$\mathbf{F} = -\frac{mM}{r^2}\mathbf{k}. \tag{6.29}$$

Integrating Eq.(6.29) we obtain:

$$v^2 = 2\frac{MG}{r}. \tag{6.30}$$

This gives:

$$c^2 - v^2 = (c - v)(c + v) = c^2 \left(1 - 2\frac{MG}{rc^2}\right) \tag{6.31}$$

and:

$$\left(c^2 - v^2\right)^{1/2} = c\left(1 - 2\frac{MG}{rc^2}\right)^{1/2} \tag{6.32}$$

If $v << c$ then:

$$v_0 = c - \frac{v}{2} \sim \left(1 - \frac{mG}{rc^2}\right)c. \tag{6.33}$$

Therefore the Newtonian result is half the result from the Schwarzschild metric.

6.3 Refraction Of Electromagnetic Radiation By Gravitation

It is first shown that the elements of the Riemann tensor appearing in equations (6.11) to (6.16) are self-consistently the non-vanishing elements of the Riemann tensor from the Schwarzschild metric. These elements are:

$$\begin{aligned} R^0{}_{101} = {} & e^{2(\beta-\alpha)}\left[\partial_0^2\beta + (\partial_0\beta)^2 - \partial_0\alpha\partial_0\beta\right] \\ & + \left[\partial_1\alpha\partial_1\beta - \partial_1^2\alpha - (\partial_1\alpha)^2\right] \end{aligned} \tag{6.34}$$

$$R^1{}_{212} = re^{-2\beta}\partial_1\beta/r^2 \tag{6.35}$$

$$R^1{}_{313} = \left(1 - e^{-2\beta}\right)\sin^2\theta/r^2 \tag{6.36}$$

$$R^2{}_{323} = \left(1 - e^{-2\beta}\right)\sin^2\theta/r^2 \tag{6.37}$$

$$R^0{}_{202} = -re^{-2\beta}\partial_1\alpha/r^2 \tag{6.38}$$

$$R^0{}_{303} = -re^{-2\beta}\sin^2\theta\partial_1\alpha/r^2. \tag{6.39}$$

(Note that in ref. [12] these elements are given incorrectly in an otherwise useful book.) In the notation of Eqs.(6.34)–(6.39):

$$e^{2\alpha} = e^{-2\beta} = 1 - 2\frac{GM}{rc^2}. \tag{6.40}$$

Furthermore:

$$\partial_0\beta = 0, \quad \partial_0\partial_1\alpha = 0, \tag{6.41}$$

$$e^{2\alpha}\left(2r\partial_1\alpha + 1\right) = 1, \tag{6.42}$$

$$\partial_1\alpha + \partial_1\beta = 0. \tag{6.43}$$

From Eq.(6.42):

$$\partial_1\alpha = \frac{1}{2r}\left(e^{-2\alpha} - 1\right) \tag{6.44}$$

therefore:

$$\begin{aligned} R^0{}_{202} &= -re^{-2\beta}\frac{1}{2r}\left(e^{-2\alpha} - 1\right)/r^2 \\ &= -\frac{GM}{c^2r^3}. \end{aligned} \tag{6.45}$$

It follows that:

$$R^0{}_{303} = R^0{}_{202}\sin^2\theta \tag{6.46}$$

in the spherical polar coordinate system. Therefore:

$$R^0{}_{202} + R^0{}_{303} = -\frac{GM}{c^2r^3}\left(1 + sin^2\theta\right). \tag{6.47}$$

The $R^0{}_{101}$ element is given by:

$$R^0{}_{101} = e^{-4\alpha}\left(-\left(\partial_1\alpha\right)^2 - \partial_1^2\alpha - \left(\partial_1\alpha\right)^2\right) \tag{6.48}$$

where:

$$\partial_1\alpha = \frac{1}{2r}\left(\frac{1}{1 - 2\frac{GM}{rc^2}} - 1\right). \tag{6.49}$$

In the weak field limit

$$M \longrightarrow 0 \tag{6.50}$$

and the inverse square law of Newton must be obtained. In the weak field limit the Schwarzschild metric reduces to the Minkowski metric:

$$ds^2 = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 \tag{6.51}$$

of special relativity. The Newtonian limit of special relativity is the one where $v << c$, in which case the scalar curvature reduces to:

$$R = -2\frac{GM}{c^2r^3}. \tag{6.52}$$

Also in the limit of special relativity it is possible to raise and lower indices with the Minkowski metric, so we obtain:

$$R^0{}_2{}^{02} = \eta^{00}\eta^{22}R^0{}_{202} = -R^0{}_{202} \tag{6.53}$$

$$R^0{}_3{}^{03} = \eta^{00}\eta^{33}R^0{}_{303} = -R^0{}_{303}. \tag{6.54}$$

By antisymmetry:

$$R^0{}_2{}^{20} = -R^0{}_2{}^{02} = R^0{}_{202} \tag{6.55}$$

$$R^0{}_3{}^{30} = -R^0{}_3{}^{03} = R^0{}_{303} \tag{6.56}$$

and so:

$$R^0{}_2{}^{20} + R^0{}_3{}^{30} = -\frac{GM}{c^2r^3}\left(1+\sin^2\theta\right). \tag{6.57}$$

If we choose

$$\phi = \frac{\pi}{2}, \quad \theta = \frac{\pi}{2} \tag{6.58}$$

in the spherical polar coordinate system then the vector points along the y axis and:

$$R^0{}_2{}^{20} + R^0{}_3{}^{30} = -2\frac{GM}{c^2r^3}. \tag{6.59}$$

Finally the Newton inverse square law is obtained from:

$$\nabla\cdot\mathbf{g} = -c^2\left(R^0{}_2{}^{20} + R^0{}_3{}^{30}\right) = -2\frac{GM}{r^3} \tag{6.60}$$

assuming that

$$R^0{}_1{}^{10} \longrightarrow 0 \tag{6.61}$$

in the weak field limit. From Eq.(6.60) we obtain:

$$\mathbf{g} = \frac{\mathbf{F}}{m} = -\frac{GM}{r^2}\mathbf{k} \tag{6.62}$$

which is the Newton inverse square law:

$$\mathbf{F} = -\frac{GmM}{r^2}\mathbf{k}. \tag{6.63}$$

Eq.(6.60) can be expressed as:

$$\nabla\cdot\mathbf{g} = -G\rho_m = -6.67\times10^{-11}\rho_m \tag{6.64}$$

and the Coulomb inverse square law as:

$$\nabla\cdot\mathbf{E}^0 = -\frac{1}{\epsilon_0}\rho_e = -1.129\times10^{11}\rho_e. \tag{6.65}$$

Here $\mathbf{g}$ is the acceleration due to gravity, ρ_{m_0} is the mass density in kgm^{-3}, is the charge density in Cm^3. In Eq.(6.65) $\mathbf{E}$ is the electric field strength in volt m^{-1} and ϵ_0 is the vacuum permittivity. From these well known equations it is clear that the electric field is twenty two orders of magnitude stronger in an earthbound laboratory than g for unit ρ_m and ρ_e. In the Evans field theory these laws are unified in terms of the scalar curvature R as follows:

$$\nabla \cdot \mathbf{E}^0 = -\frac{1}{\epsilon_0}\rho_e = -\phi^{(0)} R \tag{6.66}$$

$$\nabla \cdot \mathbf{g} = -G\rho_m = -c^2 R. \tag{6.67}$$

Here $\phi^{(0)}$ is a fundamental voltage, c is the speed of light in vacuo, and R is scalar curvature in inverse square metres. Therefore unification is achieved in terms of geometry, represented by scalar curvature R. This is the curvature of spacetime. The notions of mass density and charge density are replaced by geometry of spacetime.

The basic structure of Eqs.(6.66) and (6.67) is clear, but their interpretation requires reference to experimental data as follows.

If we use the Newtonian curvature R of Eq.(6.59) we obtain the Newton inverse square law (6.63) and also the Coulomb inverse square law:

$$\nabla \cdot \mathbf{E}^0 = 2\phi^{(0)}\frac{GM}{c^2 r^3} \tag{6.68}$$

from which the static electric field is given by:

$$\mathbf{E}^0 = \frac{\mathbf{F}}{e_1} = -\phi^{(0)}\frac{GM}{c^2 r^2}. \tag{6.69}$$

However, the Newton and Coulomb inverse square laws originate in different aspects of geometry. The former originates in curvature and the latter in torsion. The Newton law is obtained from the differential geometry:

$$q^b \wedge \widetilde{R}^a{}_b \neq 0 \tag{6.70}$$

$$T^a = 0 \tag{6.71}$$

with the constraints:

$$q^b \wedge R^a{}_b = 0 \tag{6.72}$$

$$D \wedge \omega^a{}_b = 0. \tag{6.73}$$

The Newton law is obtained from Eq.(6.70), which translates into Eq.(6.10) in the Schwarzschild metric. If it is assumed that there is a quantity $\mathbf{g}$ defined by R according to Eq.(6.67) then the inverse square law of Newton follows as Eqs.(6.62) and (6.63). The Newton law also follows, self-consistently, from the Evans wave equation in the non-relativistic limit. The quantity $\mathbf{g}$ may therefore be identified as the acceleration due to gravity.

The Coulomb law on the other hand is obtained from the geometry:

$$T^a \neq 0 \tag{6.74}$$

$$d \wedge \widetilde{T}^a \sim -q^b \wedge \widetilde{R}^a{}_b \tag{6.75}$$

which is an approximation to the IE [4]– [10]. It is seen that the spacetime torsion is zero in the Newton law and non-zero in the Coulomb law. However both laws have an inverse square dependence because both depend on scalar curvature R. The geometrical quantity common to both laws is $\widetilde{R}^a{}_b \wedge q^b$. In the Schwarzschild metric this gives Eq.(6.52). Therefore from Eq.(6.69) the force between two charges is:

$$\mathbf{F} = -\frac{e_1}{r^2}\left(\phi^{(0)}\frac{GM}{c^2}\right)\mathbf{k} = \frac{e_1 e_2}{4\pi\epsilon_0 r^2}\mathbf{k}. \tag{6.76}$$

By convention the Coulomb law of electrostatics is written without a minus sign and with a factor 4π in the denominator. The Newton inverse square law of dynamics is written with a minus sign.

From Eq.(6.76) we may express e_2 in terms of the parameter M:

$$e_2 = -\frac{4\pi\epsilon_0 G\phi^{(0)}M}{c^2} = -8.25 \times 10^{-38}\phi^{(0)}M \tag{6.77}$$

using:

$$\begin{aligned} 4\pi\epsilon_0 &= 1.112650 \times 10^{-10} J^{-1}C^2m^{-1} \\ G &= 6.67 \times 10^{-11} m^3kg^{-1}s^{-2} \\ c &= 2.997925 \times 10^8 ms^{-1}. \end{aligned} \tag{6.78}$$

Eq.(6.77) is a fundamental result which shows that charge in general relativity originates in spacetime. Eq.(6.77) means that for unit $\phi^{(0)}$ it takes 10^{38} units of mass to be equivalent to one unit of charge. This result has also been obtained self-consistently from the Evans wave equation [4]– [10]. The interpretation of this result is that in order for electromagnetism and gravitation to be mutually influential to any significant degree the homogeneous current j^a must be non-zero:

$$j^a = -\frac{A^{(0)}}{\mu_0}\left(\omega^a{}_b \wedge T^b + q^b \wedge R^a{}_b\right) \neq 0. \tag{6.79}$$

In the laboratory the Newton and Coulomb inverse square laws hold to within contemporary experimental precision so the interaction of gravitation and electromagnetism must be sought for in other ways. It is insufficient simply to change mass M in Eq.(6.77). If two charged objects of mass m and M are investigated in the laboratory then changing M has no effect on the Coulomb law to within experimental precision. This result means that the product $\phi^{(0)}M$ is a constant in the Coulomb law. Similarly, changing a charge on one of the two masses will have no effect on the Newton inverse square law. In the approximation used here this experimental fact has already been assumed in using the minimal prescription. In other words this calculation has been carried out in the approximation that the IE can be written as Eq.(6.10). This means that scalar curvature R is given by the Einstein/Hilbert theory and the SM. In this approximation it can be seen from Eq.(6.67) that the electric field has no influence on $\mathbf{g}$, as found experimentally.

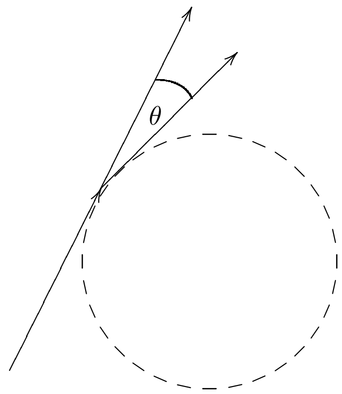

Figure 6.1: Refraction

To obtain such an influence one must use the geometry defined by the most general Bianchi identity of differential geometry [4]– [12]:

$$d \wedge T^a = -\left(\omega^a{}_b \wedge T^b + q^b \wedge R^a{}_b\right) \tag{6.80}$$

$$d \wedge \widetilde{T}^a = -\left(\omega^a{}_b \wedge \widetilde{T}^b + q^b \wedge \widetilde{R}^a{}_b\right). \tag{6.81}$$

In this geometry the torsion and curvature are both non-zero. The corresponding field equations in this geometry are:

$$d \wedge F^a = -A^{(0)}\left(\omega^a{}_b \wedge T^b + q^b \wedge R^a{}_b\right) \tag{6.82}$$

$$d \wedge \widetilde{F}^a = -A^{(0)}\left(\omega^a{}_b \wedge \widetilde{T}^b + q^b \wedge \widetilde{R}^a{}_b\right). \tag{6.83}$$

Only in this situation will electromagnetism have any effect on gravitation and vice-versa. The Newton and Coulomb inverse square laws are only approximations to these more general laws. The approximation is excellent in the laboratory but not in general in cosmology.

In the approximation to the IE given by Eq. [10], it is possible to estimate the angle of refraction in the Eddington experiment from the fact that the speed of the photon has been slowed from c to v. The resultant angle of deflection (Fig. (6.1)) from general relativity is:

$$\theta = \frac{4MG}{rc^2} \tag{6.84}$$

and this result has been verified experimentally to one part in 100,000. The same precision is obtained from the IE as a refraction problem (Fig (6.2)). With reference to the geometry of Fig (6.2) we obtain:

$$\frac{\sin(r-\theta)}{\sin r} = \frac{c}{v} = \left(1 - \frac{2MG}{rc^2}\right)^{-1}. \tag{6.85}$$

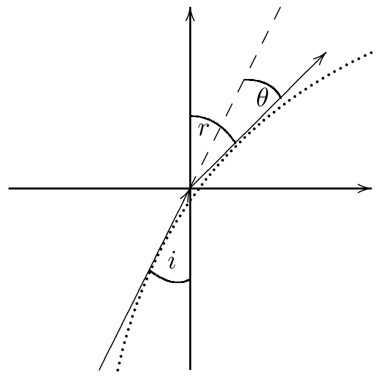

Figure 6.2: Refraction & Defraction

Using the formula:

$$\sin(r - \theta) = \sin r \sin \theta - \cos r \cos \theta \tag{6.86}$$

in the limit $\theta << 1$ radian we obtain

$$\theta \sim 1 + \tan r + \frac{2MG}{rc^2} \tag{6.87}$$

The result of general relativity is finally obtained using:

$$1 + \tan r = \frac{2MG}{rc^2}. \tag{6.88}$$

This means that the equivalent angle of refraction is almost exactly -45° . The minus sign means rotating in a certain direction, which is arbitrary. In other words the Eddington effect is explained using the IE to one part in one hundred thousand by using a set of axes inclined at almost exactly 45° to the incident light. This is an explanation based on classical electrodynamics within a unified field theory, and is missing entirely from Einsteinś analysis.

In the case of the sun therefore the approximation, Eq.(6.10), to the Evans field theory is justified within contemporary instrumental precision, meaning that our minimal prescription is in this case an excellent approximation. In the latter it has also been assumed that there is no gravitational torsion present, only gravitational curvature, and this again is an excellent approximation, the suns gravitation produces curvature and in the weak field limit is described by the Newton inverse square law. However, for objects such as pulsars, which are much more intensely gravitating than the sun, departures from this approximation must be expected, leading to anomalous gravitational shifts as observed experimentally [2, 4]– [10].

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Chapter 7

Generally Covariant Heisenberg Equation From The Evans Unified Field Theory

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Abstract

A generally covariant form of the Heisenberg equation is derived from the Cartan structure equation of differential geometry. This equation is used to suggest why the conventional Heisenberg uncertainty principle has been observed to fail qualitatively in three independent experiments, the reason is that the conventional Heisenberg equation is not generally covariant, and does not contain the correctly covariant densities of general relativity. This derivation is an illustration of the fact that general relativity and quantum mechanics are unified in the Evans field theory.

Key words: Evans field theory; Heisenberg equation; Heisenberg uncertainty principle.

7.1 Introduction

Recently the Heisenberg uncertainty principle has been shown experimentally to fail completely. Three independent experiments have demonstrated this in the past few years and all three types of experiment are rigorously reproducible and repeatable. The advanced microscopy work of Croca [1] has shown that even at moderate resolution the principle fails by nine orders of magnitude. As the resolution is increased the principle becomes more and more incorrect. Afshar [2] has carried out a series of reproducible and repeatable experiments which show that the photon and the associated electromagnetic wave can be observed simultaneously. This result implies that the commutator of conjugate variables in the uncertainty principle is zero, a more complete violation of the principles of complementarity and uncertainty is not possible. Yet this is what is observed. Thirdly a series of reproducible and repeatable experiments [3] have shown that two dimensional materials when cooled to within a millikelvin of absolute zero become conductors, whereas the uncertainty principle predicts that they become insulators. Therefore the uncertainty principle has been shown to fail completely in three entirely independent sets of experiments, all of which are rigorously reproducible and repeatable.

Theoretical advances in unified field theory have resulted in the development of a generally covariant unified field theory [4]– [12] based on differential geometry and the well known Palatini variation of general relativity in which the tetrad is the fundamental field [13]– [15]. In supersymmetry theory for example the tetrad becomes the gravitino. The tetrad postulate [13]– [15] of the Palatini variation is the metric compatibility condition of the Einstein Hilbert variation of general relativity and gravitational general relativity has recently been verified to one part in a hundred thousand by long baseline interferometric experiments at NASA [16]. This is therefore the experimental precision of the tetrad postulate in the gravitational theory. In the Evans unified field theory [4]– [12] the tetrad postulate is developed into the Evans Lemma and wave equation, from which the Dirac equation may be derived in the special relativistic limit. The Schrodinger equation is the non-relativistic limit of the Dirac equation, and the Heisenberg equation is the commutator variation of the Schrodinger equation. Therefore the conventional Heisenberg equation is a non-relativistic equation. It turns out that this is the root cause of the qualitative failure of the Heisenberg uncertainty principle described already.

In Section 7.2 a generally covariant Heisenberg equation is developed from the Cartan structure equation of differential geometry [4]– [15] by defining an angular momentum form from the torsion form defined by the structure equation. The angular momentum form is then used to construct a generally covariant commutator equation between angular momenta or rotation generators. This is the type of commutator equation which the basis of conventional quantum mechanics in the non-relativistic limit [17]. However, in the non-relativistic limit the concept of angular momentum density is missing entirely, whereas in general relativity the basic Einstein equation is a proportionality between the Einstein field tensor $G_{\mu\nu}$ and the canonical energy-momentum density $T_{\mu\nu}$. The latter is a density, so is defined with respect to volume. By introducing appropriate angular momentum densities a commutator relation is obtained which is qualitatively in accord with the experiments cited already.

Finally a discussion is given of the need to overhaul the Heisenberg uncertainty principle (the principle of indeterminacy) in the light of new experimental data.

7.2 Angular Momentum Forms And Densities

The starting point for the derivation of the generally covariant Heisenberg equation is the Cartan structure equation:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{7.1}$$

where T^a is the torsion form, $D\wedge$ is the covariant exterior derivative, q^a is the tetrad form, is the spin connection and $d\wedge$ is the ordinary exterior derivative. The torsion form is related to the Riemann form $R^a{}_b$ through the Bianchi identity:

$$D \wedge T^a = R^a{}_b \wedge q^b. \tag{7.2}$$

Reinstating the indices of the base manifold [4]– [15]:

$$T^a{}_{\mu\nu} = -T^a{}_{\nu\mu} = (D \wedge q^a)_{\mu\nu}\,. \tag{7.3}$$

Similarly the Riemann form is defined by the second Cartan structure equation:

$$R^a{}_{b\mu\nu} = -R^a{}_{b\nu\mu} = (D \wedge \omega^a{}_b)_{\mu\nu}\,. \tag{7.4}$$

Both the torsion and Riemann forms are antisymmetric in the indices of the base manifold. The torsion form is a vector-valued two-form and the Riemann form is a tensor valued two form.

The angular momentum two-form is introduced in this paper as:

$$J^a{}_{\mu\nu} = -J^a{}_{\nu\mu} = \frac{\hbar}{\kappa} T^a{}_{\mu\nu} \tag{7.5}$$

where $\hbar$ is the reduced Planck constant, the least angular momentum or action in the universe, and κ is a wavenumber. Therefore:

$$E^a = c\kappa J^a = \omega J^a = c\hbar T^a \tag{7.6}$$

is a two form with the units of energy, where c is the speed of light in vacuo. Eq.(7.6) can be interpreted as a generally covariant version of the fundamental Planck quantization:

$$E = \hbar\omega \tag{7.7}$$

where ω is the angular frequency in radians per second. Thus:

$$E^a{}_{\mu\nu} = -E^a{}_{\nu\mu} \tag{7.8}$$

is a vector valued energy two form with time-like and space-like components. More precisely it is an angular-energy / angular-momentum two form. In order to make the antisymmetric E^a generally covariant it has to be converted into a density, denoted ϵ^a, in analogy with the symmetric canonical energy momentum density appearing in the Einstein equation. The densities ϵ^a are vector valued two forms with the units of Jm^{-3}, energy divided by volume. Due to the antisymmetric structure of ϵ^a, there exist cyclic relations of the type:

$$\begin{gathered}\epsilon^a \wedge \epsilon^b = \epsilon_0 \epsilon^c \\ \text{et cyclicum}\end{gathered} \tag{7.9}$$

where ϵ^a is the least energy density magnitude of a given elementary particle.

The conventional Heisenberg equation can be expressed [17] in the non-relativistic limit as a cyclic relation between angular momenta:

$$\begin{gathered}[J_x, J_y] = i\hbar J_z \\ \text{et cyclicum}\end{gathered} \tag{7.10}$$

an equation which is independent of the choice of operator representation. Within the factor $\hbar$ Eq.(7.10) is the fundamental commutator relation between rotation generators [18]. In special relativity these are generators of the Poincarè group and in general relativity of the Einstein group. They are torsion forms within the factor h/κ Defined in Eq.(7.5). Therefore the generally covariant Heisenberg equation is a cyclic relation between torsion forms defined by the Cartan structure equation.

It is seen that volume does not enter into Eq.(7.10) in the non-relativistic limit in which this equation is written. Reinstating the wavefunction, ψ, the Heisenberg equation is usually written as:

$$[J_x, J_y]\, \psi = i\hbar J_z \psi \tag{7.11}$$

and is equivalent to the Schrodinger equation. However Eq.(7.11) is not a correctly objective or generally covariant equation of physics because it is not correctly derived from differential geometry. The wavefunction ψ is not recognized to be the correctly covariant wavefunction of the Palatini variation of general relativity, the tetrad $q^a{}_\mu$ [4]– [15]. In all situations of interest to physics the latter obeys the tetrad postulate:

$$D_\nu q^a{}_\mu = 0 \tag{7.12}$$

which is fundamental to differential geometry and can be proven rigorously in several ways. From Eq.(7.12) we obtain the identity:

$$D^\nu \left(D_\nu q^a{}_\mu \right) := 0 \tag{7.13}$$

or

$$\Box q^a{}_\mu = R q^a{}_\mu \tag{7.14}$$

where R is a well defined [4]– [12] scalar curvature. Eq.(7.14) is the Evans Lemma, the subsidiary proposition leading to the Evans wave equation:

$$(\Box + kT)\, q^a{}_\mu = 0 \tag{7.15}$$

where:

$$R = -kT. \tag{7.16}$$

Here k is Einsteinś constant and T is an index contracted canonical energy-momentum density. The Lemma and wave equation are valid for all radiated and matter fields because the tetrad postulate is valid for any connection. Thus the Evans wave equation is the fundamental wave equation of generally covariant unified field theory [4]– [12] from which all the major equations of physics are derived in well defined limits. This procedure should therefore also be used to derive the Heisenberg equation in its generally covariant or rigorously objective form. In so doing the Heisenberg uncertainty principle is abandoned, because the principle is acausal and diametrically at odds with causal and objective general relativity. The Heisenberg uncertainty principle is not objective because it asserts unknowability [17]. This is a subjective assertion introduced by Bohr and Heisenberg on the grounds of then incomplete or restricted experimental data.

The contemporary experiments with vastly improved data cited in the introduction now show that the Heisenberg uncertainty principle must be abandoned in favor of Einsteinian physics, i.e. objective and causal physics.

So in using the correctly covariant Evans wave equation, based directly on the Einsteinian principles of rigorous objectivity and rigorous causality the concept of canonical energy momentum density is introduced through T and R. The wavefunction is also correctly identified as the tetrad, the fundamental field in the Palatini variation [4]– [15] of general relativity. The correspondence principle shows that Eq.(7.15) must reduce to the Dirac equation when one particle is considered in the special relativistic limit:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) q^a{}_\mu = 0 \tag{7.17}$$

where m is the mass of the particle. Therefore in this limit:

$$kT = \frac{m^2c^2}{\hbar^2}. \tag{7.18}$$

In the rest frame of one particle:

$$T = \frac{m}{V_0} = \frac{E_0}{c^2V_0} \tag{7.19}$$

where the rest energy is:

$$E_0 = mc^2 \tag{7.20}$$

and where V_0 is a new concept [4]– [12], the REST VOLUME:

$$V_0 = \frac{k\hbar^2}{mc^2} = \frac{k\hbar^2}{E_0}. \tag{7.21}$$

For the electron:

$$V_0 = 2.56 \times 10^{-81} m^3. \tag{7.22}$$

Every elementary particle, including the photon and the neutrinos, has a rest volume inversely proportional to its mass. This is a new law of physics derived from the Evans unified field theory.

Using the de Broglie equation for the rest frequency ω_0:

$$E_0 = mc^2 = \hbar\omega_0 \tag{7.23}$$

we obtain:

$$V_0 = \frac{\hbar k}{\omega_0} \tag{7.24}$$

a simple inverse proportionality between V_0 and ω_0 and a new statement of wave particle duality: a particle with rest volume V_0 is also a wave of rest frequency ω_0. In the rest frame in the special relativistic limit therefore:

$$\epsilon_0 = \frac{\hbar\omega}{V_0} \tag{7.25}$$

which is the quantum of energy density for an elementary particle, including the photon. The energy densities ϵ^a, ϵ^b, ϵ^c appearing in Eq.(7.9) must in general be defined with respect to a volume V. It is plausible to define V in the special relativistic limit as the sample volume or volume occupied by the apparatus, while V_0 is the particles rest volume, or minimum volume. Therefore:

$$\epsilon^a = E^a / V \tag{7.26}$$

and so on, where:

$$E^a = \omega J^a \tag{7.27}$$

and so on. The fundamental rotation generator relation at the root of the Heisenberg equation is therefore expressed as:

$$\left[V\epsilon^a, V\epsilon^b\right] = i\left(V_0\epsilon_0\right)\left(V\epsilon^c\right) \tag{7.28}$$

or

$$\left[J^a, J^b\right] = i\hbar\frac{V_0}{V}J^c. \tag{7.29}$$

Eq.(7.29) is a plausible development of the Heisenberg equation to include the concept of angular momentum density. Essentially the geometry is proportional to a density in physics through Eq.(7.16). In general:

$$V \gg V_0 \tag{7.30}$$

and so it is possible that:

$$\left[J^a, J^b\right] \sim 0 \tag{7.31}$$

as observed in the experiments cited already. Eq.(7.31) means that a particle and wave may be observed simultaneously for all practical purposes, as in the

Afshar experiments. If for example V is one cubic meter V_0 for the photon is many orders of magnitude smaller, so V_0/V is essentially zero as observed experimentally [1]– [3]. In the Afshar experiments for example the volume used for the photon is V_0, and V is the effective volume of the apparatus. The wave spreads throughout the apparatus as also observed in the Croca experiments and in the Aspect experiments [19], but the volume associated with the photon as particle is V_0. The photon therefore appears simultaneously as a particle and also as a wave, as observed experimentally. The Croca and Aspect experiments show that the wave can occupy the whole of the apparatus and spread out indefinitely. In the conventional Copenhagen interpretation a photon is either a particle or a wave, but never a particle and wave simultaneously. This idea leads to all kinds of difficulties as is well known, and so should be abandoned..

7.3 Discussion

In this discussion we give a brief summary of the three types of experiment which show independently that the Heisenberg uncertainty principle fails qualitatively. The Croca experiments are summarized in ref. (1). A tunneling super-resolution microscope and apertureless optical microscope are used to demonstrate conclusively that the principle fails even at moderate resolutions. The experiments are summarized in pp. 109 ff. of ref. (1). Other types of experiment summarized by Croca include a photon ring experiment and Franson type experiments. In eq. (4.7.9), for example, of ref. (1) it is shown experimentally that:

$$\delta x \delta v = 10^{-9} \frac{\hbar}{2m} \tag{7.32}$$

where x is position and v is velocity. The theoretical result according to the Heisenberg uncertainty principle is:

$$\delta x \delta v \geqslant \frac{\hbar}{2m}. \tag{7.33}$$

Therefore ref. (1) is in itself the source of several independent tests of the Heisenberg uncertainty principle and these tests all violate the principle dramatically.

The Afshar experiments were reproduced at Harvard University and are summarized in ref. (2) and are rigorously reproducible and repeatable, showing that a photon and a wave can be observed simultaneously. One of them is a modified Young experiment. It is well known that a beam consisting of one photon produces interference in a Young experiment [2], therefore a photon cannot be a localized particle, it is both particulate and wavelike. In one of the Afshar experiments [2] a laser is directed at two pinholes in an opaque screen in a Young experiment. Photon detectors are used to record the rate at which photons are coming through each pinhole. An interference pattern is observed SIMULTANEOUSLY by use of wire grids. The latter are arranged so that the wires coincide precisely with the dark fringes of the interference pattern.

One pinhole is then closed, and the interference pattern disappears. The light spreads out from the one open pinhole, some of the light hits the wire grid and is scattered. Less light reaches the photon detector corresponding to the open pinhole.

The pinhole is now reopened and it is observed that the light intensity at each detector returns to its value before the wires were set in place. This is because the wires coincide precisely with the dark fringes, or minima of the interferogram and so little or no light is reflected from the wire grids. This proves experimentally the existence of an interference pattern or interferogram coming from the wave-like nature of the laser light. At the same time the intensity of light from each slit can be measured by the photon detectors, so it is possible to count the number of photons emerging from each pinhole. So it is possible to observe both the particulate and wavelike components of the laser light.

The Bohr Heisenberg complementarity principle on the other hand asserts that these experimental results are NOT possible, because a photon and wave cannot be measured at the same time. The principle asserts that physics is not causal, in direct contradiction of Einsteinś causal general relativity. According to Bohr nothing exists until it is measured. This means that physics is not objective, again in direct contradiction of rigorously objective general relativity. Bohr claimed that observations always influence results, so that results are different according to the way they are measured or differently influenced. Einstein rejected this assertion and recent data at NASA [16]show that the 1915 gravitational general relativity is indeed accurate to one part on one hundred thousand as discussed in the introduction of this paper. The Afshar [2] and Croca [1] experiments show that Bohr and Heisenberg were entirely (i.e. qualitatively) incorrect. The tetrad postulate of the Palatini variation of general relativity is therefore accurate experimentally to one part in one hundred thousand. Therefore the Evans unified field theory is based on the tetrad postulate and produces causal and objective quantum mechanics unified with general relativity. The Afshar experiment is explained as in Section 7.2 of this paper.

The third independent type of experiment [3] is based on the wave particle duality of electrons. The experiments were carried out by Kravchenko et al. at Northeastern University on two dimensional silicon films and are rigorously reproducible and repeatable. Within millikelvins of absolute zero these silicon films become conductors. The Heisenberg uncertainty principle asserts the complete opposite, that the films should become perfect insulators [3]. The same result is obtained with superconductors [3], which become metals within millikelvins of absolute zero, again in violation of the Heisenberg uncertainty principle. The latter asserts [3] that there are only two possibilities for Cooper pairs in superconductors: insulating or superconducting. The conjugate variables in this case are phase and particle number, so the Heisenberg uncertainty principle asserts that of the phase is known exactly the particle number is completely unknown. The latter is interpreted conventionally as indicating large fluctuations in particle number or flow of electrons as in a superconductor. However within millikelvins of absolute zero this is not observed, an ordinary metallic conductor is observed [3].

The Evans unified field theory would attempt to explain this in terms of general relativity, and abandons the Heisenberg uncertainty principle and the principle of complementarity.

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Chapter 8

Metric Compatibility Condition And Tetrad Postulate

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Abstract

The metric compatibility condition of Riemann geometry and the tetrad postulate of differential geometry are cornerstones of general relativity in respectively its Einstein Hilbert and Palatini variations. In the latter the tetrad tensor is the fundamental field, in the former the metric tensor is the fundamental field. In the Evans unified field theory the tetrad becomes the fundamental field for all types of matter and radiation, and the tetrad postulate leads to the Evans Lemma, the Evans wave equation, and to all the fundamental wave equations of physics in various well defined limits. The tetrad postulate is a fundamental requirement of differential geometry, and this is proven in this paper in seven ways. For centrally directed gravitation therefore both the metric compatibility condition and the tetrad postulate are accurate experimentally to one part in one hundred thousand.

Key words: Metric compatibility; tetrad postulate; Einstein Hilbert variation of general relativity; Palatini variation of general relativity; Evans unified field theory.

8.1 Introduction

The theory of general relativity was formulated originally in 1915 by Einstein and independently by Hilbert. It was developed for centrally directed gravitation, and was first verified by the Eddington experiment [1]. Recently [2] the precision of the Eddington experiment has been improved to one part in one hundred thousand. Therefore the basic geometrical assumptions used by Einstein and Hilbert have also been verified experimentally to one part in one hundred thousand. One of these is the metric compatibility condition [3]– [5] of Riemann geometry, a condition which asserts that the covariant derivative of the metric tensor vanishes. The metric tensor is the fundamental field in the Einstein Hilbert variation of general relativity. It is defined by:

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{\mu\nu} \tag{8.1}$$

where $q^a{}_\mu$ is the tetrad [3]– [5], a mixed index rank two tensor. The Latin superscript of the tetrad tensor refers to the spacetime of the tangent bundle at a point P of the base manifold indexed by the Greek subscript of the tetrad. In eqn.(8.1) η_{ab} is the Minkowski metric:

$$\eta_{ab} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{8.2}$$

The metric compatibility condition is then [3]– [5], for any spacetime:

$$D_\rho g^{\mu\nu} = D_\rho g_{\mu\nu} = 0. \tag{8.3}$$

Using the Leibnitz Theorem [3]– [5]Eq.(8.1) and (8.3) imply:

$$q^b{}_\nu D_\rho q^a{}_\mu + q^a{}_\mu D_\rho q^b{}_\nu = 0 \tag{8.4}$$

one possible solution of which is:

$$D_\rho q^a{}_\mu = D_\rho q^b{}_\nu = 0. \tag{8.5}$$

Eq.(8.5) is the tetrad postulate of the Palatini variation [3]– [8] of general relativity. In Section 8.2 it will be shown in various complementary ways that Eq.(8.5) is the unique solution of Eq.(8.4). It follows that for central gravitation, the tetrad postulate has been verified experimentally [2] to one part in one hundred thousand.

In Section 8.3 a brief discussion is given of the physical meaning of the metric compatibility condition used by Einstein and Hilbert in 1915 to describe centrally directed gravitation. In 1915 the original metric compatibility condition was supplemented by the additional assumption that the spacetime of gravitational general relativity is free of torsion:

$$T^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\mu\nu} - \Gamma^\kappa{}_{\nu\mu} = 0 \tag{8.6}$$

where $T^{\kappa}{}_{\mu\nu}$ is the torsion tensor and where $\Gamma^{\kappa}{}_{\mu\nu}$ is the Christoffel symbol. The latter is symmetric in its lower two indices and is also known as the Levi-Civita or Riemann connection [3]– [5]. For the centrally directed gravitation of the sun these assumptions hold to one part in one hundred thousand [2]. However, the Evans unified field theory [9]– [15] has recently recognized that electromagnetism is the torsion form of differential geometry [3]– [5], gravitation being the Riemann form, and has shown how electromagnetism interacts with gravitation in a spacetime in which the torsion tensor is not in general zero. Therefore in Section 8.3 we discuss the implications for the metric compatibility condition of the 1915 theory, and summarize the conditions needed for the interaction of gravitation and electromagnetism.

8.2 Seven Proofs Of The Tetrad Postulate

It has been shown in the introduction that for any spacetime (whether torsion free or not) the tetrad postulate is a possible solution of the metric compatibility condition. In this section it is shown in seven ways that it is the unique solution.

1. Proof from Fundamental Matrix Invertibility.

   Consider the following basic properties of the tetrad tensor [3]– [5]:

$$q^b{}_\nu q^\nu{}_b = 1 \tag{8.7}$$
$$q^a{}_\mu q^\mu{}_a = 1 \tag{8.8}$$
$$q^\mu{}_a q^a{}_\nu = \delta^\mu{}_\nu \tag{8.9}$$
$$q^a{}_\mu q^\mu{}_b = \delta^a{}_b \tag{8.10}$$

   where $\delta^\mu{}_\nu$ and $\delta^a{}_b$ are Kronecker delta functions. Differentiate Eqs.(8.7) to (8.10) covariantly with the Leibnitz Theorem:

$$q^\nu{}_b D_\rho q^b{}_\nu + q^b{}_\nu D_\rho q^\nu{}_b = 0 \tag{8.11}$$
$$q^a{}_\mu D_\rho q^\mu{}_a + q^\mu{}_a D_\rho q^a{}_\mu = 0 \tag{8.12}$$
$$q^\mu{}_a D_\rho q^a{}_\nu + q^a{}_\nu D_\rho q^\mu{}_a = 0 \tag{8.13}$$
$$q^a{}_\mu D_\rho q^\mu{}_b + q^\mu{}_b D_\rho q^a{}_\mu = 0. \tag{8.14}$$

   Rearranging dummy indices in Eq(8.11) ($a \to b, \mu \to \nu$):

$$q^\mu{}_a D_\rho q^a{}_\mu + q^b{}_\nu D_\rho q^\nu{}_b = 0. \tag{8.15}$$

   Rearranging dummy indices in Eq.(8.14) ($\mu \to \nu$):

$$q^\mu{}_b D_\rho q^a{}_\mu + q^a{}_\nu D_\rho q^\nu{}_b = 0. \tag{8.16}$$

   Multiply Eq.(8.15) by $q^a{}_\mu$:

$$D_\rho q^a{}_\mu + q^a{}_\mu q^b{}_\nu D_\rho q^\nu{}_b = 0. \tag{8.17}$$

Multiply Eq.(8.16) by $q^b{}_\mu$:

$$D_\rho q^a{}_\mu + q^b{}_\mu q^a{}_\nu D_\rho q^\nu{}_b = 0. \tag{8.18}$$

It is seen that Eq.(8.17) is of the form:

$$x + ay = 0 \tag{8.19}$$

and Eq.(8.18) is of the form:

$$x + by = 0 \tag{8.20}$$

where

$$a \neq b. \tag{8.21}$$

The only possible solution is:

$$x = y = 0. \tag{8.22}$$

This gives the tetrad postulate, Q.E.D.:

$$D_\rho q^a{}_\mu = D_\rho q^\nu{}_b = 0, \tag{8.23}$$

which is therefore the unique solution of Eq.(8.4). Note the tetrad postulate is true for any connection, whether torsion free or not.

2. Proof from Coordinate Independence of Tensors.

A tensor of any kind is independent of the way it is written [3]– [5]. Consider the covariant derivative of any tensor X in two different bases 1 and 2. It follows that:

$$(DX)_1 = (DX)_2. \tag{8.24}$$

In the coordinate basis [3]:

$$\begin{aligned}(DX)_1 &= (D_\mu X^\nu) dx^\mu \otimes \partial_\nu \\ &= (\partial_\mu X^\nu + \Gamma^\nu{}_{\mu\lambda} X^\lambda) dx^\mu \otimes \partial_\nu.\end{aligned} \tag{8.25}$$

In the mixed basis:

$$\begin{aligned}(DX)_2 &= (D_\mu X^a)\, dx^\mu \otimes \hat{e}_{(a)} \\ &= \left(\partial_\mu X^a + \omega^a{}_{\mu b} X^b\right) dx^\mu \otimes \hat{e}_{(a)} \\ &= q^\sigma{}_a \left(q^a{}_\nu \partial_\mu X^\nu + X^\nu \partial_\mu q^a{}_\nu \right.\\ &\quad \left. + \, \omega^a{}_{\mu b} q^b{}_\lambda X^\lambda\right) dx^\mu \otimes \partial\sigma\end{aligned} \tag{8.26}$$

where we have used the commutation rule for tensors. Now switch dummy indices σ to μ and use:

$$q^\nu{}_a q^a{}_\nu = 1 \tag{8.27}$$

to obtain:

$$(DX)_1 = \left(\partial_\mu X^\nu + q^\nu{}_a \partial_\mu q^a{}_\lambda X^\lambda + q^\nu{}_a q^b{}_\lambda \omega^a{}_{\mu b} X^\lambda\right) dx^\mu \otimes \partial_\nu \qquad (8.28)$$

Now compare Eq.(8.25) and Eq.(8.28) to give:

$$\Gamma^\nu{}_{\mu\lambda} = q^\nu{}_a \partial_\mu q^a{}_\lambda + q^\nu{}_a q^b{}_\lambda \omega^a{}_{\mu b} \qquad (8.29)$$

Multiply both sides of Eq.(8.29) by $q^a{}_\nu$:

$$q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} = \partial_\mu q^a{}_\lambda + q^b{}_\lambda \omega^a{}_{\mu b} \qquad (8.30)$$

to obtain the tetrad postulate, Q. E. D.:

$$D_\mu q^a{}_\lambda = \partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu = 0. \qquad (8.31)$$

3. Proof from Basic Definition.

For any vector V^a [3]:

$$V^a = q^a{}_\nu V^\nu \qquad (8.32)$$

and using the Leibnitz Theorem:

$$D_\mu V^a = q^a{}_\nu D_\mu V^\nu + V^\nu D_\mu q^a{}_\nu. \qquad (8.33)$$

Using the result:

$$D_\mu q^a{}_\nu = 0 \qquad (8.34)$$

obtained in proofs (1) and (2), it is proven here that Eqs.(8.32) and (8.34) imply:

$$D_\mu q^a{}_\lambda = \partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu \qquad (8.35)$$

From Eqs.(8.33) and (8.34):

$$\partial_\mu V^a + \omega^a{}_{\mu b} V^b = q^a{}_\nu \left(\partial_\mu V^\nu + \Gamma^\nu{}_{\mu\lambda} V^\lambda\right). \qquad (8.36)$$

From Eq.(8.32):

$$\partial_\mu V^a = V^\nu \partial_\mu q^a{}_\nu + q^a{}_\nu \partial_\mu V^\nu \qquad (8.37)$$

and

$$\omega^a{}_{\mu b} V^b = \omega^a{}_{\mu b} q^b{}_\nu V^\nu. \qquad (8.38)$$

Add Eqs.(8.37) and (8.38):

$$\partial_\mu V^a + \omega^a{}_{\mu b} V^b = q^a{}_\nu \partial_\mu V^\nu + V^\nu \partial_\mu q^a{}_\nu + \omega^a{}_{\mu b} q^b{}_\nu V^\nu \qquad (8.39)$$

Comparing Eqs.(8.36) and (8.39):

$$q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} V^\lambda = V^\nu \left(\partial_\mu q^a{}_\nu + \omega^a{}_{\mu b} q^b{}_\nu\right) \qquad (8.40)$$

and switching dummy indices $\nu \to \lambda$, we obtain:

$$\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} = 0. \qquad (8.41)$$

This equation has been obtained from the assumption (8.34), so it follows that:

$$D_\mu q^a{}_\nu = \partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} = 0 \qquad (8.42)$$

Q.E.D.

4. Proof from the First Cartan Structure Equation [9].

   This proof has been given in all detail in ref. [9] and is summarized here for convenience. Similarly for Proofs (5) to (7). The first Cartan structure equation [3]– [8] is a fundamental equation of differential geometry first derived by Cartan. It defines the torsion form as the covariant exterior derivative of the tetrad form:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{8.43}$$

   i.e.

$$T^a{}_{\mu\nu} = \partial_\mu q^a{}_\nu - \partial_\nu q^a{}_\mu + \omega^a{}_{\mu b} q^b{}_\nu - \omega^a{}_{\nu b} q^b{}_\mu . \tag{8.44}$$

   Here $T^a{}_{\mu\nu}$ is the torsion two-form, $q^a{}_\mu$ is the tetrad one-form and $\omega^a{}_{\mu b}$ is the spin connection. The torsion tensor of Riemann geometry is defined [3]– [5] as:

$$T^\lambda{}_{\mu\nu} = q^\lambda{}_a T^a{}_{\mu\nu} . \tag{8.45}$$

   Using the tetrad postulate (8.31) in the form:

$$\Gamma^\lambda{}_{\mu\nu} = q^\lambda{}_a \partial_\mu q^a{}_\nu + q^\lambda{}_a q^b{}_\nu \omega^a{}_{\mu b} \tag{8.46}$$

   it is seen from Eqs.(8.44) to (8.46) that:

$$\begin{aligned} T^\lambda{}_{\mu\nu} &= q^\lambda{}_a \left(\partial_\mu q^a{}_\nu + \omega^a{}_{\mu b} q^b{}_\nu \right) \\ &\quad - q^\lambda{}_a \left(\partial_\nu q^a{}_\mu + \omega^a{}_{\nu b} q^b{}_\mu \right) \\ &= \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu} . \end{aligned} \tag{8.47}$$

   Eq.(8.47) is the torsion tensor of Riemann geometry Q.E.D. Given the Cartan structure equation (8.43), therefore, the tetrad postulate is needed to derive the torsion tensor of Riemann geometry. The converse is also true.

5. Proof from the Second Cartan Structure Equation [3].

   Similarly this proof has been given in complete detail elsewhere [9]– [15] and is an elegant illustration of the tetrad postulate being used as the link between differential and Riemann geometry. The second Cartan structure equation defines the Riemann form as the covariant exterior derivative of the spin connection:

$$R^a{}_b = D \wedge \omega^a{}_b \tag{8.48}$$

   or

$$R^a{}_{b\nu\mu} = \partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} + \omega^a{}_{\nu c} \omega^c{}_{\mu b} - \omega^a{}_{\mu c} \omega^c{}_{\nu b} . \tag{8.49}$$

   To establish this link the tetrad postulate is used in the form:

$$\omega^a{}_{\mu b} = q^a{}_\nu q^\lambda{}_b \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_b \partial_\mu q^a{}_\lambda \tag{8.50}$$

to write the spin connection in terms of the gamma connection. The Riemann tensor is defined as [3]– [5]:

$$R^{\sigma}{}_{\lambda\nu\mu} = q^{\sigma}{}_{a} q^{b}{}_{\lambda} R^{a}{}_{b\nu\mu} \tag{8.51}$$

and using the invertibility property of the tetrad tensor [3]:

$$q^{\lambda}{}_{c} q^{c}{}_{\lambda} = 1 \tag{8.52}$$

the Riemann tensor is correctly obtained [9]– [15] as:

$$R^{\sigma}{}_{\lambda\nu\mu} = \partial_\nu \Gamma^{\sigma}{}_{\mu\lambda} - \partial_\mu \Gamma^{\sigma}{}_{\nu\lambda} + \Gamma^{\sigma}{}_{\nu\rho}\Gamma^{\rho}{}_{\mu\lambda} - \Gamma^{\sigma}{}_{\mu\rho}\Gamma^{\rho}{}_{\nu\lambda} \tag{8.53}$$

Q. E. D. Therefore it has been shown that the Riemann form and the Riemann tensor are linked by the tetrad postulate. The Riemann form is defined by the second Cartan structure equation (8.48). The first and second Cartan structure equations are also known as the first and second Maurer - Cartan structure equations [3]. They are true for any type of spin connection.

6. Proof from the First Bianchi Identity.

The first Bianchi identity of differential geometry [3] is:

$$D \wedge T^{a} = R^{a}{}_{b} \wedge q^{b}. \tag{8.54}$$

This condensed notation denotes [9]– [15]:

$$(d \wedge T)^{a}{}_{\mu\nu\rho} = \partial_\mu T^{a}{}_{\nu\rho} + \partial_\nu T^{a}{}_{\rho\mu} + \partial_\rho T^{a}{}_{\mu\nu} \tag{8.55}$$

$$(\omega \wedge T)^{a}{}_{\mu\nu\rho} = \omega^{a}{}_{\mu b} T^{b}{}_{\nu\rho} + \omega^{a}{}_{\nu b} T^{b}{}_{\rho\mu} + \omega^{a}{}_{\rho b} T^{b}{}_{\mu\nu}\,. \tag{8.56}$$

The torsion form is defined as:

$$T^{a}{}_{\mu\nu} = \left(\Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}\right) q^{a}{}_{\lambda}. \tag{8.57}$$

Similarly:

$$R^{a}{}_{b} \wedge q^{b} = \left(R^{\sigma}{}_{\mu\nu\rho} + R^{\sigma}{}_{\nu\rho\mu} + R^{\sigma}{}_{\rho\mu\nu}\right) q^{a}{}_{\sigma}. \tag{8.58}$$

Use of the Leibnitz Theorem and the tetrad postulate in the form:

$$\partial_\mu q^{a}{}_{\sigma} + \omega^{a}{}_{\mu b} q^{b}{}_{\sigma} = \Gamma^{\lambda}{}_{\mu\sigma} q^{a}{}_{\lambda} \tag{8.59}$$

leads correctly [9]– [15] to:

$$\begin{aligned}
&\partial_\mu \Gamma^{\lambda}{}_{\nu\rho} - \partial_\nu \Gamma^{\lambda}{}_{\mu\rho} + \Gamma^{\lambda}{}_{\mu\sigma}\Gamma^{\sigma}{}_{\nu\rho} - \Gamma^{\lambda}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\mu\rho} \\
&+\partial_\nu \Gamma^{\lambda}{}_{\rho\mu} - \partial_\rho \Gamma^{\lambda}{}_{\nu\mu} + \Gamma^{\lambda}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\rho\mu} - \Gamma^{\lambda}{}_{\rho\sigma}\Gamma^{\sigma}{}_{\nu\mu} \\
&+\partial_\rho \Gamma^{\lambda}{}_{\nu\nu} - \partial_\mu \Gamma^{\lambda}{}_{\rho\nu} + \Gamma^{\lambda}{}_{\rho\sigma}\Gamma^{\sigma}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\rho\nu} \\
&= R^{\lambda}{}_{\rho\mu\nu} + R^{\lambda}{}_{\mu\nu\rho} + R^{\lambda}{}_{\nu\rho\mu}
\end{aligned} \tag{8.60}$$

allowing the identification of the Riemann tensor for any gamma connection:

$$R^{\lambda}{}_{\rho\mu\nu} = \partial_{\mu}\Gamma^{\lambda}{}_{\nu\rho} - \partial_{\nu}\Gamma^{\lambda}{}_{\mu\rho} + \Gamma^{\lambda}{}_{\mu\sigma}\Gamma^{\sigma}{}_{\nu\rho} - \Gamma^{\lambda}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\mu\rho} \tag{8.61}$$

Q.E.D. Therefore it has been shown that the tetrad postulate is the necessary and sufficient condition to link the first Bianchi identity (8.54) and the equivalent in Riemann geometry, Eq.(8.60).

7. Proof from the Second Bianchi Identity

The second Bianchi identity of differential geometry is [3, 9]– [15]:

$$\begin{aligned} D \wedge R^{a}{}_{b} &= d \wedge R^{a}{}_{b} + \omega^{a}{}_{c} \wedge R^{c}{}_{b} + \omega^{c}{}_{b} \wedge R^{a}{}_{c} \\ &= 0. \end{aligned} \tag{8.62}$$

Using the results of Proof (7), and using by implication the tetrad postulate again, we correctly obtain [9]– [15] the second Bianchi identity of Riemann geometry:

$$D_{\rho}R^{\kappa}{}_{\sigma\mu\nu} + D_{\mu}R^{\kappa}{}_{\sigma\nu\rho} + D_{\nu}R^{\kappa}{}_{\sigma\rho\mu} = 0 \tag{8.63}$$

Q. E.D. Therefore it has been shown that the tetrad postulate is the necessary and sufficient link between the second Bianchi identity of differential geometry [3] and the second Bianchi identity of Riemann geometry.

8.3 Physical Meaning Of The Metric Compatibility Condition And The Tetrad Postulate

The metric compatibility condition of Riemann geometry means that the metric tensor is covariantly constant [3, 9]– [15]: the covariant derivative of the metric tensor vanishes. If the metric is not covariantly constant then the metric is not compatible. The Einstein Hilbert variation of general relativity (the original 1915 theory) is based on metric compatibility [3,9]– [15]. The theory is accurate for central gravitation of the sun to one part in one hundred thousand [2]. Metric compatibility is used and also the assumption that the torsion tensor vanishes. These assumptions lead to the definition of the Christoffel symbol used by Einstein in his original theory of general relativity. Metric compatibility can also be assumed without the assumption of zero torsion. In this case we obtain the Palatini variation of general relativity in which metric compatibility becomes the tetrad postulate as described in Sections 8.1 and 8.2. The advantages of the Palatini variation are well known and the tetrad postulate has recently been shown to be the geometrical origin of all the wave equations of physics [9]– [15]. In a unified field theory a non-zero torsion form and torsion tensor are always needed to describe the electromagnetic sector. Only when the gravitational and electromagnetic sectors become independent can we use the original Einstein Hilbert variation of gravitational general relativity, with its vanishing torsion tensor and symmetric or Christoffel connection.

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Chapter 9

Derivation Of The Evans Lemma And Wave Equation From The First Cartan Structure Relation

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Abstract

A new theorem of differential geometry is proven: the first Cartan structure equation is the commutator of the tetrad postulate. Conversely the appropriate interchange of base manifold indices in the tetrad postulate gives the first Cartan structure equation. The latter can be written as an equality of two tetrad postulates with reversed indices. Therefore the Evans Lemma and wave equation can be obtained directly from the first Cartan structure equation, which is thereby shown to be the source of all wave equations in generally covariant physics, both relativity and quantum mechanics.

Key words: Cartan structure equation; tetrad postulate; Evans Lemma; Evans wave equation.

9.1 Introduction

Recently a generally covariant unified field theory has been developed which unifies general relativity and quantum mechanics in terms of standard differential geometry [1]– [12]. In this theory the wave equations of physics are derived from the tetrad postulate and the field equations of electrodynamics and gravitation from the Cartan structure equations and the Bianchi identities of differential geometry [13]. In this paper a simple theorem of differential geometry is proven in Section 9.2 which shows that the first Cartan structure equation is an equality of two tetrad postulates. If the appropriate base manifold indices are interchanged in the tetrad postulate, the result is the first Cartan structure equation. The tetrad postulate is the source of the Evans Lemma of differential geometry [1]– [12], an identity which states that scalar curvature is the eigenvalue of the tetrad eigenfunction. The Eigen operator in the Evans Lemma is the d'Alembertian and the tetrad is the fundamental field of the Palatini variation of general relativity [1, 15]. The Lemma is a theorem of differential geometry which serves as the subsidiary proposition that gives the Evans wave equation using the Einstein field equation in index contracted form. The Evans wave equation gives all the well known wave equations of physics in appropriate limits [1]– [12], notably the Dirac equation in the special relativistic limit.

Therefore it is shown in this paper that the source of all the wave equations of quantum mechanics is the first Cartan structure equation itself. The latter is also the source of the tetrad postulate and vice versa. This means that physics is geometry, as inferred by Einstein and many others.

In Section 9.3 an example of this new inference at work is given in the context of the Aharonov Bohm effects, which are described straightforwardly in the Evans unified field theory [1]– [12].

9.2 Proof Of The Theorem

In condensed notation the first Cartan structure equation is

$$D \wedge q^a = T^a \tag{9.1}$$

which is shorthand for

$$d \wedge q^a + \omega^a{}_b \wedge q^b = T^a. \tag{9.2}$$

Reinstating the unwritten [13] indices of the base manifold

$$d \wedge q^a{}_\lambda + \omega^a{}_{\mu b} \wedge q^b{}_\lambda = T^a{}_{\mu\lambda} \tag{9.3}$$

and writing out Eq.(9.3) in full we obtain

$$\partial_\mu q^a{}_\lambda - \partial_\lambda q^a{}_\mu + \omega^a{}_{\mu b} q^b{}_\lambda - \omega^a{}_{\lambda b} q^b{}_\mu = T^a{}_{\mu\lambda}\,. \tag{9.4}$$

In these equations $D\wedge$ is the covariant exterior derivative, q^a is the tetrad one-form and T^a is the torsion two-form of differential geometry. Therefore the first Cartan structure equation states that the torsion form is the covariant exterior derivative of the tetrad form. Eq.(9.4) is Eq.(9.1) written out in full using the spin connection $\omega^a{}_{\mu b}$. In order to define the torsion form correctly the spin connection must be identically non-zero. It is seen from Eq.(9.4) that the torsion form $T^a{}_{\mu\nu}$ is defined in terms of commutators or wedge products.

Now express the torsion form in terms of the torsion tensor $T^\nu{}_{\mu\lambda}$ of Riemann geometry [13]:

$$T^\nu{}_{\mu\lambda} = \Gamma^\nu{}_{\mu\lambda} - \Gamma^\nu{}_{\lambda\mu}. \tag{9.5}$$

The relation between the torsion form $T^a{}_{\mu\lambda}$ of differential geometry and the torsion tensor $T^\nu{}_{\mu\lambda}$ of Riemann geometry is [13]

$$T^a{}_{\mu\lambda} = T^\nu{}_{\mu\lambda}\, q^a{}_\nu \tag{9.6}$$

i.e.

$$T^a{}_{\mu\lambda} = \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \Gamma^\nu{}_{\lambda\mu} q^a{}_\nu. \tag{9.7}$$

From Eqs.(9.4) and (9.7):

$$\begin{aligned} \partial_\mu q^a{}_\lambda - \partial_\lambda q^a{}_\mu + \omega^a{}_{\mu b} q^b{}_\lambda - \omega^a{}_{\lambda b} q^b{}_\mu \\ = \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \Gamma^\nu{}_{\lambda\mu} q^a{}_\nu. \end{aligned} \tag{9.8}$$

Eq.(9.8) is the difference of two tetrad postulates [1]– [15]:

$$\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda = \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu \tag{9.9}$$

and

$$\partial_\lambda q^a{}_\mu + \omega^a{}_{\lambda b} q^b{}_\mu = \Gamma^\nu{}_{\lambda\mu} q^a{}_\nu. \tag{9.10}$$

Eqs.(9.9) and (9.10) are respectively:

$$D_\mu q^a{}_\lambda = 0 \tag{9.11}$$

and

$$D_\lambda q^a{}_\mu = 0. \tag{9.12}$$

Therefore the first Cartan structure equation is a commutator of two tetrad postulates

$$D_\mu q^a{}_\lambda - D_\lambda q^a{}_\mu = 0 \tag{9.13}$$

i.e.

$$D_\mu q^a{}_\lambda = D_\lambda q^a{}_\mu = 0 \tag{9.14}$$

Q.E.D.

The Evans Lemma is obtained [1]– [12] from the identity:

$$D^\mu D_\mu q^a{}_\lambda = D^\lambda D_\lambda q^a{}_\mu := 0 \tag{9.15}$$

and so the Lemma is obtained directly from the first Cartan structure equation. The Lemma is the subsidiary geometrical proposition:

$$\Box q^a{}_\lambda = R q^a{}_\lambda \tag{9.16}$$

where R is a the scalar curvature defined in the Einstein field equation [13]. The index contracted form of the latter equation is [13]:

$$R = -kT \tag{9.17}$$

where T is the index contracted canonical energy-momentum tensor and where k is the Einstein constant. Note that Eq.(9.17) is valid for all radiated and matter fields [16] not just gravitation. Using Eq.(9.17) in Eq.(9.16) gives the Evans wave equation of generally covariant unified field theory [1]– [12]:

$$(\Box + kT)\, q^a{}_\mu = 0. \tag{9.18}$$

The source of the Evans wave equation has therefore been shown in this paper to be the first Cartan structure equation itself.

9.3 Application To The Class Of Aharonov Bohm Effects

The fundamental ansatz that transforms from geometry to physics in the unified field theory is [1]– [12]:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu \tag{9.19}$$

where $A^a{}_\mu$ is the vector potential magnitude. Similarly the electromagnetic field tensor follows from Eq.(9.19):

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu}\,. \tag{9.20}$$

Thus, the electromagnetic potential $A^a{}_\mu$ is the tetrad form within a premultiplier $A^{(0)}$,and the electromagnetic field is the torsion form within the same premultiplier $A^{(0)}$. Using the the ansatz (9.19) the first Cartan structure equation gives the relation between field and potential in two ways. Firstly

$$F^a{}_{\mu\nu} = (d \wedge A^a)_{\mu\nu} + \omega^a{}_{\mu b} \wedge A^b{}_\nu \tag{9.21}$$

and secondly, using Eq.(9.6):

$$F^a{}_{\mu\nu} = T^\rho{}_{\mu\nu}\, A^a{}_\rho. \tag{9.22}$$

The class of Aharonov Bohm effects have been explained straightforwardly [1]– [12] using Eq.(9.21) as being due to the term $\omega^a{}_{\mu b} \wedge A^b{}_\nu$. This term is also responsible for the Evans spin field [1]– [12] and is the origin of polarization and magnetization [1]– [12]. In simple analogy, the iron whisker of the Chambers experiment, for example, acts as a stirring rod, and sets up a whirlpool of spacetime in its vicinity, i.e. in regions where the magnetic field does not exist, the term $\omega^a{}_{\mu b} \wedge A^b{}_\nu$ results in the observed electron diffraction fringe shift. This explanation means that there exists a hitherto unobserved electromagnetic

Aharonov Bohm effect due to the Evans spin field [1]– [12]. This is a close relative of the inverse Faraday effect and would be of probable interest for RADAR and stealth technology.

The additional inference into the Aharonov Bohm effects given by the new geometrical theorem of this paper is summarized in Eq.(9.22), which shows that the electromagnetic field is the inner product of the torsion tensor and the electromagnetic potential. The torsion tensor vanishes in the Maxwell Heaviside field theory, because the latter is constructed in a flat or Minkowski spacetime, but Eq.(9.22) is generally covariant as required by relativity theory. The combined result of Eqs.(9.21) and (9.22) is therefore:

$$F^a{}_{\mu\nu} = T^\rho{}_{\mu\nu} A^a{}_\rho = (d \wedge A^a)_{\mu\nu} + \omega^a{}_{\mu b} \wedge A^b{}_\nu \tag{9.23}$$

and shows that the Aharonov Bohm effects are due to a term $\omega^a{}_{\mu b} \wedge A^b{}_\nu$ which does not exist in Maxwell Heaviside theory and does not exist in the standard model. Nevertheless this term is the result of a rigorously objective theory of electromagnetism based on general relativity, not special relativity. This is the generally covariant unified field theory, which is therefore preferred experimentally and philosophically over the Maxwell Heaviside field theory.

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Chapter 10

Proof Of The Evans Lemma From The Tetrad Postulate

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Abstract

Two rigorous proofs of the Evans Lemma are developed from the fundamental tetrad postulate of differential geometry. Both proofs show that the Lemma is the subsidiary geometrical proposition upon which is based the Evans wave equation. The latter is the source of all wave equations in physics and generally covariant quantum mechanics.

Key words: Evans lemma; tetrad postulate; unified field theory; Evans wave equation.

10.1 Introduction

Recently a generally covariant unified field theory has been developed [1]– [12], a theory which is based rigorously on standard differential geometry. The basic theorems of standard differential geometry [13]– [15] include the Cartan structure relations, the Bianchi identities, and the tetrad postulate. Recently it has been proven [16] that the first Cartan structure equation is an equality of two tetrad postulates. Cartan geometry seems to be entirely sufficient for a unified field theory based on Einstein's idea that physics is geometry. This is the fundamental idea of relativity theory - that all physics must be both objective to all observers and rigorously causal. This includes quantum mechanics, and it has recently been shown experimentally in many ways, summarized in ref [10], that the Heisenberg uncertainty principle is incorrect qualitatively. The generally covariant unified field theory [1]– [12] suggests a causal quantum mechanics based on differential geometry, and suggests a development [10] of the Heisenberg uncertainty principle to make it compatible with experimental data.

Section 10.2 gives an outline of the advantages of Cartan geometry over Riemann geometry in the development of a unified field theory. Section 10.3 gives the first proof of the Lemma, and this is followed in Section 10.4 by a second proof which reveals the existence of a subsidiary condition.

10.2 Advantages Of Cartan Geometry

Without Cartan geometry it is much more difficult, if not impossible, to develop an objective unified field theory. The reason is that the fundamental structure equations of Cartan, and the fundamental Bianchi identities, are easily recognized as having the structure of generally covariant electromagnetic theory. This is by no means clear in Riemann geometry. To illustrate the advantage of Cartan geometry we discuss the four fundamental equations below, first in Cartan geometry and then in Riemann geometry.

The first Cartan structure equation gives the relation between the electromagnetic field and the electromagnetic potential [1]– [12]. In Cartan geometry it is:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{10.1}$$

and in Riemann geometry it is:

$$T^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\mu\nu} - \Gamma^\kappa{}_{\nu\mu}. \tag{10.2}$$

Here T^a is the torsion form, q^a is the tetrad form, $\omega^a{}_b$ is the spin connection, $T^\kappa{}_{\mu\nu}$ is the torsion tensor and $\Gamma^\kappa{}_{\mu\nu}$ is the gamma connection of Riemann geometry. Eq.(10.1) in generally covariant electromagnetic theory [1]– [12] becomes:

$$F^a = D \wedge A^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{10.3}$$

where F^a is the electromagnetic field two-form and A^a is the electromagnetic potential. In the special relativistic limit Eq.(10.3) reduces to the familiar relation between field and potential in Maxwell Heaviside field theory [17]:

$$F = d \wedge A. \tag{10.4}$$

It is seen by comparison of Eq.(10.1) and (10.3) that Cartan geometry leads almost directly to the correctly and generally covariant theory of electrodynamics, Eq.(10.3). However the equivalent of Eq.(10.1) in Riemann geometry, Eq.(10.2), leads to no such inference.

The field equations of electrodynamics in the Evans unified field theory are based on the first Bianchi identity of differential geometry, which in its most condensed form may be written as:

$$D \wedge T^a = R^a{}_b \wedge q^b. \tag{10.5}$$

Eq.(10.5) is equivalent to:

$$d \wedge T^a = - \left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b \right). \tag{10.6}$$

Here $R^a{}_b$ is the Riemann form [1]– [15]. Using the Evans ansatz

$$A^a = A^{(0)} q^a \tag{10.7}$$

Eq.(10.6) becomes the homogeneous field equation of generally covariant electrodynamics:

$$d \wedge F^a = -A^{(0)} \left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b \right) = \mu_0 j^a \tag{10.8}$$

where j^a is the homogeneous current. Eq.(10.8) leads to the Faraday law of induction and the Gauss law of magnetism when j^a is very small:

$$j^a \sim 0. \tag{10.9}$$

Eq.(10.9) in turn leads to the free space condition:

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b \tag{10.10}$$

one possible solution of which is circular polarization [1]– [12].

None of these inferences are clear, however, from the equivalent of Eq.(10.5) in Riemann geometry [1]– [15]:

$$\begin{aligned} R^\lambda{}_{\rho\mu\nu} = \partial_\mu \Gamma^\lambda{}_{\nu\rho} - \partial_\nu \Gamma^\lambda{}_{\mu\rho} + \Gamma^\lambda{}_{\mu\sigma} \Gamma^\sigma{}_{\nu\sigma} - \Gamma^\lambda{}_{\nu\sigma} \Gamma^\sigma{}_{\mu\rho} \\ et \quad cyclicum \end{aligned} \tag{10.11}$$

and so it would be difficult if not impossible to construct a unified field theory from Riemann geometry.

Cartan geometry also helps to clarify and simplify the structure of Einstein's original gravitational theory. This is accomplished using the second Cartan structure equation and the second Bianchi identity of Cartan geometry. The former is:

$$R^a{}_b = D \wedge \omega^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{10.12}$$

and the latter is:

$$D \wedge R^a{}_b = d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b + \omega^c{}_b \wedge R^a{}_c \quad = 0. \tag{10.13}$$

Eq.(10.12) bears an obvious similarity to Eq.(10.1) and again this is indicative of the fact that electromagnetism and gravitation are parts of a unified field based on Cartan geometry. The equivalent of Eq.(10.12) in Riemann geometry is, however:

$$R^\sigma{}_{\lambda\nu\mu} = \partial_\nu \Gamma^\sigma{}_{\mu\lambda} - \partial_\mu \Gamma^\sigma{}_{\nu\lambda} + \Gamma^\sigma{}_{\nu\rho} \Gamma^\rho{}_{\mu\lambda} - \Gamma^\sigma{}_{\nu\rho} \Gamma^\rho{}_{\nu\lambda} \tag{10.14}$$

and there is no resemblance to Eq.(10.2), the first Cartan structure equation written in terms of Riemann geometry. The same is true of the second Bianchi identity (10.13) of Cartan geometry, which is closely similar to the first Bianchi identity (10.5). However, the equivalent of Eq.(10.13) in Riemann geometry is:

$$D_\rho R^\kappa{}_{\sigma\mu\nu} + D_\mu R^\kappa{}_{\sigma\nu\rho} + D_\nu R^\kappa{}_{\sigma\rho\mu} = 0 \tag{10.15}$$

and is entirely different in structure form Eq.(10.11), the first Bianchi identity in Riemann geometry.

10.3 First Proof Of The Evans Lemma From The Tetrad Postulate

This first proof of the Evans Lemma [1]– [12] is based on the tetrad postulate of Cartan geometry [13]– [16], which is proven in eight ways in refs. [12] and [16]. The proof in ref (10.16) is a particularly clear demonstration of the fundamental nature of the tetrad postulate, because it is basically a restatement of the first Cartan structure equation without which there would be no Cartan geometry. The tetrad postulate is:

$$D_\mu q^a{}_\nu = 0 \tag{10.16}$$

where D_μ denotes covariant derivative [13] and may be thought of as the equivalent in Cartan geometry of the metric compatibility condition of Riemann geometry [1]– [16]. The metric compatibility condition has been tested experimentally to one part in one hundred thousand [16] by the NASA Cassini experiments designed to test the original 1915 theory of general relativity. The latter is based on the metric compatibility condition and the concomitant relation between the symmetric Christoffel symbol and the symmetric metric. However, the tetrad postulate is more generally applicable and is valid for all types of connection [1]– [16].

The Evans Lemma [1]– [12] is obtained from the identity:

$$D^\mu \left(D_\mu q^a{}_\nu\right) := 0 \tag{10.17}$$

i.e. from:

$$D^\mu (0) := 0. \tag{10.18}$$

Eq (10.18) implies [1]– [13] that:

$$\partial^\mu (0) := 0 \tag{10.19}$$

and so we obtain:

$$\partial^\mu \left(D_\mu q^a{}_\nu\right) := 0 \tag{10.20}$$

or

$$\partial^\mu \left(\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu\right) := 0 \tag{10.21}$$

where $\omega^a{}_{\mu b}$ is the spin connection and $\Gamma^\nu{}_{\mu\lambda}$ is the general gamma connection [1]– [16]. The specialized Christoffel connection of the 1915 theory is obtained when the gamma connection is symmetric in its lower two indices; in order to develop a unified field theory we need the most general gamma connection, which is asymmetric in its lower two indices. Thus, for a given upper index, the most general gamma connection is a sum of a gamma connection symmetric in its lower two indices and a gamma connection antisymmetric in its lower two indices. The former is the Christoffel connection of gravitation and the latter is the gamma connection for electromagnetism [1]– [16].

To obtain the Evans Lemma rewrite Eq.(10.21) as follows:

$$\Box q^a{}_\mu = \partial^\mu \left(\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) := R q^a{}_\mu \tag{10.22}$$

where

$$R = q^\lambda{}_a \partial^\mu \left(\Gamma^\nu{}_{\mu\lambda} q^a{}_\mu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{10.23}$$

is scalar curvature, with units of inverse square metres. The Evans Lemma is therefore the prototypical wave equation of Cartan geometry:

$$\Box q^a{}_\mu = R q^a{}_\mu \tag{10.24}$$

Q.E.D.

Now consider the Einstein field equation [13]:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\nu\mu} \tag{10.25}$$

where $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the symmetric metric tensor, $T_{\mu\nu}$ is the symmetric canonical energy momentum tensor, and k is the Einstein constant. Multiply both sides of Eq. (10.25) by $g^{\mu\nu}$ and define the following scalars by index contraction:

$$R = g^{\mu\nu} R_{\mu\nu} \tag{10.26}$$

$$T = g^{\mu\nu} T_{\mu\nu} \tag{10.27}$$

Use:

$$g^{\mu\nu} g_{\mu\nu} = 4 \tag{10.28}$$

to obtain:

$$R = -kT. \tag{10.29}$$

From Eq.(10.24) and (10.29) we obtain the Evans wave equation [1]– [12]:

$$\left(\Box + kT \right) q^a{}_\mu = 0 \tag{10.30}$$

where:

$$R = -kT = g^{\mu\nu} R_{\mu\nu} = q^\lambda{}_a \partial^\mu \left(\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{10.31}$$

From the correspondence principle the Dirac equation is obtained in the limit:

$$kT \longrightarrow \left(\frac{mc}{\hbar} \right)^2 \tag{10.32}$$

with

$$T = \frac{m}{V_0} \tag{10.33}$$

giving the volume of the elementary particle in its rest frame:

$$V_0 = \frac{\hbar^2 k}{mc^2} = \frac{\hbar^2 k}{En_0} \tag{10.34}$$

This procedure also shows that the Dirac spinor is derived from the tetrad, which is the fundamental field [1]– [16] of the Palatini variation of general relativity [13]– [15]. In the unified field theory [1]– [12, 16] the tetrad is the fundamental field for all radiated and matter fields, (gravitation, electromagnetism, weak and strong fields and particle fields).

The Lemma is therefore a subsidiary condition based on Cartan geometry, a condition that leads via Eq.(10.29) to the Evans wave equation of physics. The various fields of physics are all defined by various tetrads [1]– [12], and all sectors of the unified field theory are generally covariant and rigorously objective. The need for general covariance in electrodynamics for example introduces the spin connection into electrodynamics, and with it the Evans spin field observed in the inverse Faraday effect [1]– [12]. Thereby electrodynamics is recognized as spinning spacetime, gravitation as curving spacetime. The two fields interact when the homogeneous current j^a is non zero [1]– [12]. A non-zero j^a however implies the experimental violation of the Faraday Law of induction and Gauss law of magnetism, and within contemporary experimental precision, this has not been observed in the laboratory.

10.4 Second Proof Of The Evans Lemma From The Tetrad Postulate

Rewrite Eq.(10.17) as:

$$D^\mu \left(\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu\right) = 0 \tag{10.35}$$

and use the Leibnitz Theorem for covariant derivatives [13] to obtain:

$$\begin{aligned}(D^\mu \partial_\mu) q^a{}_\lambda + \partial_\mu (D^\mu q^a{}_\lambda) + \left(D^\mu \omega^a{}_{\mu b}\right) + \omega^a{}_{\mu b} \left(D^\mu q^b{}_\lambda\right) \\ - \left(D^\mu \Gamma^\nu{}_{\mu\lambda}\right) q^a{}_\nu - \Gamma^\nu{}_{\mu\lambda} (D^\mu q^a{}_\nu) = 0.\end{aligned} \tag{10.36}$$

Using Eq.(10.16) in Eq. (10.36) we obtain:

$$(D^\mu \partial_\mu) q^a{}_\lambda + \left(D^\mu \omega^a{}_{\mu b}\right) q^b{}_\lambda - \left(D^\mu \Gamma^\nu{}_{\mu\lambda}\right) q^a{}_\nu = 0. \tag{10.37}$$

Now use:

$$D^\mu \partial_\mu = \partial^\mu \partial_\mu + \dots \tag{10.38}$$

$$D^\mu \omega^a{}_{\mu b} = \partial^\mu \omega^a{}_{\mu b} + \dots \tag{10.39}$$

$$D^\mu \Gamma^\nu{}_{\mu\lambda} = \partial^\mu \Gamma^\nu{}_{\mu\lambda} + \dots \tag{10.40}$$

to rewrite Eq.(10.37) as

$$\partial^\mu \left(\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu\right) + \dots = 0 \tag{10.41}$$

i.e.

$$\partial^{\mu}\left(D_{\mu}q^{a}{}_{\lambda}\right)+\ldots=0 \tag{10.42}$$

By comparison of Eq.(10.20) and (10.42) it is seen that there must be a subsidiary condition which ensures that the remainder term on the left hand side of Eq.(10.42) be zero. This condition provides a useful analytical constraint on the geometry of the Lemma.

In order to proceed we define the covariant derivatives:

$$D_{\mu}V^{\nu}=\partial_{\mu}V^{\nu}+\Gamma^{\nu}{}_{\mu\lambda}V^{\lambda} \tag{10.43}$$

$$D_{\mu}V_{\nu}=\partial_{\mu}V_{\nu}-\Gamma_{\nu\mu}{}^{\lambda}V_{\lambda} \tag{10.44}$$

Therefore if

$$V^{\nu}=0 \tag{10.45}$$

then

$$V^{0}=V^{1}=V^{2}=V^{3}=0 \tag{10.46}$$

and in Eq.(10.46):

$$\begin{aligned} D_{\mu}V^{0}&=\partial_{\mu}V^{0}+\Gamma^{\nu}{}_{\mu\lambda}V^{\lambda}\\ &\vdots\\ D_{\mu}V^{3}&=\partial_{\mu}V^{3}+\Gamma^{\nu}{}_{\mu\lambda}V^{\lambda}. \end{aligned} \tag{10.47}$$

However:

$$V^{\lambda}=0,\lambda=0,1,2,3 \tag{10.48}$$

which implies

$$\begin{aligned} D_{\mu}V^{0}&=\partial_{\mu}V^{0}=0\\ &\vdots\\ D_{\mu}V^{3}&=\partial_{\mu}V^{3}=0 \end{aligned} \tag{10.49}$$

and implies the subsidiary condition:

$$\Gamma^{\nu}{}_{\mu\lambda}V^{\lambda}=0. \tag{10.50}$$

The same reasoning applies to the subsidiary condition in Eq.(10.42). Therefore when all components of a vector or tensor are zero:

$$D^{\mu}D_{\mu}=\partial^{\mu}D_{\mu}. \tag{10.51}$$

The components of a vector or tensor are not scalars, so in general D^{μ} acting on a vector component or tensor component is not the same as ∂^{μ} acting on the same component. This is clear from Eqs.(10.43) and (10.44).

The covariant divergence is now defined from the expression for the covariant divergence of a vector:

$$D_{\mu}V^{\mu}=\partial_{\mu}V^{\mu}+\Gamma^{\mu}{}_{\mu\lambda}V^{\lambda} \tag{10.52}$$

i.e.:

$$D_\mu \partial^\mu = \partial_\mu \partial^\mu + \Gamma^\mu{}_{\mu\lambda} \partial^\lambda. \tag{10.53}$$

Rewriting dummy indices inside the connection:

$$D_\mu \partial^\mu = \Box + \Gamma^\nu{}_{\nu\mu} \partial^\mu. \tag{10.54}$$

The wave equation (10.37) is therefore:

$$\left(\Box + \Gamma^\nu{}_{\nu\mu} \partial^\mu\right) q^a{}_\lambda - R_1 q^a{}_\lambda = 0 \tag{10.55}$$

where:

$$-R_1 q^a{}_\lambda := \left(D^\mu \omega^a{}_{\mu b}\right) q^b{}_\lambda - \left(D^\mu \Gamma^\nu{}_{\mu\lambda}\right) q^a{}_\nu. \tag{10.56}$$

Using the tetrad postulate:

$$\partial_\mu q^a{}_\lambda = -\omega^a{}_{\mu b} q^b{}_\lambda + \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu \tag{10.57}$$

so

$$\partial^\mu q^a{}_\lambda = -\omega^{\mu a}{}_b q^b{}_\lambda + \Gamma^{\mu\nu}{}_\lambda q^a{}_\nu. \tag{10.58}$$

Therefore in Eq.(10.55):

$$\Box q^a{}_\lambda - \Gamma^\nu{}_{\nu\mu} \omega^{\mu a}{}_b q^b{}_\lambda + \Gamma^\nu{}_{\nu\mu} \Gamma^{\mu\nu}{}_\lambda q^a{}_\nu - R_1 q^a{}_\lambda. \tag{10.59}$$

Finally define:

$$-R_2 q^a{}_\lambda := -\Gamma^\nu{}_{\nu\mu} \omega^{\mu a}{}_b q^b{}_\lambda + \Gamma^\nu{}_{\nu\mu} \Gamma^{\mu\nu}{}_\lambda q^a{}_\nu \tag{10.60}$$

to obtain the Evans Lemma:

$$\begin{aligned} \Box q^a{}_\lambda &= R q^a{}_\lambda, \\ R &= R_1 + R_2, \end{aligned} \tag{10.61}$$

Q.E.D.

10.5 Discussion

The Evans lemma is an eigenequation for R, and is accompanied by Eq.(10.23), which is a subsidiary condition for R. These geometrical equations provide a source for all the wave equations of physics in general relativity. Such a concept does not exist in the standard model, where quantum mechanics and general relativity are mutually incompatible. Develop Eq.(10.23) using [13]:

$$\partial^\mu = g^{\mu\sigma} \partial_\sigma \tag{10.62}$$

to give:

$$R = g^{\mu\sigma} q^\lambda{}_a \left(\Gamma^\nu{}_{\mu\lambda} \partial_\sigma q^a{}_\nu + q^a{}_\nu \partial_\sigma \Gamma^\nu{}_{\mu\lambda} - \omega^a{}_{\mu b} \partial_\sigma q^b{}_\lambda - q^b{}_\lambda \partial_\sigma \omega^a{}_{\mu b}\right). \tag{10.63}$$

Now use the tetrad postulates:

$$\partial_\sigma q^a{}_\nu + \omega^a{}_{\sigma b} q^b{}_\nu - \Gamma^\rho{}_{\sigma\nu} q^a{}_\rho = 0, \tag{10.64}$$

$$\partial_\sigma q^b{}_\lambda + \omega^b{}_{\sigma c} q^c{}_\lambda - \Gamma^\nu{}_{\sigma\lambda} q^a{}_\nu = 0, \tag{10.65}$$

to find:

$$\begin{aligned} R = g^{\mu\sigma} \big(& q^\lambda{}_a q^a{}_\rho \Gamma^\nu{}_{\mu\lambda} \Gamma^\rho{}_{\sigma\nu} - q^\lambda{}_a q^b{}_\nu \Gamma^\nu{}_{\mu\lambda} \omega^a{}_{\sigma b} \\ & + q^\lambda{}_a q^a{}_\nu \partial_\sigma \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_a q^b{}_\nu \omega^a{}_{\mu b} \Gamma^\nu{}_{\sigma\lambda} \\ & + q^\lambda{}_a q^c{}_\lambda \omega^a{}_{\mu b} \omega^b{}_{\sigma c} - q^\lambda{}_a q^b{}_\lambda \partial_\sigma \omega^a{}_{\mu b} \big) . \end{aligned} \tag{10.66}$$

Eliminate the tetrads using:

$$R^\sigma{}_{\lambda\nu\mu} = q^\sigma{}_a q^b{}_\lambda R^a{}_{b\nu\mu} \tag{10.67}$$

$$R^a{}_{b\nu\mu} = q^a{}_\sigma q^\lambda{}_b R^\sigma{}_{\lambda\nu\mu} \tag{10.68}$$

$$R^a{}_{b\nu\mu} = \partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} + \omega^a{}_{\nu c} \omega^c{}_{\mu b} - \omega^a{}_{\mu c} \omega^c{}_{\nu b} \tag{10.69}$$

$$R^\sigma{}_{\lambda\nu\mu} = \partial_\nu \Gamma^\sigma{}_{\mu\lambda} - \partial_\mu \Gamma^\sigma{}_{\nu\lambda} + \Gamma^\sigma{}_{\nu\rho} \Gamma^\rho{}_{\mu\lambda} - \Gamma^\sigma{}_{\mu\rho} \Gamma^\rho{}_{\nu\lambda} . \tag{10.70}$$

The Riemann tensor $R^\sigma{}_{\lambda\nu\mu}$ and the Riemann form $R^a{}_{b\nu\mu}$ are antisymmetric respectively in σ and λ and in a and b. Using this antisymmetry Eq.(10.66) reduces to:

$$R = -g^{\mu\sigma} q^\lambda{}_a q^b{}_\nu \left(\Gamma^\nu{}_{\mu\lambda} \omega^a{}_{\sigma b} + \omega^a{}_{\mu b} \Gamma^\nu{}_{\sigma\lambda} \right) . \tag{10.71}$$

Now simplify and remove the tetrads using:

$$q^\lambda{}_a \Gamma^\nu{}_{\mu\lambda} = \Gamma^\nu{}_{\mu a} \tag{10.72}$$

$$q^b{}_\nu \omega^a{}_{\sigma b} = \omega^a{}_{\sigma\nu} \tag{10.73}$$

$$q^\lambda{}_a \omega^a{}_{\mu b} = \omega^\lambda{}_{\mu b} \tag{10.74}$$

$$q^b{}_\nu \Gamma^\nu{}_{\sigma\lambda} = \Gamma^b{}_{\sigma\lambda} \tag{10.75}$$

to give:

$$R = -g^{\mu\sigma} \left(\Gamma^\nu{}_{\mu a} \omega^a{}_{\sigma\nu} + \omega^\lambda{}_{\mu b} \Gamma^b{}_{\sigma\lambda} \right) . \tag{10.76}$$

Finally re-arrange dummy indices $b \to a, \lambda \to \nu$ to give:

$$R = -g^{\mu\sigma} \left(\Gamma^\nu{}_{\mu a} \omega^a{}_{\sigma\nu} + \omega^\nu{}_{\mu b} \Gamma^a{}_{\sigma\nu} \right) . \tag{10.77}$$

Eq.(10.77) is the required generalization to unified field theory of the equation for the scalar curvature R used in the original 1915 theory of gravitational general relativity, i.e. is the generalization of

$$R = g^{\mu\sigma} R_{\mu\sigma} \tag{10.78}$$

where $R_{\mu\sigma}$ is the Ricci tensor.

Comparing Eqs.(10.77) and (10.78):

$$R_{\mu\sigma} = - \left(\Gamma^\nu{}_{\mu a} \omega^a{}_{\sigma\nu} + \omega^\nu{}_{\mu a} \Gamma^a{}_{\sigma\nu} \right) \tag{10.79}$$

is the Ricci tensor in the unified field theory. Using Eq.(10.29) it is found that:

$$kT = g^{\mu\sigma} \left(\Gamma^\nu{}_{\mu a} \omega^a{}_{\sigma\nu} + \omega^\nu{}_{\mu a} \Gamma^a{}_{\sigma\nu} \right) \tag{10.80}$$

Eq.(10.80) is the subsidiary condition for the Evans wave equation (10.30), which in general relativity must be solved simultaneously with Eq.(10.80). In the limit of special relativity however, there is only one equation to solve - the Dirac equation - because:

$$g^{\mu\sigma}\left(\Gamma^{\nu}{}_{\mu a}\omega^{a}{}_{\sigma\nu}+\omega^{\nu}{}_{\mu a}\Gamma^{a}{}_{\sigma\nu}\right)=\left(\frac{mc}{\hbar}\right)^{2}. \tag{10.81}$$

In conclusion the Evans lemma and wave equation have been derived rigorously form Cartan geometry and the Einstein equation (10.29). As inferred by Einstein (10.17) the latter must be interpreted as being valid for all fields, not only the gravitational field. Eq.(10.29) is more fundamental than the more well known Einstein field equation (10.25) because various field equation can be constructed [1]– [12] from Eq. (10.29).

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Chapter 11

Self Consistent Derivation Of The Evans Lemma And Application To The Generally Covariant Dirac Equation

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Abstract

The self consistency of two derivations of the Evans Lemma is demonstrated rigorously and the Evans wave equation derived therefrom. The wave equation is reduced to the Dirac equation in the appropriate limit and the meaning discussed of the generally covariant Dirac and Pauli spinors. The effect of gravitation on particle physics may be investigated with the Evans equation.

Key words : Evans unified field theory, lemma and wave equation; generally covariant Dirac equation; effect of gravitation on particle physics.

11.1 Introduction

Recently a generally covariant unified field theory has been developed [1]–[14]which gives a plausible description of all radiated and matter fields in terms of the tetrad. The latter is the fundamental field of the Palatini variation of general relativity [15]– [17]. The Evans lemma [3, 4] is an identity of Cartan geometry which is the subsidiary proposition to the Evans wave equation. The latter unifies general relativity and quantum mechanics and is a wave equation of causal and objective physics [3, 4]. It has been demonstrated experimentally [11,12] beyond reasonable doubt that the Heisenberg uncertainty principle is an intellectual aberration, so should be abandoned in favor of a causal and generally covariant interpretation [1]– [14] of quantum mechanics based on Cartan geometry. The lemma and wave equation are therefore fundamentally important to a consistent interpretation of natural philosophy as an objective subject in which every event has a cause. It is therefore necessary to demonstrate the lemma rigorously in more than one way, and to demonstrate its geometrical self consistency.

In Section 11.2 the self consistency of the Lemma is demonstrated with two independent methods. In Section 11.3 the Lemma is transformed into a wave equation using the index contracted form of the Einstein field equation, and the resulting equation reduced to the single particle Dirac equation in the appropriate limit, proving that the origin of the Dirac four spinor is Cartan geometry. The effect of gravitation on the single particle Dirac equation can therefore be calculated from the generally covariant single particle Dirac equation derived directly from the Evans wave equation.

11.2 Geometrical Self Consistency Of The Evans Lemma

The lemma is an identity which is derived from the standard tetrad postulate of Cartan geometry [18]:

$$D_\mu q^a{}_\nu = 0. \tag{11.1}$$

Covariant differentiation of Eq.(11.1) gives the identity:

$$D^\mu \left(D_\mu q^a{}_\nu\right) := 0. \tag{11.2}$$

The covariant derivative is defined [18] such that:

$$D^\mu \left(\phi\right) = \partial^\mu \left(\phi\right) \tag{11.3}$$

where ϕ is a scalar. Thus for every scalar element defined in Eq.(11.1), Eq.(11.3) applies. It follows that:

$$\partial^\mu \left(D_\mu q^a{}_\nu\right) := 0. \tag{11.4}$$

The tetrad postulate is expanded out next as follows [18]:

$$D_\mu q^a{}_\lambda = \partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu = 0 \tag{11.5}$$

where $\omega^a{}_{\mu b}$ is the spin connection and where $\Gamma^\nu{}_{\mu\lambda}$ is the gamma connection for a spacetime both with curvature and torsion. Using the inverse tetrad relation:

$$q^a{}_\mu q^\mu{}_a = 1 \tag{11.6}$$

it follows directly from Eq.(11.4) that:

$$\Box q^a{}_\mu = R q^a{}_\mu \tag{11.7}$$

where

$$R = q^\lambda{}_a \partial^\mu \left(\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{11.8}$$

and where the d'Alembertian operator is defined by:

$$\Box = \partial^\mu \partial_\mu . \tag{11.9}$$

Eq.(11.7) is the lemma, or subsidiary geometrical proposition, that leads to the Evans wave equation. It is a simple identity of Cartan geometry and is the structure that leads directly to the generally covariant, causal and thus objective wave equations of physics. From the tetrad postulate (11.1):

$$\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda = \partial_\mu q^a{}_\lambda \tag{11.10}$$

and using Eq.(11.10) in Eq.(11.8) it is found self-consistently that:

$$R = q^\lambda{}_a \partial^\mu \left(\partial_\mu q^a{}_\lambda \right) = q^\lambda{}_a \Box q^a{}_\lambda \tag{11.11}$$

which leads back self consistently to Eq. (11.7) upon use of Eq.(11.6). It has therefore been shown that the most basic structure of Cartan geometry is the wave equation (11.7). This wave equation of geometry is the source of quantum mechanics in physics. The importance of the lemma is therefore clear, it indicates that all of physics is derived from Cartan geometry. Geometry is transformed into physics using:

$$R = -kT. \tag{11.12}$$

Eq.(11.12) is the most fundamental equation of relativity, and is the simplest way in which geometry can be translated into physics via the scalar energy-momentum density T and the Einstein constant k. Here R is the scalar curvature in inverse square meters. Eq.(11.12) applies to all radiated and matter fields as intended originally by Einstein himself [19]. Not only can we recover the Einstein Hilbert field equation from Eq.(11.12) but also a number of other field equations [1]– [14]. The Einstein Hilbert field equation is derived from the second Bianchi identity of Riemann geometry on the assumption [18] of the Christoffel connection which is symmetric in its lower two indices. This assumption implies that the torsion tensor is zero. Therefore the Einstein Hilbert field theory assumes that there is spacetime curvature but no spacetime torsion. In some circumstances this assumption is perfectly adequate, for example for the sun (Cassini experiments at NASA, 2002 to present), but in other circumstances it is well known that there are cosmological anomalies [20], some of them appear to be very large anomalies. So the Einstein Hilbert field equation appears to be only partially successful in a cosmological context when we take all the data into account.

In the Evans field theory on the other hand curvature and torsion are both present in general [1]– [14] and the connection is the spin connection of the Palatini variation of general relativity in which the fundamental field is the tetrad and not the symmetric metric of the EinsteinHilbert field theory. The symmetric metric is the dot product of two tetrads, as is well known [18]:

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{ab} \tag{11.13}$$

where η_{ab} is the Minkowski metric of the tangent spacetime. It follows immediately that there always exists an antisymmetric metric - the wedge product of two tetrads:

$$g^c{}_{\mu\nu} = q^a{}_\mu \wedge q^b{}_\nu . \tag{11.14}$$

The antisymmetric metric is a vector valued two-form of differential geometry. The most general metric is the outer product of two tetrads [1]– [14]. The outer product is a matrix, and therefore can always be written [21] as the sum of a symmetric and antisymmetric matrix. The trace of the symmetric matrix is essentially the dot product and the antisymmetric traceless part is essentially the cross product. A simple example is vector analysis in three dimensional Euclidean space. If the dot product $\mathbf{A}\cdot\mathbf{B}$ is defined of two vectors, we can always define a cross product $\mathbf{A}\times\mathbf{B}$. This rule can be generalized to n dimensional non-Euclidean geometry through the use of tetrads. The dot product is generalized to Eq.(11.13) and the cross product is generalized to Eq.(11.14).

The antisymmetric metric is missing from the Einstein Hilbert field theory of gravitation, but is a special case of the Evans field theory when the spin connection is dual to the tetrad [3, 4]. In this special case the wedge product of the spin connection and the tetrad that appears in the first Cartan structure equation:

$$T^a{}_{\mu\nu} = (d \wedge q^a)_{\mu\nu} + \omega^a{}_{\mu b} \wedge q^b{}_\nu \tag{11.15}$$

reduces to the antisymmetric metric within a factor κ with the dimensions of wavenumber. This duality condition:

$$\omega^a{}_{\mu b} = -\frac{1}{2}\kappa \epsilon^a{}_{bc} q^c{}_\mu \tag{11.16}$$

defines free space electromagnetic radiation [1]– [14] decoupled from gravitation - a special case of the general Evans unified field theory. $T^a{}_{\mu\nu}$ is the torsion form (a vector valued two-form) $d\wedge$ denotes the exterior derivative, and $\omega^a{}_{\mu b}$ denotes the spin connection of the well known Palatini variation [15]– [17] of relativity theory. Therefore the Einstein-Hilbert field theory of gravitation, although well known and well used, is severely constrained by its fundamental geometrical assumption of a Christoffel (symmetric) connection:

$$\Gamma^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\nu\mu} . \tag{11.17}$$

This could well be the source of the well observed [20] anomalies of cosmology and the Evans field theory should be used to address these anomalies. If relativity theory is abandoned, objective physics is abandoned, leaving essentially no physics at all. The use of the Christoffel connection means that:

$$R_{\rho\sigma\mu\nu} + R_{\rho\mu\nu\sigma} + R_{\rho\nu\sigma\mu} = 0 \tag{11.18}$$

whereas more generally [1]– [14] Eq.(11.18) is the first Bianchi identity of Cartan geometry:

$$\left(d \wedge T^a\right)_{\mu\nu\sigma} + \omega^a{}_{\mu b} \wedge T^b{}_{\nu\sigma} := R^a{}_{b\mu\nu} \wedge q^b{}_{\sigma}. \tag{11.19}$$

It has been shown [1]– [14] that Eq.(11.19) is the same as the following identity of Riemann geometry:

$$\begin{aligned} &\partial_\mu \Gamma^\lambda{}_{\mu\rho} - \partial_\nu \Gamma^\lambda{}_{\mu\rho} + \Gamma^\lambda{}_{\mu\sigma}\Gamma^\sigma{}_{\nu\rho} - \Gamma^\lambda{}_{\nu\sigma}\Gamma^\sigma{}_{\mu\rho} \\ +&\partial_\nu \Gamma^\lambda{}_{\rho\mu} - \partial_\rho \Gamma^\lambda{}_{\nu\mu} + \Gamma^\lambda{}_{\nu\sigma}\Gamma^\sigma{}_{\rho\mu} - \Gamma^\lambda{}_{\rho\sigma}\Gamma^\sigma{}_{\nu\mu} \\ +&\partial_\rho \Gamma^\lambda{}_{\mu\nu} - \partial_\mu \Gamma^\lambda{}_{\rho\nu} + \Gamma^\lambda{}_{\rho\sigma}\Gamma^\sigma{}_{\mu\nu} - \Gamma^\lambda{}_{\mu\sigma}\Gamma^\sigma{}_{\rho\nu} \\ &:= R^\lambda{}_{\rho\mu\nu} + R^\lambda{}_{\mu\nu\rho} + R^\lambda{}_{\nu\rho\mu} \end{aligned} \tag{11.20}$$

using both the Riemann and torsion tensors, both being non-zero in general. In general the Riemann form of Cartan geometry is [1]– [14]:

$$R^a{}_{b\mu\nu} = q^a{}_{\sigma} q^\tau{}_b R^\sigma{}_{\tau\mu\nu} \tag{11.21}$$

where $R^\sigma{}_{\tau\mu\nu}$ is the Riemann tensor of Riemann geometry. The symmetries:

$$R^a{}_{b\mu\nu} = -R^a{}_{b\nu\mu} \tag{11.22}$$

$$R^\sigma{}_{\tau\mu\nu} = -R^\sigma{}_{\tau\nu\mu} \tag{11.23}$$

are always true, but in general the Riemann form and Riemann tensor are asymmetric in their first two indices. The Riemann tensor becomes antisymmetric in its first two indices if and only if Eq.(11.18) is true [18]. This is another illustration of the rather severe geometrical constraints on the Einstein Hilbert field theory. In the Evans field theory these constraints are lifted and a lot of new physics awaits exploration.

A simple example of a new field equation from Eq.(11.12) is:

$$R q^a{}_{\mu} = -kT q^a{}_{\mu} \tag{11.24}$$

which is a classical field equation closely similar to the Evans wave equation of generally covariant quantum mechanics:

$$\left(\Box + kT\right) q^a{}_{\mu} = 0 \tag{11.25}$$

obtained from Eqs.(11.7) and (11.12). Therefore the Evans lemma of geometry translates into physics using Eq.(11.12). To solve Eq.(11.25) it is possible for example to first define T and then derive the eigenfunctions $q^a{}_{\mu}$ if possible analytically or otherwise computationally. There are various model situations that may be used for T. One of the simplest is the single particle special relativistic limit where:

$$T \longrightarrow \frac{m}{V_0} \tag{11.26}$$

in the particle rest frame. Here m is the mass of an elementary particle and V_0 is its rest volume, a new fundamental concept introduced by the Evans field theory [1]– [14]. The correspondence principle states essentially that general relativity reduces to special relativity under well defined conditions. The wave equation of special relativistic quantum mechanics is the experimentally well tested Dirac equation:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) q^a{}_\mu = 0 \tag{11.27}$$

where c is the speed of light in vacuo and $\hbar$ the reduced Planck constant. It is deduced therefore that the Dirac four spinor, the wavefunction of the Dirac equation, is a tetrad. The latter is the fundamental field of the Palatini variation of general relativity and as such must remain the fundamental field in special relativity. This important conclusion is a direct consequence of the correspondence principle.

Using this argument and comparing Eqs.(11.25) and (11.27) it follows that the fundamental rest volume is defined by:

$$V_0 = \frac{\hbar^2 k}{mc^2} := \frac{\hbar^2 k}{En_0} \tag{11.28}$$

for all elementary particles, including the photon, neutrinos, gravitons and gravitinos. This is one of the major discoveries of the Evans field theory because it removes the necessity for Feynman calculus and renormalization in quantum electrodynamics and quantum chromodynamics. It also removes the unphysical infinities of classical electrodynamics, infinities which originate in the notion of point electron without volume. From Eq.(11.28) it is deduced from general relativity that there are no point particles in nature, and that every elementary particle must have mass. From this deduction it follows that the Higgs mechanism must be abandoned and that theories based on the Higgs mechanism, such as the GWS theory, must be modified. The first steps towards such a modification have been taken [1]– [14].

By deriving the Dirac equation from Cartan geometry spin has been introduced into general relativity, and this is a key step towards towards the unification of gravitational theory with electromagnetic theory and the theory of the weak and strong fields. Spin enters into consideration through the tetrad. The four fundamental fields of physics presently thought to exist: gravitation, electromagnetic, the weak and strong, are all mathematical representations of the fundamental tetrad field.

By use of the Leibnitz Theorem [1]– [14, 18] we also obtain from the tetrad postulate:

$$\left(D^\mu \partial_\mu\right) q^a{}_\lambda + \left(D^\mu \omega^a{}_{\mu b}\right) q^b{}_\lambda - \left(D^\mu \Gamma^\nu{}_{\mu\lambda}\right) q^a{}_\nu = 0. \tag{11.29}$$

Eqs.(11.4) and (11.29) may be used to cross check the derivation of the Evans Lemma as follows. In Eq.(11.29) use the results:

$$D_\mu \partial^\mu = \Box + \Gamma^\mu{}_{\mu\lambda} \partial^\lambda \tag{11.30}$$

$$D^\mu = g^{\mu\nu} D_\nu \tag{11.31}$$

$$\partial_\mu = g_{\mu\nu} \partial^\nu \tag{11.32}$$

implying that:

$$D^\mu \partial_\mu = g^{\mu\nu} D_\nu g_{\mu\nu} \partial^\nu = 4 D_\mu \partial^\mu . \tag{11.33}$$

From Eq.(11.33) in Eq.(11.29):

$$4 \left(D^\mu \partial_\mu \right) q^a{}_\lambda + \left(D^\mu \omega^a{}_{\mu b} \right) q^b{}_\lambda - \left(D^\mu \Gamma^\nu{}_{\mu\lambda} \right) q^a{}_\nu = 0. \tag{11.34}$$

Using the Leibnitz rule:

$$\left(D_\mu \partial^\mu \right) q^a{}_\lambda = D_\mu \left(\partial^\mu q^a{}_\lambda \right) + \partial^\mu \left(D_\mu q^a{}_\lambda \right), \tag{11.35}$$

therefore in Eq.(11.34)

$$4 \left(D_\mu \left(\partial^\mu q^a{}_\lambda \right) + \partial^\mu \left(D_\mu q^a{}_\lambda \right) \right) + \left(D^\mu \omega^a{}_{\mu b} \right) q^b{}_\lambda - \left(D^\mu \Gamma^\nu{}_{\mu\lambda} \right) q^a{}_\nu = 0. \tag{11.36}$$

By comparison of Eq.(11.36) and (11.4):

$$4 D_\mu \left(\partial^\mu q^a{}_\lambda \right) + D^\mu \omega^a{}_{\mu b} - \left(D^\mu \Gamma^\nu{}_{\mu\lambda} \right) q^a{}_\nu = 0 \tag{11.37}$$

i.e.

$$D^\mu \left(\partial_\nu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu \right) = 0 \tag{11.38}$$

which is

$$D^\mu \left(D_\mu q^a{}_\lambda \right) = 0. \tag{11.39}$$

Eq.(11.2) is recovered self consistently, Q.E.D., thus proving the self consistency and correctness of both derivations of the Evans Lemma.

Therefore the set of equations to solve in Cartan geometry is as follows:

$$\Box q^a{}_\mu = R q^a{}_\lambda \tag{11.40}$$

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{11.41}$$

$$d \wedge T^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \tag{11.42}$$

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{11.43}$$

$$D \wedge R^a{}_b = 0. \tag{11.44}$$

There are five equations, and the unknowns are R, q^a and $\omega^a{}_b$. Eqs.(11.41) and (11.42) give a relation between q^a and $\omega^a{}_b$:

$$\begin{aligned} & d \wedge \left(d \wedge q^a + \omega^a{}_b \wedge q^b \right) \\ & + \omega^a{}_b \wedge \left(d \wedge q^b + \omega^b{}_c \wedge q^c \right) \\ & = \left(d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \right) \wedge q^b . \end{aligned} \tag{11.45}$$

Eq.(11.44) is an equation in $\omega^a{}_b$:

$$D \wedge \left(d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \right) = 0. \tag{11.46}$$

If for a simple model, $\omega^a{}_b$ is found from Eq.(11.46), the second Bianchi identity, then q^a may be found from Eq.(11.45). Given this q^a, R and T may be found as eigenvalues of Eq.(11.40). In order to translate this Cartan geometry into electrodynamics we use the Evans Ansatz:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu, \tag{11.47}$$

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu}\,, \tag{11.48}$$

where $A^{(0)}$ is the scalar magnitude of the vector potential of electrodynamics.

11.3 Structure Of The Dirac Equation

The wave equation (11.25) becomes the generally covariant Dirac equation when the appropriate representation space is used. In n dimensional non-Euclidean geometry the tetrad is defined in general [18] by:

$$V^a = q^a{}_\mu V^\mu \tag{11.49}$$

where V^a is a vector in the tangent spacetime, and V^μ is a vector in the base manifold. These vectors are in general n dimensional. The tetrad appropriate to the Dirac equation is given by $n = 2$. Thus V^a and V^μ are two-vectors and the tetrad is a 2×2 matrix. For electrodynamics on the other hand $n = 4$, and V^a and V^μ are four-vectors. The potential field of generally covariant electrodynamics is:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu. \tag{11.50}$$

The intrinsic spin of the electromagnetic field can be described by the three space-like components of $A^a{}_\mu$ and so we can restrict attention to $n = 3$ for this illustrative purpose. More generally, $A^a{}_\mu$ always has a fourth, time-like dimension which defines the scalar potential.

The three dimensional ($n = 3$) representation space can be characterized by two sets of basis vectors, each with $O(3)$ symmetry in contrast with the $SU(2)$ symmetry of the $n = 2$ representation space of the Dirac equation. The first basis of the $n = 3$ space is the Cartesian (X, Y, Z) and the second is the complex circular [1]– [14] $((1), (2), (3))$. The intrinsic spin of electrodynamics is therefore described by assigning:

$$a = (1), (2), (3), \tag{11.51}$$

$$\mu = X, Y, Z, \tag{11.52}$$

and the electromagnetic potential is, within $A^{(0)}$, the tetrad constructed from Eqs.(11.51) and (11.52), with components such as:

$$A_X{}^{(1)}, \ldots, A_Z{}^{(3)}. \tag{11.53}$$

By introduction of the electromagnetic phase factor $e^{i\phi}$, one frame spins and translates with respect to the other. This is electromagnetism - spinning spacetime. Gravitation is curving spacetime. For electromagnetism, the tangent spacetime labeled a is a static Minkowski spacetime and the base manifold labeled μ is a spacetime that spins and translates with respect to the Minkowski spacetime. For gravitation the tangent spacetime is the same, but the base manifold curves with respect to the tangent spacetime. For the unified field the base manifold, spins, translates and curves with respect to the tangent spacetime.

The experimentally observed circular polarization of electromagnetism is described by the following type of tetrad in vector notation [1]– [14]:

$$\mathbf{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} - i\mathbf{j}\right) e^{i\phi}, \tag{11.54}$$

$$\mathbf{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} + i\mathbf{j}\right) e^{-i\phi}, \tag{11.55}$$

together with:

$$\mathbf{A}^{(3)} = A^{(0)}\mathbf{k}. \tag{11.56}$$

Here $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are Cartesian unit vectors defined by the $O(3)$ symmetry rule:

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}. \tag{11.57}$$

These are related to the unit vectors of the complex circular basis by:

$$\mathbf{e}^{(1)} = \frac{1}{\sqrt{2}} \left(\mathbf{i} - i\mathbf{j}\right) \tag{11.58}$$

$$\mathbf{e}^{(2)} = \frac{1}{\sqrt{2}} \left(\mathbf{i} + i\mathbf{j}\right) \tag{11.59}$$

$$\mathbf{e}^{(3)} = \mathbf{k}. \tag{11.60}$$

implying the $O(3)$ symmetry rule:

$$\mathbf{e}^{(1)} \times \mathbf{e}^{(2)} = i\mathbf{e}^{(3)*} \tag{11.61}$$

The tetrads in vector notation are therefore:

$$\mathbf{q}^{(1)} = \mathbf{e}^{(1)} e^{i\phi}, \tag{11.62}$$

$$\mathbf{q}^{(2)} = \mathbf{e}^{(2)} e^{-i\phi}, \tag{11.63}$$

$$\mathbf{q}^{(3)} = \mathbf{e}^{(3)}, \tag{11.64}$$

and obey the $O(3)$ symmetry rule:

$$\mathbf{q}^{(1)} \times \mathbf{q}^{(2)} = i\mathbf{q}^{(3)*}. \tag{11.65}$$

The electromagnetic phase ϕ is in general the generally covariant Evans phase [1]– [14]. Thus the tetrad can be used straightforwardly for electromagnetism as well as for gravitation. This is the key to the unified field theory.

These considerations can now be applied to the single particle Dirac equation, in which $n = 2$. As for the intrinsic spin of the electromagnetic field, $q^a{}_\mu$ in the Dirac equation represents a spinning and translating of spacetime, the helicity of the elementary fermion. The photon of the electromagnetic field is a boson. In the generally covariant Dirac equation the spin is superimposed on a curving of the base manifold (the Evans spacetime [1]– [14]), and this curving is gravitation. The generally covariant Dirac equation therefore describes the effect of gravitation on the fermion, i.e. the influence of gravitation on elementary particles and anti-particles that are fermions. The famous Dirac equation of special relativity is recovered as discussed already in this paper, i.e. in the limit defined by Eq.(11.26). As for electrodynamics the spin is introduced through a phase factor $e^{i\phi}$ and as for electrodynamics there are right and left handed helicities. For the electromagnetic potential right and left handed senses of circular polarization are defined by:

$$\mathbf{A}_R^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} - i\mathbf{j}\right) e^{i\phi}, \tag{11.66}$$

$$\mathbf{A}_L^{(2)} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} + i\mathbf{j}\right) e^{i\phi}. \tag{11.67}$$

For the same phase factor $e^{i\phi}$, the right basis vector is $\left(\mathbf{i} - i\mathbf{j}\right)/\sqrt{2}$ and the left basis vector is $\left(\mathbf{i} + i\mathbf{j}\right)/\sqrt{2}$. These are complex conjugate basis vectors for the same phase factor $e^{i\phi}$, and therefore for a given phase factor a right left basis may be defined being the complex conjugate basis defined by $\left(\mathbf{i} - i\mathbf{j}\right)/\sqrt{2}$ and $\left(\mathbf{i} + i\mathbf{j}\right)/\sqrt{2}$

It follows that such a basis can also be defined for the Dirac equation, and that there exists a two-dimensional column vector in the tangent spacetime with complex conjugate components:

$$V^a = \begin{bmatrix} V^R \\ V^L \end{bmatrix}. \tag{11.68}$$

In the base manifold there exists a column vector:

$$V^\mu = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{11.69}$$

which is spinning and translating with respect to V^a. The column vectors are linked by:

$$\begin{bmatrix} V^R \\ V^L \end{bmatrix} = \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^L{}_1 & q^L{}_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{11.70}$$

where the tetrad is:

$$q^a{}_\mu = \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^L{}_1 & q^L{}_2 \end{bmatrix} \tag{11.71}$$

This is a matrix consisting of a row vector $\begin{bmatrix} q^R{}_1 & q^R{}_2 \end{bmatrix}$ superimposed on a row vector $\begin{bmatrix} q^L{}_1 & q^L{}_2 \end{bmatrix}$. Transposition of the two row vectors gives the left and right

Pauli spinors in the limit defined by Eq.(11.26):

$$\xi^R = \begin{bmatrix} q^R{}_1 \\ q^R{}_2 \end{bmatrix}, \xi^L = \begin{bmatrix} q^L{}_1 \\ q^L{}_2 \end{bmatrix} \tag{11.72}$$

and the Dirac spinor is:

$$\psi = \begin{bmatrix} \xi^R \\ \xi^L \end{bmatrix}. \tag{11.73}$$

Since:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^L{}_1 & q^L{}_2 \end{bmatrix} = 0 \tag{11.74}$$

it follows that:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) \begin{bmatrix} q^R{}_1 \\ q^R{}_2 \\ q^L{}_1 \\ q^L{}_2 \end{bmatrix} = 0 \tag{11.75}$$

and this is the single particle Dirac equation. This equation is generalized in the Evans field theory to:

$$(\Box + kT)\,\psi = 0 \tag{11.76}$$

The whole of Dirac algebra may be recovered from Cartan geometry as exemplified by:

$$\overline{\psi}\psi = \xi^{L+}\xi^R + \xi^{R+}\xi^L \tag{11.77}$$

where $\overline{\psi}$ is the adjoint of the Dirac spinor ψ. In order to investigate the effect of gravitation on the single particle Dirac equation a model is chosen for T and Eq.(11.76) solved for the eigenfunctions, which are components of the Dirac spinor. The Dirac equation in general relativity is a matter field equation and reduces in the non-relativistic limit to the Schrodinger equation and in the classical limit to the Newton equation of motion. Through the Poisson equation we recover simultaneously the Newton inverse square law in the appropriate weak field and non-relativistic limit. This shows why gravitational and inertial acceleration are identical, both are derived from Cartan geometry.

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Chapter 12

Quark Gluon Model In The Evans Unified Field Theory

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Abstract

The interaction of quarks and gluons inside a given elementary particle is described through the use of two or more simultaneous Evans wave equations and the equivalent minimal prescription. Approximate quark flavor symmetry in the quark gluon standard model is replaced by the mathematically required exact quark flavor symmetry perturbed by quark gluon momentum exchange. Therefore the apparently different observed quark masses are the result of the interaction of a confined quark with a massive gluon field inside an elementary particle or between confined quarks in different elementary particles. Therefore the six quarks initially have the same mass but different flavors in the hypothetical free state (single quark state of the Evans wave equation), but the different interactions of quark and gluon inside a given elementary particle result in the apparently different confined quark masses observed experimentally. These masses are more accurately the average result of different and transient momentum exchanges of massive quark and massive gluon. Quarks have only been observed to date in a confined state, where quark gluon interaction is always present inside an elementary particle or between two elementary particles. This is essentially a multi particle momentum exchange problem between quark and gluon. Many such interactions are possible because there are six quark flavors of $SU(n)$ symmetry and three quark colors of $SU(3)$ symmetry in general, giving rise to many possible permutations and combinations and therefore to many types of elementary particle as observed experimentally. The Evans uni-

fied field theory is rigorously objective (i.e. generally covariant) throughout and in consequence there can be no massless particles, the radiated gluon is therefore massive and not massless as in the standard model. The gluon field has $SU(3)$ symmetry and is also described by an Evans wave equation. The many possible types of interaction between quarks and gluons is therefore always described by simultaneous Evans wave equations defining momentum exchange. These equations must be solved numerically and simultaneously in general with given initial and boundary conditions.

Keywords: Evans unified field theory, quark gluon model, flavor symmetry, color symmetry, gluon potential field.

12.1 Introduction

In the standard model of quark gluon interaction [1] there are six quark flavors u, d, s, c, t and b and three quark colors R, W, and B. The quarks are matter fields. The potential of the radiated gluon field also has $SU(3)$ symmetry [1] and in the standard model the various gluons are considered to be a massless particles. The masses of the six quarks are not the same experimentally: u and d for example have approximately the same mass but the mass of s is very different. In contrast the masses of the left and right electron appearing in the Dirac equation and observed in the Stern Gerlach experiment [2] (the effect of a magnetic field of right design on an electron beam) are exactly the same within contemporary instrumental precision. The right and left electrons are therefore said to be degenerate in the absence of a magnetic field [1]. Therefore they can be described by an exact symmetry, in this case the $SU(2)$ symmetry of the appropriate representation space of the Dirac equation. This $SU(2)$ symmetry implies the use of two Pauli spinors, one right and one left. These are both column two vectors, which when superimposed on each other define the Dirac four spinor, a column vector with four components. The Evans unified field theory [3]– [18] shows that the Dirac equation is a limit of the Evans wave equation defined by:

$$kT = \frac{m^2c^2}{\hbar^2} = \frac{mk}{V}. \tag{12.1}$$

Here T is the scalar energy momentum density defined by the fundamental field equation of relativity theory for all matter and radiated fields:

$$R = -kT \tag{12.2}$$

where R is scalar curvature and k is Einsteins constant. In Eq.(12.1) m is mass, $\hbar$ is the reduced Planck constant and c is the speed of light. Eqs.(12.1) and (12.2) imply that every elementary particle and every quark and radiated particle such as a photon and gluon have a rest volume defined by:

$$V = \frac{\hbar^2 k}{mc^2}. \tag{12.3}$$

The Evans field theory shows that the Dirac spinor is a special relativistic example or limit of the tetrad, the fundamental field of the Palatini variation of general relativity [19]– [21]. The tetrad is the eigenfunction of the Evans lemma:

$$\Box q^a{}_\mu = R q^a{}_\mu \tag{12.4}$$

which gives the Evans wave equation:

$$(\Box + kT)\, q^a{}_\mu = 0 \tag{12.5}$$

using Eq.(12.2). The Dirac equation is obtained straightforwardly from the Evans wave equation using Eq. (12.1) and a 2×2 tetrad:

$$q^a{}_\mu = \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^L{}_1 & q^L{}_2 \end{bmatrix}. \tag{12.6}$$

Transposition of the two row vectors of the tetrad into two column vectors gives the column four vector which is the Dirac spinor:

$$\psi = \begin{bmatrix} q^R{}_1 \\ q^R{}_2 \\ q^L{}_1 \\ q^L{}_2 \end{bmatrix} = \begin{bmatrix} \xi^R \\ \xi^L \end{bmatrix}. \tag{12.7}$$

The Pauli spinors are therefore identified as:

$$\xi^R = \begin{bmatrix} q^R{}_1 \\ q^R{}_2 \end{bmatrix}, \xi^L = \begin{bmatrix} q^L{}_1 \\ q^L{}_2 \end{bmatrix}. \tag{12.8}$$

Therefore the Dirac equation is a result of differential geometry, because the lemma (12.4) is an identity obtained straightforwardly from the standard tetrad postulate [22] of Cartan's differential geometry:

$$D_\mu q^a{}_\nu = 0 \tag{12.9}$$

where D_μ denotes the covariant derivative. This result is one of the major advances of Evans field theory because it allows the generally covariant description of momentum exchange between any radiated and matter fields in nature. This result yields an objective (i.e. generally covariant) description of all nature, from quarks to cosmological objects, i.e. of any type of matter fields interacting with any type of radiated field.

In Section 12.2 the approximate quark flavor symmetries of the standard model are replaced by exact quark flavor symmetries perturbed by quark gluon momentum exchange processes (of which very many are possible giving rise to many observed elementary particles [1]). The interacting quark field and gluon field are described by two or more simultaneous Evans wave equations which must be solved numerically and simultaneously with given initial and boundary conditions. In Section 12.3 consideration is extended to include the quark colors, for each flavor there are three colors.

12.2 Perturbation Of Exact Flavor Symmetry By Momentum Exchange

In the standard model the use is made of approximate flavor symmetries. The simplest is $SU(2)$, in which the Dirac type spinor is [1]:

$$\xi = \begin{bmatrix} u \\ d \end{bmatrix}. \tag{12.10}$$

This is approximate because u and d do not have the same masses and are therefore only approximately degenerate in the absence of a perturbing field. In contrast the right and left electrons of the original Dirac equation are exactly degenerate in the absence of a magnetic field as discussed already. This is a severe conceptual problem for the standard model because in group theory and in nature there can only be exact symmetries, no approximate symmetries. The three quark model has $SU(3)$ symmetry, the four quark model has $SU(4)$ symmetry and so on up to the $SU(6)$ symmetry of the six quark model. These symmetries are group symmetries and again cannot be approximate. In the three quark model the Dirac type spinor is a three spinor:

$$\xi = \begin{bmatrix} u \\ d \\ s \end{bmatrix}. \tag{12.11}$$

and so on up to the six spinor of the six quark model. The problem of approximate symmetry becomes worse and worse because the six quark masses are not even approximately the same experimentally. In the Evans unifed field theory each spinor is governed by the wave equation, for example:

$$(\Box + kT) \begin{bmatrix} u \\ d \end{bmatrix} = 0 \tag{12.12}$$

for the two quark model, and

$$(\Box + kT) \begin{bmatrix} u \\ d \\ s \end{bmatrix} = 0 \tag{12.13}$$

for the three quark model and so on up to the six quark model:

$$(\Box + kT) \begin{bmatrix} u \\ d \\ s \\ c \\ t \\ b \end{bmatrix} = 0. \tag{12.14}$$

Thus, in the Evans unified field theory, there is gravitational interaction between quarks inside an elementary particle, or between quarks in two different elementary particles. This occurs in addition to the interaction between quarks mediated by gluons. Similarly, there is gravitational interaction between electrons in the Evans unified field theory in addition to the interaction mediated by photons. In the presence of gravitational interaction:

$$kT \neq \frac{m^2c^2}{\hbar^2}. \tag{12.15}$$

In the absence of gravitational interaction Eq.(12.1) applies. The problem at hand is therefore simplified if we neglect gravitational interaction to one of interaction between quarks and gluons. In the standard model the $SU(3)$ quark color symmetry (Section 12.3) is considered to be exact, and the quark color spinor is [1]:

$$\psi = \begin{bmatrix} R \\ W \\ B \end{bmatrix}. \tag{12.16}$$

This three spinor plays a role analogous to right and left spin in the Pauli spinors of the right and left electrons, and is introduced following considerations [1] similar to the Pauli exclusion principle for electrons. The gluon field in the standard model is the radiated field of $SU(3)$ symmetry that mediates the strong nuclear interaction. The gauge potential $A^a{}_\mu$ of the gluon field has eight components. In the Evans field theory each component of $A^a{}_\mu$ obeys the Evans wave equation:

$$(\Box + kT)\, A^a{}_\mu = 0. \tag{12.17}$$

Therefore the interaction of a gluon with a quark is described by a momentum exchange process in the Evans field theory, in which each type of gluon has mass as described by Eq.(12.17). In the special relativistic limit (12.1), Eq.(12.17) reduces to:

$$\left(\Box + \frac{m_g^2c^2}{\hbar^2}\right) A^a{}_\mu = 0 \tag{12.18}$$

where m_g is the mass of a given gluon. In the standard model there is no gluon mass, and no photon mass, in contradiction to the observation of photon mass in the Eddington and NASA Cassini experiments, precise to one part in one hundred thousand. The absence of photon and gluon mass from the standard model is therefore another major conceptual problem for that model.

The free quark flavors in the absence of gravitational interaction are described by:

$$\left(\Box + \frac{m_q^2c^2}{\hbar^2}\right) \psi = 0 \tag{12.19}$$

and the free gluons in the absence of gravitational interaction by:

$$\left(\Box + \frac{m_g^2c^2}{\hbar^2}\right) A = 0. \tag{12.20}$$

In Eq.(12.19) an exact symmetry is used in the Evans field theory, as required by basic group theory and in contrast to the meaningless approximate symmetries of the standard model. In other words the six quarks flavors have the same mass in the free state.

Eqs.(12.19) and (12.20) can be factorized [3]– [18] into first order differential equations:

$$(i\gamma^a \partial_a - m_q c/\hbar)\,\psi = 0 \tag{12.21}$$

$$(i\gamma^a \partial_a - m_g c/\hbar)\,A = 0 \tag{12.22}$$

where γ^a is the Dirac matrix.

The momentum exchange between any type of quark and any type of gluon is given through a minimal prescription as follows:

$$(i\hbar\gamma^a\,(\partial_a - igA_a) - m_q c)\,\psi = 0 \tag{12.23}$$

$$(i\hbar\gamma^a\,(\partial_a + igA_a) - m_g c)\,A = 0. \tag{12.24}$$

Here g is a coupling parameter analogous to the e used in describing momentum exchange between photon and electron in quantum electrodynamics in the Evans field theory [3]– [18]. Thus Eqs.(12.23) and (12.24) describe quantum chromodynamics in the Evans unified field theory in the absence of any consideration of gravitational interaction. There are six quark flavors, three quark colors and eight types of gluon in general, so there is a total of $6 \times 3 \times 8 = 144$ different coupling parameters g in general. The effective mass generated in each type of interaction is defined by [3]– [18]:

$$kT = \left(\frac{m_q c}{\hbar}\right)^2_{eff} = \left(\frac{m_q c}{\hbar}\right)^2 + \frac{g m_q c}{\hbar^2}\gamma^a\,(A_a + A_a^*) + \frac{g^2}{\hbar^2}A_a^* A^a. \tag{12.25}$$

The experimental observation of apparently different confined quark masses is therefore explained generically by Eq.(12.25), the apparently different confined quark masses of the standard model being in Evans field theory a well defined combination of free quark and free gluon mass and appropriate coupling parameter g. In various elementary particles there are different quark combinations [1]. Baryons are bound states of three quarks, and mesons are quark anti-quark states. Baryons participate in the strong interactions and have overall half integral spins and so interaction between baryons is mediated by gluons according to Eqs.(12.23) and (12.24). In the standard model basic concepts such as the degeneracy of multiplets of hadrons are based on the approximate quark degeneracy. Hadrons participate in the strong interaction and so the interaction between hadrons takes place through gluon exchange. A given representation of $SU(3)$ for example contains several representations of $SU(2)$ [1], and from this it is concluded in the standard model that an $SU(3)$ supermultiplet contains several isospin multiplets of different strangeness S. This group theoretical reasoning is the basis of for example the Gell-Mann Nishijima relation used in the GWS theory of the standard model. However, the fundamental but approximate quark flavor degeneracy, as we have argued, is meaningless, bringing into question all of these basic concepts of the standard model. In the Evans field theory an exact quark flavor degeneracy is used, and this is self consistent, as well as objective, physics.

12.3 Quark Color Symmetry

The quark color symmetry of R, W and B was introduced to address the problem posed by Fermi Dirac statistics [1]. In contrast to the flavor symmetry the color symmetry is exact. The relevant spinor is:

$$\psi = \begin{bmatrix} u^R \\ u^W \\ u^B \end{bmatrix} etc. \tag{12.26}$$

and has $SU(3)$ symmetry. Therefore for each quark flavor there are three colors. The standard model therefore uses a mixture of approximate and exact symmetries for flavor and color wavefunctions. In the Evans field theory in contrast, exact group theoretical symmetries are used throughout, the theory is generally covariant throughout, and the basic contradictions between quantum mechanics and general relativity are removed through the use of massive photons and gluons and a geometrically based approach to the whole of physics.

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Chapter 13

The Origin of Intrinsic Spin and the Pauli Exclusion Principle in the Evans Unified Field Theory

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Abstract

The tetrad of the Evans unified field theory is shown to be the wavefunction for electromagnetism, the Dirac equation, strong force theory and the Majorana/Weinberg spin equations for any particle and field in physics. The origin of intrinsic spin in physics is shown to be a basis set of elements in the tangent spacetime to a base manifold at point P. The tangent spacetime is a Minkowski spacetime and the base manifold an Evans spacetime. The origin of the Pauli exclusion principle is the half integral intrinsic spin described by an appropriate basis set of elements. Right and left intrinsic spin in electrodynamics are the two states of circular polarization which are again described by an appropriate basis set. Similar reasoning applies for the origin of quark color and for general spin in the Majorana Weinberg equations. In the Evans unified field theory there is therefore a self-consistent description of intrinsic spin in physics and gravitational theory.

Key words: Evans field theory; intrinsic spin; right and left circular polarization; Pauli exclusion principle; quark color; Majorana Weinberg equations.

13.1 Introduction

A true unified field theory must be able to trace the origin of intrinsic spin in physics, and describe the various manifestation of spin in all radiated and matter fields. Furthermore it must be able to integrate this type of theory with gravitational theory and also with quantum mechanics. This is a formidable problem which appears to have been given one plausible solution lately [1]– [17] in the Evans unified field theory. In this paper the origin of intrinsic spin is discussed in terms of the tetrad, which is the fundamental field in the Evans theory for all material matter and radiation. It is shown in Section 13.2 that there exists a basis set of elements in tangent spacetime at a point P in the base manifold, a basis set which defines the existence of intrinsic spin. In electrodynamics the basis set defines left and right circular polarization and the intrinsic spin field of generally covariant electrodynamics. In Section 13.3 the intrinsic left and right spin of a fermionic field in the Dirac equation is defined in terms of the appropriate basis set, and the origin of the Pauli exclusion principle revealed. In Section 13.4 the origin of quark color in strong field theory is defined by a color basis set in the tangent spacetime, and this is related to quark flavor in the base manifold by the tetrad field of strong force theory. This is the matter field of the six quarks currently postulated to exist and the tetrad in this case is a transformation matrix linking quark color and flavor. Finally in Section 13.5 the Majorana Weinberg equations for arbitrary spin are set up using the same principles of differential geometry which underpin the Evans unified field theory. In each case the wavefunction is the tetrad $q^a{}_\mu$, and the tangent spacetime label a is the index of the elements of the basis set. The index a is the index of intrinsic spin.

13.2 Electrodynamics

The existence of intrinsic spin in electrodynamics was discovered experimentally by Arago in 1811 and is referred to as left and right circular polarization. The existence of the Evans spin field, observed in the inverse Faraday effect is indicated conclusively by general relativity [1]– [17]. The vector potentials for left and right circular polarization are:

$$\mathbf{A}_R{}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} - i\mathbf{j}\right) e^{i\phi} \tag{13.1}$$

$$\mathbf{A}_L{}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} + i\mathbf{j}\right) e^{i\phi}, \tag{13.2}$$

where ϕ is the electromagnetic phase and where the (1) index denotes complex conjugation as follows:

$$\mathbf{A}_R{}^{(1)} = \mathbf{A}_R{}^{(2)*} \tag{13.3}$$

$$\mathbf{A}_L{}^{(1)} = \mathbf{A}_L{}^{(2)*}. \tag{13.4}$$

The left and right spin field is then:

$$\mathbf{A}^R = \frac{A^{(0)}}{\sqrt{2}}\,(\mathbf{i} - i\mathbf{j})\,e^{i\phi} \tag{13.5}$$

$$\mathbf{A}^L = \frac{A^{(0)}}{\sqrt{2}}\,(\mathbf{i} + i\mathbf{j})\,e^{i\phi}. \tag{13.6}$$

For our present purposes we may simplify the argument by writing:

$$\mathbf{A}_R{}^{(1)} = \mathbf{A}^R \tag{13.7}$$

$$\mathbf{A}_L{}^{(1)} = \mathbf{A}^L. \tag{13.8}$$

The basis vectors for the complex circular basis are defined by:

$$\mathbf{e}^{(1)} = \frac{1}{\sqrt{2}}\,(\mathbf{i} - i\mathbf{j}) \tag{13.9}$$

$$\mathbf{e}^{(2)} = \frac{1}{\sqrt{2}}\,(\mathbf{i} + i\mathbf{j}) \tag{13.10}$$

$$\mathbf{e}^{(3)} = \mathbf{k} \tag{13.11}$$

where **i**, **j**, and **k** are Cartesian unit vectors. Therefore:

$$\mathbf{A}^R = A^{(0)}e^{i\phi}\mathbf{e}^{(1)} \tag{13.12}$$

$$\mathbf{A}^L = A^{(0)}e^{i\phi}\mathbf{e}^{(2)}. \tag{13.13}$$

It follows that the right and left basis vectors may be defined as:

$$\mathbf{e}^R = e^{i\phi}\mathbf{e}^{(1)} \tag{13.14}$$

$$\mathbf{e}^L = e^{i\phi}\mathbf{e}^{(2)}. \tag{13.15}$$

Within the phase factor $\mathbf{e}^{i\phi}$ these are the $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ basis vectors of the complex circular basis. The components of the right and left basis vectors define a tetrad matrix:

$$q^a{}_\mu = \begin{bmatrix} e^R{}_x & e^R{}_y \\ e^L{}_x & e^L{}_y \end{bmatrix} \tag{13.16}$$

where

$$\begin{aligned} e^R{}_x &= \frac{e^{i\phi}}{\sqrt{2}} \quad , \quad e^R{}_y = \frac{-ie^{i\phi}}{\sqrt{2}} \\ e^L{}_x &= \frac{e^{i\phi}}{\sqrt{2}} \quad , \quad e^L{}_y = \frac{ie^{i\phi}}{\sqrt{2}}. \end{aligned} \tag{13.17}$$

The tetrad in Eq.(13.16) obeys the Evans wave equation in the limit of zero photon mass:

$$kT = \left(\frac{mc}{\hbar}\right)^2 \longrightarrow 0 \tag{13.18}$$

so that:

$$\Box q^a{}_\mu = 0. \tag{13.19}$$

With the Evans Ansatz:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu \tag{13.20}$$

Eq.(13.19) is the d'Alembert wave equation in free space:

$$\Box A^a{}_\mu = 0. \tag{13.21}$$

The tetrad $q^a{}_\mu$ is always defined geometrically [18] by:

$$V^a = q^a{}_\mu V^\mu \tag{13.22}$$

where V^a is a vector in the tangent spacetime and V^μ is a vector in the base manifold.

Define

$$V^\mu = \begin{bmatrix} e_x \\ e_y \end{bmatrix} = e^{-i\phi} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{13.23}$$

and

$$V^a = \begin{bmatrix} e^R \\ e^L \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1-i \\ 1+i \end{bmatrix} \tag{13.24}$$

and it follows from Eqs.(13.16) and (13.22) to (13.24) that:

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1-i \\ 1+i \end{bmatrix} = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \begin{bmatrix} e^{-i\phi} \\ e^{-i\phi} \end{bmatrix} \tag{13.25}$$

i.e.

$$V^a = q^a{}_\mu V^\mu \tag{13.26}$$

Q.E.D.

From Eq.(13.25) it is seen that the basis set for the intrinsic spin of electromagnetism is:

$$\mathbf{e}^{(1)} \times \mathbf{e}^{(2)} = i\mathbf{e}^{(3)*} \tag{13.27}$$

$$\mathbf{e}^{(2)} \times \mathbf{e}^{(3)} = i\mathbf{e}^{(1)*} \tag{13.28}$$

$$\mathbf{e}^{(3)} \times \mathbf{e}^{(1)} = i\mathbf{e}^{(2)*} \tag{13.29}$$

i.e. the basis set is made up of the complex circular unit vectors. Eq.(13.27) to (13.29) have $O(3)$ symmetry. This reasoning may be extended to find the origin and meaning of intrinsic spin in other contexts.

13.3 Fermionic Matter Field And The Dirac Equation

The tetrad field for the Dirac equation is

$$q^a{}_\mu = \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^L{}_1 & q^L{}_2 \end{bmatrix} \tag{13.30}$$

where the Pauli spinors are defined by:

$$\phi^R = \begin{bmatrix} q^R{}_1 \\ q^R{}_2 \end{bmatrix}, \quad \phi^L = \begin{bmatrix} q^L{}_1 \\ q^L{}_2 \end{bmatrix}. \tag{13.31}$$

The tetrad field is defined by:

$$V^a = q^a{}_\mu V^\mu \tag{13.32}$$

where

$$V^a = \begin{bmatrix} e^R \\ e^L \end{bmatrix}, \quad V^\mu = \begin{bmatrix} e^1 \\ e^2 \end{bmatrix}. \tag{13.33}$$

The column vector V^μ is a two dimensional column vector in the base manifold and transforms under $SU(2)$ symmetry [19]. Similarly the column vector V^a is a two dimensional column vector in the tangent spacetime.

The tetrad field $q^a{}_\mu$ is defined by Eq.(13.30) and obeys the Evans wave equation [1]– [17]:

$$(\square + kT)\, q^a{}_\mu = 0. \tag{13.34}$$

The Dirac equation is recovered in the limit:

$$kT \longrightarrow \left(\frac{mc}{\hbar}\right)^2, T \longrightarrow \frac{m}{V} \tag{13.35}$$

where m is the mass of the fermion, $\hbar$ is the reduced Planck constant, c is the velocity of light and V is the rest volume of the fermion:

$$V = \frac{\hbar^2 k}{mc^2}. \tag{13.36}$$

In the limit (13.35) the Dirac spinor is defined [1]– [17] by:

$$\psi = \begin{bmatrix} q^R{}_1 \\ q^R{}_2 \\ q^L{}_1 \\ q^L{}_2 \end{bmatrix} \tag{13.37}$$

and the Dirac equation is:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right)\psi = 0. \tag{13.38}$$

This is a free particle equation, and in this limit no gravitational attraction exists between fermions in Eq.(13.38). To describe gravitational attraction between fermions we need the Evans wave equation (13.34), in general without approximation.

13.4 Strong Field Theory

In contemporary strong field theory [19] there are thought to exist six quark flavors and three quark colors. If we accept this view uncritically the Evans unified field theory can be applied to the n-quark models, where $n = 2, \ldots, 6$. These models transform under $SU(n)$ symmetry [19]. In the 2-quark model there are two flavors, u and d, and three colors, R, W and B. Define the following column two-vector (a two-spinor) in the base manifold:

$$V^\mu = \begin{bmatrix} u \\ d \end{bmatrix} = \begin{bmatrix} e^1 \\ e^2 \end{bmatrix} \tag{13.39}$$

and the following column three-vector (a three-spinor) in the tangent spacetime to the base manifold at point P:

$$V^a = \begin{bmatrix} e^R \\ e^W \\ e^B \end{bmatrix}. \tag{13.40}$$

The flavors u and d represent two physically distinct quarks, each of which has color R, W and B. The u and d particles are analogous to the two distinct electrons of Dirac theory. The electrons are distinct because they are left and right handed, with half integral spin. Similarly, R, W and B in strong field theory plays the role of half integral spin in electron theory. It is seen that strong field theory is built up by direct analogy with Dirac theory, and quarks also have half integral spin [19].

Now define the tetrad matrix linking quark color and quark flavor. This must be a 2×3 matrix:

$$\begin{bmatrix} e^R \\ e^W \\ e^B \end{bmatrix} = \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^W{}_1 & q^W{}_2 \\ q^B{}_1 & q^B{}_2 \end{bmatrix} \begin{bmatrix} e^1 \\ e^2 \end{bmatrix}. \tag{13.41}$$

Therefore the color-flavor tetrad for the two-quark model is:

$$q^a{}_\mu = \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^W{}_1 & q^W{}_2 \\ q^B{}_1 & q^B{}_2 \end{bmatrix} \tag{13.42}$$

and is the eigenfunction of the Evans wave equation [1]– [17]:

$$(\Box + kT)\, q^a{}_\mu = 0. \tag{13.43}$$

This means that $q^a{}_\mu$ is the quark matter field. The quarks interact through gluons, which are the radiated fields [19] of strong field theory.

Similarly, in the three-quark model the tetrad is defined by:

$$\begin{bmatrix} e^R \\ e^W \\ e^B \end{bmatrix} = \begin{bmatrix} q^R{}_1 & q^R{}_2 & q^R{}_3 \\ q^W{}_1 & q^W{}_2 & q^W{}_3 \\ q^B{}_1 & q^B{}_2 & q^B{}_3 \end{bmatrix} \begin{bmatrix} e^1 \\ e^2 \\ e^3 \end{bmatrix} \tag{13.44}$$

and is a 3×3 matrix. The name "tetrad" is used generically [18]. As a final example the tetrad of the four-quark model is a 4×3 matrix defined by:

$$\begin{bmatrix} e^R \\ e^W \\ e^B \end{bmatrix} = \begin{bmatrix} q^R{}_1 & q^R{}_2 & q^R{}_3 & q^R{}_4 \\ q^W{}_1 & q^W{}_2 & q^W{}_3 & q^W{}_4 \\ q^B{}_1 & q^B{}_2 & q^B{}_3 & q^B{}_4 \end{bmatrix} \begin{bmatrix} e^1 \\ e^2 \\ e^3 \\ e^4 \end{bmatrix} \tag{13.45}$$

and it is possible to proceed in this way up to the six-quark model, where the tetrad is a 6×3 matrix.

13.5 Majorana And Weinberg Equations

The Majorana equation [20, 21] represents the free space equations of electromagnetism as Weyl equations, i.e. a Dirac equation with no mass term. The equations of electromagnetism used originally by Majorana in the nineteen twenties were the Maxwell Heaviside equations. In order to derive the generally covariant Majorana equation the unified field theory is needed. The Weinberg equation [22] for any spin is a generalization of the Majorana equation for any half-integral or integral spin. All these spin equations are special cases of the Evans unified field theory. In order to illustrate this consider the Maxwell Heaviside field equations in free space. In S.I. units:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \tag{13.46}$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mathbf{0} \tag{13.47}$$

where $\mathbf{B}$ is magnetic flux density and $\mathbf{E}$ is electric field strength. These equations are used simply for the sake of illustration. The generally covariant equations of electrodynamics from the Evans unified field theory [1]– [17] include the fundamental Evans spin field - which is absent from the Maxwell Heaviside field theory but which is observed experimentally in the inverse Faraday effect. Eqs.(13.46) and (13.47) can be written as:

$$\nabla \times (\mathbf{E} - ic\mathbf{B}) + \frac{i}{c}\frac{\partial}{\partial t}(\mathbf{E} - ic\mathbf{B}) = \mathbf{0}. \tag{13.48}$$

Now consider the right and left circularly polarized solutions of Eq.(13.48):

$$\mathbf{E}^R = \frac{E^{(0)}}{\sqrt{2}}(\mathbf{i} - i\mathbf{j})\, e^{i\phi} \tag{13.49}$$

$$\mathbf{B}^R = \frac{B^{(0)}}{\sqrt{2}}(i\mathbf{i} + \mathbf{j})\, e^{i\phi} \tag{13.50}$$

and

$$\mathbf{E}^L = \frac{E^{(0)}}{\sqrt{2}}(\mathbf{i} + i\mathbf{j})\, e^{i\phi} \tag{13.51}$$

$$\mathbf{B}^L = \frac{B^{(0)}}{\sqrt{2}} \left(-i\mathbf{i} + \mathbf{j}\right) e^{i\phi}. \tag{13.52}$$

Use

$$E^{(0)} = cB^{(0)} = \omega A^{(0)} \tag{13.53}$$

to obtain

$$\mathbf{E}^R - ic\mathbf{B}^R = 2\omega \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} - i\mathbf{j}\right) e^{i\phi}. \tag{13.54}$$

Define the potential field as:

$$\mathbf{A}^R = \frac{A^{(0)}}{\sqrt{2}} i \left(\mathbf{i} - i\mathbf{j}\right) e^{i\phi} \tag{13.55}$$

so that

$$\mathbf{E}^R - ic\mathbf{B}^R = 2\frac{\omega}{i}\mathbf{A}^R. \tag{13.56}$$

Similarly:

$$\mathbf{E}^L + ic\mathbf{B}^L = 2\frac{\omega}{i}\mathbf{A}^L \tag{13.57}$$

where

$$\mathbf{A}^L = \frac{A^{(0)}}{\sqrt{2}} i \left(\mathbf{i} + i\mathbf{j}\right) e^{i\phi}. \tag{13.58}$$

Eqs.(13.56) and (13.57) define the right and left handed potential fields. These obey the equations:

$$\left(\nabla \times + \frac{i}{c}\frac{\partial}{\partial t}\right) \mathbf{A}^R = \mathbf{0} \tag{13.59}$$

$$\left(\nabla \times - \frac{i}{c}\frac{\partial}{\partial t}\right) \mathbf{A}^L = \mathbf{0}. \tag{13.60}$$

The components of Eq.(13.59) are:

$$\frac{\partial {A^R}_z}{\partial y} - \frac{\partial {A^R}_y}{\partial z} + \frac{i}{c}\frac{\partial {A^R}_x}{\partial t} = 0 \tag{13.61}$$

$$\frac{\partial {A^R}_x}{\partial z} - \frac{\partial {A^R}_z}{\partial x} + \frac{i}{c}\frac{\partial {A^R}_y}{\partial t} = 0 \tag{13.62}$$

$$\frac{\partial {A^R}_y}{\partial x} - \frac{\partial {A^R}_x}{\partial y} + \frac{i}{c}\frac{\partial {A^R}_z}{\partial t} = 0. \tag{13.63}$$

Now use the quantum condition [19]:

$$p^\mu = i\hbar\partial^\mu \tag{13.64}$$

where

$$p^\mu = \left(\frac{En}{c}, \mathbf{p}\right), \quad \partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right). \tag{13.65}$$

Thus:

$$En = i\hbar\frac{\partial}{\partial t}, \quad \mathbf{p} = -i\hbar\nabla. \tag{13.66}$$

Eqs.(13.61) to (13.63) therefore become:

$$EnA^R{}_x + ic\left(p_y A^R{}_z - p_z A^R{}_y\right) = 0 \tag{13.67}$$

$$EnA^R{}_y + ic\left(p_z A^R{}_x - p_x A^R{}_z\right) = 0 \tag{13.68}$$

$$EnA^R{}_z + ic\left(p_x A^R{}_y - p_y A^R{}_x\right) = 0. \tag{13.69}$$

Define the three-spinor:

$$\phi^R = \begin{bmatrix} A^R{}_x \\ A^R{}_y \\ A^R{}_z \end{bmatrix} \tag{13.70}$$

and:

$$\begin{aligned} \alpha\cdot\mathbf{p} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} p_x + \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{bmatrix} p_y + \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} p_z \\ &= i\begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix}. \end{aligned} \tag{13.71}$$

Then Eqs.(13.67) to (13.63) are:

$$\left(\frac{En}{c}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + i\begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix}\right)\begin{bmatrix} A^R{}_x \\ A^R{}_y \\ A^R{}_z \end{bmatrix} = 0 \tag{13.72}$$

or

$$\left(\frac{En}{c} + \alpha\cdot\mathbf{p}\right)\phi^R = 0. \tag{13.73}$$

Similarly:

$$\left(\frac{En}{c} - \alpha\cdot\mathbf{p}\right)\phi^L = 0 \tag{13.74}$$

where the three-spinors are defined as:

$$\phi^R = \begin{bmatrix} A^R{}_x \\ A^R{}_y \\ A^R{}_z \end{bmatrix}, \quad \phi^L = \begin{bmatrix} A^L{}_x \\ A^L{}_y \\ A^L{}_z \end{bmatrix}. \tag{13.75}$$

Eqs.(13.73) and (13.74) are the Majorana equations [20, 21]. They are Weyl-type equations, i.e. a Dirac equation with no mass term. Instead of Pauli matrices however, the $O(3)$ symmetry rotation matrices of Eq.(13.71) are used. Eqs.(13.73) and (13.74) are limits of

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\psi = 0. \tag{13.76}$$

when $m \longrightarrow 0$. Here

$$\psi = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \tag{13.77}$$

is a six-spinor analogous to the Dirac four-spinor of Eq.(13.37). Eq.(13.76) is a limit of the Evans wave equation:

$$(\Box + kT)\,\psi = 0. \tag{13.78}$$

The spinor ψ is obtained from the tetrad:

$$q^a{}_\mu = \begin{bmatrix} A^R{}_1 & A^R{}_2 & A^R{}_3 \\ A^L{}_1 & A^L{}_2 & A^L{}_3 \end{bmatrix} \tag{13.79}$$

defined by:

$$A^{(0)} \begin{bmatrix} e^R \\ e^L \end{bmatrix} = \begin{bmatrix} A^R{}_1 & A^R{}_2 & A^R{}_3 \\ A^L{}_1 & A^L{}_2 & A^L{}_3 \end{bmatrix} \begin{bmatrix} e^1 \\ e^2 \\ e^3 \end{bmatrix}. \tag{13.80}$$

This illustration shows that the Maxwell-Heaviside electromagnetism of the standard model is an example of a spin equation which is the massless special relativistic limit of the Evans wave equation. The symmetry in this case can be either $O(3)$ or $SU(3)$. Finally the Weinberg equation [22] is the spin equation for any integral or half integral spin, and the Weinberg equation is also a limit of the generally covariant Evans wave equation.

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Chapter 14

On The Origin Of Dark Matter As Spacetime Torsion

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Abstract

It is suggested qualitatively that the origin of dark matter in the universe is spacetime torsion in the gravitational sector of the Evans unified field theory. In the general spacetime manifold containing both curvature and torsion, the Newton inverse square law is affected by the fact that the first Bianchi identity used in the 1915 Einstein/Hilbert field theory is no longer obeyed geometrically. Consequent departures from the inverse square law can be interpreted as due to the effective and unseen mass of dark matter in a cosmological context. Torsion introduces mass - dark matter. Since dark matter does not radiate electromagnetically, its presence can be detected only indirectly. The torsion tensor in the Evans unified field theory obeys the same laws as electromagnetism within a C negative factor. Therefore the characteristics of dark matter may be similar to those of electromagnetism.

Key words: Evans field theory, torsion tensor, dark matter.

14.1 Introduction

A substantial fraction of the mass of the universe is thought to be made up of dark matter, which does not radiate and can therefore be detected only indirectly. It is suggested qualitatively in this paper that dark matter may be due to departures from the 1915 Einstein / Hilbert field theory due to the presence of spacetime torsion and the interaction of torsion with curvature. Gravitational general relativity [1,2] is almost always developed without torsion, in a spacetime containing curvature only. In Wald [1] for example there is only a brief mention of torsional theories of gravitation, and no development thereof. Similarly for Carroll [2]. The 1915 theory of Einstein and Hilbert is accurate to one part in about one hundred thousand for the sun [3] but in other cosmological contexts appears to be qualitatively unable to account for reproducible and repeatable observation [4], in particular, dark matter. In order to construct a generally covariant unified field theory [4]– [22] the torsion tensor becomes fundamentally important because it is the electromagnetic field within a fundamental vector potential magnitude $A^{(0)}$. The Palatini variation of general relativity [1, 2] is also required for a unified field theory, because in this variation the fundamental field is the tetrad [1,2] and not the symmetric metric of the Einstein Hilbert variation of general relativity (the original 1915 theory). The interaction of various types of radiated and matter fields is described [4]– [22] by Cartanś differential geometry, in which the torsion and curvature are related by the two Cartan structure equations [2] and the two Bianchi identities of differential geometry. The latter are more general than the two Bianchi identities of Riemann geometry used in the 1915 Einstein Hilbert field theory of pure gravitation. Therefore in a spacetime or base manifold where there is both torsion and curvature present simultaneously, departures from the 1915 theory are expected in general. It is well known [23] that Einstein himself thought of the 1915 theory as a beginning only, and from about 1925 to 1955 sought a unified field theory that is both objective (generally covariant) and causal. It is generally agreed [24] that the Evans unified field theory achieves this aim in one relatively straightforward way [4]– [22].

In Section 14.2 the fundamental differential geometry is defined of a field theory of gravitation in which torsion and curvature are both present in general and interact in general. This field theory is the gravitational sector of the Evans unified field theory [4]– [22]. In Section 14.3 the approximations are defined which are needed to reduce the general field theory (gravitational sector of the Evans unified field theory) to the Einstein Hilbert field theory of 1915. The major approximation is that the torsion tensor is assumed to vanish. This is not true in general of differential geometry, and in Riemann geometry the torsion tensor vanishes if and only if the Christoffel connection is assumed [2]. In general therefore the Newton inverse square law is obtained in the weak field limit if and only if the torsion vanishes. Reinstate the torsion and new physics is expected. A part of this new physics may be dark matter physics. It is already known [4]– [22] that the torsion tensor multiplied by $A^{(0)}$ is the field tensor of electromagnetism in the Evans unified field theory.

14.2 The Gravitational Sector Of The Evans Unified Field Theory

In the manifold with torsion and curvature both present, the Evans field theory is described by standard Cartan geometry [2,4]– [22] in terms of the two Cartan structure equations of differential geometry:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{14.1}$$

$$R^a{}_b = D \wedge \omega^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{14.2}$$

and the two Bianchi identities of differential geometry:

$$D \wedge T^a := R^a{}_b \wedge q^b \tag{14.3}$$

$$D \wedge R^a{}_b := 0. \tag{14.4}$$

In Eq.(14.1) T^a is the torsion form, q^a is the tetrad form, $\omega^a{}_b$ is the spin connection, $\omega^a{}_b$ is the Riemann form, $d\wedge$ denotes exterior derivative and $D\wedge$ denotes covariant exterior derivative. Cartan geometry is always defined by the Evans Lemma [4]– [22]:

$$\Box q^a{}_\mu := R q^a{}_\mu \tag{14.5}$$

where R is a well defined scalar curvature. Using the fundamental equation of general relativity:

$$R = -kT \tag{14.6}$$

in Eq.(14.5) produces the Evans wave equation:

$$(\Box + kT)\, q^a{}_\mu = 0 \tag{14.7}$$

which is the fundamental wave equation of all radiated and matter fields. Here k is Einsteins constant and T is the canonical energy momentum density. The wave equation (14.7) straightforwardly quantizes the gravitational field, which is the tetrad field.

14.3 Approximations To The General Theory

The original theory of general relativity, developed independently by Einstein and Hilbert in 1915, assumes that:

$$T^a = 0. \tag{14.8}$$

Therefore the Cartan structure equation (14.1) in this limit reduces to:

$$d \wedge q^a + \omega^a{}_b \wedge q^b = 0 \tag{14.9}$$

and the Bianchi identity (14.3) reduces to:

$$R^a{}_b \wedge q^b = 0. \tag{14.10}$$

The Newton inverse square law is obtained in the weak field limit of the geometry defined by Eqs.(14.8), (14.9) and (14.10). The presence of even an infinitesimal amount of spacetime torsion will produce a perturbation in the Bianchi identity (14.10) such that:

$$R^a{}_b \wedge q^b \neq 0 \tag{14.11}$$

implying that the connection is no longer a Christoffel connection. The perturbation will therefore lead to departures from the Newton inverse square law in the weak field limit and to departures from the Einstein Hilbert field theory in cosmological contexts. Evidently [3] such departures are too small to be observed in an earthbound laboratory because the Newton inverse square law is valid to within contemporary instrumental precision. They are also too small to be measured in the solar system, because the 1915 law is valid for the sun to within contemporary instrumental precision (NASA Cassini experiments [3]).

Dark matter is well known to exist in the universe, however, and it is suggested qualitatively that dark matter is due to the interaction of torsion with curvature in the more general theory of Section 14.2. There are numerous other cosmological anomalies [25] which are reproducible and repeatable. They are anomalies because they cannot be described by the 1915 theory, and so become candidates for investigation with the gravitational sector of the Evans unified field theory (Section 14.2) rather than by the 1915 field theory of Einstein and Hilbert (Section 14.3).

The Newtonian limit of the Evans field theory is a very special approximation of the general wave equation (14.7). Newton's theory happens to work well because of the instrumental limits of contemporary physics. With sufficiently sensitive instruments departures from the Newtonian laws would be observable in the laboratory and departures form the 1915 theory would be observable in the solar system. The well known force law of Newton:

$$\mathbf{F} = m\mathbf{g} \tag{14.12}$$

where $\mathbf{F}$ is the force on a particle of mass m and $\mathbf{g}$ is the acceleration due to gravity, is the non-relativistic, classical, limit of the Dirac equation. The latter is now known [4]– [22] to be the limit of Eq.(14.7) when:

$$kT \longrightarrow \frac{km}{V_0} = \frac{m^2c^2}{\hbar^2}. \tag{14.13}$$

Here m is the mass of a particle, $\hbar$ is the reduced Planck constant and c the speed of light. The Evans rest volume V_0 [4]– [22] is defined for all elementary particles (i.e. all radiated and matter fields now known) by:

$$V_0 = \frac{\hbar^2 k}{mc^2}. \tag{14.14}$$

The tetrad of the Dirac equation is [4]– [22]:

$$q^a{}_\mu = \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^L{}_1 & q^L{}_2 \end{bmatrix} \tag{14.15}$$

from which we obtain the Dirac spinor:

$$\psi = \begin{bmatrix} q^R{}_1 \\ q^R{}_2 \\ q^L{}_1 \\ q^L{}_2 \end{bmatrix} = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \tag{14.16}$$

where ϕ^R and ϕ^L are the right and left Pauli spinors.

Thus:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right)\psi = 0 \tag{14.17}$$

is the well known Dirac wave equation for a free particle. The free particle Schrödinger equation is the non-relativistic quantum limit of the Dirac equation:

$$\frac{\hbar^2}{2m}\nabla^2\psi = -i\hbar\frac{\partial\psi}{\partial t}. \tag{14.18}$$

Using the operator equivalence:

$$En = i\hbar\frac{\partial}{\partial t}, \quad \mathbf{p} = -i\hbar\nabla \tag{14.19}$$

the Newtonian limit of the Schrödinger equation is obtained:

$$En = \frac{p^2}{2m} \tag{14.20}$$

where En is Newtonian kinetic energy and p is Newtonian momentum. Eq.(14.20) may also be expressed as:

$$En = \frac{1}{2}mv^2 \tag{14.21}$$

and may be found from Eq.(14.12).

A series of approximations is therefore needed to reduce the Evans wave equation to the Dirac equation, and thence to the Schrödinger and Newton equations. One of these approximations is that the torsion tensor has to vanish in order to recover Newtonian dynamics. Reinstate the torsion tensor as in Section 14.2 and all of these well known equations of dynamics are affected, leading to a considerable amount of new physics given the instrumental precision required.

Similarly the Poisson equation of Newtonian dynamics is found from the Evans wave equation in the limit:

$$V^a \to V^\mu, \quad V^a = q^a{}_\mu V^\mu \tag{14.22}$$

and

$$q^a{}_\mu \longrightarrow 1 \tag{14.23}$$

where 1 in Eq(14.23) is the unit diagonal matrix. The limit (14.22) means that the base manifold approaches a Minkowski spacetime. The latter is the spacetime in differential geometry of the tangent spacetime to the base manifold at a point P [2]. If it is assumed that $q^a{}_\mu$ is essentially time independent, and when:

$$T \rightarrow \frac{m}{V} = \rho \tag{14.24}$$

the Evans wave equation (14.7) becomes the Poisson equation:

$$\nabla^2 q = k\rho \tag{14.25}$$

where we have written:

$$q = q^1{}_1 = q^2{}_2 = q^3{}_3 \sim 1. \tag{14.26}$$

Using:

$$k = \frac{8\pi G}{c^2} \tag{14.27}$$

and

$$\Phi = \frac{1}{2}c^2 q \tag{14.28}$$

we recover the standard Poisson equation used in Newtonian dynamics [2]:

$$\nabla^2 \Phi = 4\pi G\rho \tag{14.29}$$

from which the Newton inverse square law follows directly.

Therefore we have recovered the force law (14.12) and the inverse square law from the same equation, the Evans wave equation. This shows why gravitational and inertial mass is the same, they are both approximations to the same differential geometry. In the presence of torsion all of these well known laws of physics are affected, and so dark matter enters into consideration through the interaction of torsion with curvature. In order to describe dark matter physics, the Evans wave equation must be solved with given initial and boundary conditions for spin and gamma connections which are in general asymmetric in their lower two indices. This is a problem for the computer in general, although analytical solutions may be found to the Evans wave equation analogous to the Schwarzschild solution of the 1915 field equation of Einstein and Hilbert.

The Dirac, Schrödinger, Newton and Poisson equations of dynamics are all limits of the Evans wave equation when torsion is zero.

The torsion is described within a factor $A^{(0)}$ by the same equations as those of electromagnetism in the Evans unified field theory [4]– [22], i.e. by:

$$d \wedge T^a = -\left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b\right) \tag{14.30}$$

$$d \wedge \widetilde{T}^a = -\left(q^b \wedge \widetilde{R}^a{}_b + \omega^a{}_b \wedge \widetilde{T}^b\right). \tag{14.31}$$

Therefore if dark matter is described by torsion, the former behaves like electromagnetism without the presence of electric charge. Dark matter cannot be detected by electromagnetic radiation, and does not obey the 1915 theory in general. If there is a large amount of torsion present in a given region of the universe, then the 1915 theory will appear to be highly anomalous. Such anomalies are well known experimentally [25] and are reproducible and repeatable. There is no reason to expect the 1915 theory to be valid in regions of intense spacetime torsion, such as near a pulsar or example.

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Chapter 15

Generally Covariant Quantum Mechanics

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"Dedicated to the Late Jean-Pierre Vigier"

Abstract

In order to make quantum mechanics compatible with general relativity the Heisenberg uncertainty principle is given a generally covariant (or objective) and causal interpretation. The fundamental conjugate variables are shown to be quantities such as momentum density, angular momentum density and energy density in general relativity instead of quantities such as momentum, angular momentum and energy in special relativity. The generally covariant interpretation of quantum mechanics given in this paper agrees with the repeatable and reproducible experimental data of Croca et al. and of Afshar, data which show that the conventional Heisenberg uncertainty principle is qualitatively incorrect, and with it all the arguments of the Copenhagen school throughout the twentieth century. The correctly objective interpretation of quantum mechanics is given by the deterministic school of Einstein, de Broglie, Vigier and others. This is a direct result of the Evans unified field theory.

Key words: Evans unified field theory; generally covariant Heisenberg equation and Heisenberg uncertainty principle.

15.1 Introduction

Generally covariant quantum mechanics does not exist in the standard model because general relativity is causal and objective, the Copenhagen interpretation of quantum mechanics is acausal and subjective. This is the central issue of the great twentieth century debate in physics between the Copenhagen and deterministic schools of thought, an issue which is resolved conclusively in favor of the deterministic school by the Evans unified field theory [1]– [19]. At the root of the debate is the Heisenberg equation of motion [20], which in its simplest form is a rewriting of the operator equivalence condition of quantum mechanics:

$$p^\mu = i\hbar\partial^\mu. \tag{15.1}$$

Eq.(15.1) is an equation of special relativity where:

$$p^\mu = \left(\frac{En}{c}, \mathbf{p}\right) \tag{15.2}$$

is the energy momentum four vector, and:

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right). \tag{15.3}$$

Here En denotes energy, c is the vacuum speed of light, $\mathbf{p}$ is the linear momentum,$\hbar$ is the reduced Planck constant, and the partial four derivative ∂^μ is defined in terms of the time derivative and the ∇ operator in Eq.(15.3).

In Section 15.2 the operator equivalence (15.1) is rewritten as the Heisenberg equation of motion [20]. The Heisenberg uncertainty principle is a direct mathematical consequence of the Heisenberg equation and therefore is a direct mathematical consequence of Eq.(15.1). However, recent reproducible and repeatable experimental data [21]– [23] show that the Heisenberg uncertainty principle is violated completely in several independent ways. This means that the principle as it stands is completely incorrect, i.e. by many orders of magnitude. For example in the various experiments of Croca et al. [21] the principle is incorrect by nine orders of magnitude even at moderate microscope resolution. As the resolution is increased it becomes qualitatively wrong, i.e. the conjugate variable of position and momentum appear to commute precisely within experimental precision. The uncertainty principle states that if the conjugate variables are for example the position x and the component p_x of linear momentum then [20]:

$$\delta x\delta p_x \geq \frac{\hbar}{2}. \tag{15.4}$$

This means that the δx and δp_x variables cannot commute and the conventional Copenhagen interpretation is that they cannot be simultaneously observable [20] and cannot be simultaneously knowable. This subjective assertion violates causality and general relativity (i.e. objective physics) and has led to endless confusion for students. The experimental results of Croca et al [21] show that:

$$\delta x\delta p_x = 10^{-9}\frac{\hbar}{2} \tag{15.5}$$

at moderate microscope resolution. As the latter is increased, the experimental result is:

$$\delta x \delta p_x \longrightarrow 0 \tag{15.6}$$

in direct experimental contradiction of the uncertainty principle (15.4).

In the independent series of experiments by Afshar [22], carried out at Harvard and elsewhere, the photon and electromagnetic wave are shown to be simultaneously observable, indicating independently that the position and momentum or time and energy conjugate variables commute precisely to within experimental precision. In a third series of independent experiments [23], on two dimensional materials near absolute zero, the Heisenberg uncertainty principle again predicts diametrically the incorrect experimental result to within instrumental precision. Each type of experiment [21]– [23] is independently reproducible and repeatable to high precision.

In Section 15.3 this major crisis for the standard model is resolved straightforwardly through the use of the appropriate densities of conjugate variables in general relativity. The experimentally well tested Eq.(15.1) is retained, but the fundamental conjugate variables are carefully redefined within the generally covariant Evans unified field theory, which derives generally covariant quantum mechanics from Cartans differential geometry [1]– [19] for all radiated and matter fields. The result is a generally covariant quantum mechanics in agreement with the most recent experiments [21]– [23] and philosophically compatible with the causal and objective Evans unified field theory. The latter denies subjectivity and acausality in natural philosophy.

15.2 A Simple Derivation Of The Heisenberg Equation Of Motion

In its simplest form the Heisenberg equation of motion [20] is:

$$[x, p_x]\, \psi = i\hbar\psi \tag{15.7}$$

where ψ is the wave-function. Eq.(15.7) means that the commutator:

$$[x, p_x] = xp_x - p_x x \tag{15.8}$$

operates on the wave-function. From Eq.(15.1):

$$p_x = -i\hbar \frac{\partial}{\partial x} \tag{15.9}$$

and so p_x becomes a differential operator acting on ψ . The position x is interpreted as a simple multiple, i.e. x multiplies anything that follows it. Using the Leibnitz Theorem and these operator rules it follows that:

$$
\begin{aligned}
[x, p_x]\,\psi &= (xp_x - p_x x)\,\psi \\
&= x\,(p_x\psi) - p_x\,(x\psi) \\
&= x\,(p_x\psi) - (p_x x)\,\psi - x\,(p_x\psi) \\
&= -\,(p_x x)\,\psi \\
&= \left(i\hbar\frac{\partial x}{\partial x}\right)\psi \\
&= i\hbar\psi
\end{aligned}
\tag{15.10}
$$

and it is seen that the Heisenberg equation is a rewriting of Eq.(15.1), a component of which is Eq.(15.9). Using standard methods [20] we obtain the famous Heisenberg uncertainty principle

$$\delta x \delta p_x \geq \frac{\hbar}{2} \tag{15.11}$$

from Eq.(15.9) and thus from Eq.(15.1), the famous wave particle duality of de Broglie. This derivation is given to show that the uncertainty principle, which has dominated thought in physics for nearly a century, is another statement of the de Broglie wave particle duality.

The subjective and acausal interpretations inherent in the Heisenberg uncertainty principle were rejected immediately by Einstein and his followers, as is well known, and later also rejected by de Broglie and his follower Vigier. These are the acknowledged masters of the deterministic school of physics in the twentieth century. The data [21]– [23] now show with pristine clarity that the deterministic school is right, the Copenhagen school is wrong. It is important to realize that the deterministic school accepts quantum mechanics, i.e accepts Eq.(15.1) but rejects the INTERPRETATION of Eq.(15.7) by the Copenhagen school. The ensuing debate became protracted due to a lack of a unified field theory and a lack of experimental data. Both are now available.

15.3 The Generally Covariant Heisenberg Equation

Neither school discovered the reason why Eq.(15.11) is so wildly incorrect. With the emergence of the Evans unified field theory (2003 to present), more than a hundred years after special relativity (1892 - 1905), we now know why the experiments [21]– [23] give the results they do. The error in the Copenhagen school's philosophy is the obvious one - the subjective reliance on conjugate variables which are not correctly objective (generally covariant). They are not DENSITIES, as required by the fundamentals of general relativity and the Evans unified field theory. In order to develop a correctly objective quantum mechanics the momentum p_x has to be replaced by a momentum density $\overline{p_x}$ and the angular momentum $\hbar$ by an angular momentum density $\overline{\hbar}$. The reason is that the fundamental law of general relativity [1]– [19] is:

$$R = -kT \tag{15.12}$$

where T is the scalar valued canonical energy-momentum density, R is a well defined scalar curvature, and k is Einstein's constant. In the rest frame T reduces to the mass density of an elementary particle:

$$T \rightarrow \frac{m}{V_0} \tag{15.13}$$

and within a factor c^2 this is the rest energy density:

$$\overline{En_0} = \frac{mc^2}{V_0}. \tag{15.14}$$

Here V_0 is the Evans rest volume

$$V_0 = \frac{\hbar^2 k}{mc^2} \tag{15.15}$$

where m is the elementary particle mass.

Define the experimental momentum density for a given instrument by:

$$\overline{p_x} = \frac{p_x}{V} \tag{15.16}$$

and define the fundamental angular momentum or action density by:

$$\overline{\hbar} = \frac{\hbar}{V_0}. \tag{15.17}$$

Here V is a macroscopic volume defined by the apparatus being used, or the volume occupied by the momentum component p_x. The quantum $\overline{\hbar}$ is the fundamental density of the reduced Planck constant:

$$\overline{\hbar} = \frac{mc^2}{\hbar k} \tag{15.18}$$

i.e. the rest energy divided by the product of $\hbar$ and k. Eq(15.18) means that the quantum of action occupies the Evans rest volume V_0. For any particle, including the six quarks, the photon and the neutrinos, the graviton and the gravitino. This deduction follows from the special relativistic limit of the Evans wave equation for one particle is [1]– [19]:

$$kT = \frac{km}{V_0} = \frac{m^2c^2}{\hbar^2}. \tag{15.19}$$

In general relativity therefore Eq.(15.9) becomes:

$$\overline{p_x} = -i\overline{\hbar}\frac{\partial}{\partial x} \tag{15.20}$$

i.e.:

$$\overline{p_x}\psi = -i\overline{\hbar}\frac{\partial\psi}{\partial x}. \tag{15.21}$$

Eq.(15.9), which is precisely verified experimentally in quantum mechanics, is therefore the same as:

$$\overline{p_x} = -i\frac{V_0}{V}\overline{\hbar}\frac{\partial}{\partial x} \tag{15.22}$$

which is a special case of the fundamental wave particle duality in general relativity:

$$\overline{p}^\mu = i\frac{V_0}{V}\overline{\hbar}\partial^\mu. \tag{15.23}$$

The generally covariant Heisenberg equation is therefore:

$$[x, \overline{p_x}] = i\frac{V_0}{V}\overline{\hbar} \tag{15.24}$$

and the fundamental conjugate variables of generally covariant quantum mechanics are x and $\overline{p_x}$. The fundamental quantum is $\overline{\hbar}$.

Experimentally for a macroscopic volume V:

$$V_0 \ll V \tag{15.25}$$

and so

$$[x, \overline{p_x}] \sim 0 \tag{15.26}$$

which implies

$$\delta x \sim 0, \quad \delta\overline{p_x} \sim 0 \tag{15.27}$$

is quite possible experimentally. Therefore what is being observed experimentally in the Croca and Afshar experiments is $\overline{p_x}$ and not p_x, and $\overline{\hbar}$ and not $\hbar$. This deduction means that a particle coexists with its matter wave, as inferred by de Broglie. For electromagnetism this coexistence has been clearly observed by Afshar [22] in modified Young experiments which are precise, reproducible and repeatable.

The fundamental conjugate variables are therefore position and momentum density, or time and energy density, and not position and momentum, or time and energy as in the conventional theory [20] and as in the Copenhagen interpretation. The wave function is always the tetrad, and this is always defined causally and objectively by Cartans differential geometry [1]– [19]. There is no uncertainty or acausality in geometry and none in physics. The fundamental wave equation of generally covariant quantum mechanics is the Evans wave equation [1]– [19]:

$$(\Box + kT)\, q^a{}_\mu = 0 \tag{15.28}$$

which reduces to all other wave equations of physics, and thence to other equations of physics in well defined limits.

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Chapter 16

General Covariance And Coordinate Transformation In Classical And Quantum Electrodynamics

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Abstract

In a unified field theory classical and quantum electrodynamics must be generally covariant, and not Lorentz covariant as in the contemporary standard model. This means that electrodynamics must be objective under the general coordinate transformation: equivalently the effect of gravitation on electrodynamics must be considered. As an illustration of this general principle the Lorentz force law is derived from a general coordinate transformation of the torsion tensor of standard differential geometry. In the limit of special relativity the general coordinate transformation becomes a Lorentz transformation and the Lorentz force law is recovered in the absence of gravitation.

Key words: Evans unified field theory, general coordinate transformation, general covariance, Lorentz covariance, Lorentz force law.

16.1 Introduction

In the contemporary standard model neither classical nor quantum electrodynamics is an objective investigation in natural philosophy. In consequence the effect of gravitation on electromagnetism cannot be investigated in the standard model, a major weakness of contemporary physics. Recently an objective or generally covariant unified field theory has been developed [1]– [20], a theory which shows how gravitation and electromagnetism may be able to influence each other mutually. In this paper the Evans unified field theory is illustrated through the general coordinate transformation of the torsion tensor in differential geometry [21]. Within a factor $A^{(0)}$, the torsion tensor is the electromagnetic field tensor. In Section 16.2 the generally covariant form of the Lorentz force law is obtained through a general coordinate transformation of the electromagnetic field tensor. In the limit of special relativity Section 16.3 shows that Lorentz force law of the standard model is obtained as a well defined limit of the generally covariant, or objective, Lorentz force law of the Evans unified field theory. The correctly objective Lorentz force law shows how gravitation affects the Lorentz force law of the standard model.

16.2 General Coordinate Transformation

The vector transformation law of general relativity shows that the vector field:

$$V = V^{\mu}\hat{e}_{(\mu)} \tag{16.1}$$

is invariant under the general coordinate transformation

$$V^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} V^{\mu}, \quad \partial_{\mu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \partial_{\mu} \tag{16.2}$$

where

$$\hat{e}_{(\mu)} = \partial_{\mu}. \tag{16.3}$$

Here V^{μ} denotes the vector components [21] and $\hat{e}_{(\mu)}$ the set of basis vectors. The vector components x^{μ} are those of the position four vector, and ∂_{μ} is the partial derivative four vector. Therefore Eq.(16.3) defines the coordinate basis. In Eqs.(16.1) to (16.3) the primed frame is related to the unprimed frame through the general coordinate transformation. The coordinate basis (3) is used conventionally [21] in gravitational general relativity. This is the Einstein Hilbert variation of general relativity, where the fundamental field is the symmetric metric tensor. The Lorentz transform of special relativity is the special case of Eq.(16.2) where:

$$V^{\mu'} = \Lambda^{\mu'}{}_{\mu} V^{\mu}, \quad x^{\mu'} = \Lambda^{\mu'}{}_{\mu} x^{\mu} \tag{16.4}$$

Here $\Lambda^{\mu'}{}_{\mu}$ is the well known [21] Lorentz transform matrix.

The tensor transformation law of general relativity [21] is

$$T^{\mu'_1\cdots\mu'_k}{}_{\nu'_1\ldots\nu'_l} = \left(\frac{\partial x^{\mu'_1}}{\partial x^{\mu_1}}\cdots\frac{\partial x^{\mu'_k}}{\partial x^{\mu_k}}\right)\left(\frac{\partial x^{\nu}}{\partial x^{\nu'}}\cdots\frac{\partial x^{\nu_l}}{\partial x^{\nu'_l}}\right)T^{\mu_1\cdots\mu_k}{}_{\nu_1\ldots\nu_l} \tag{16.5}$$

in a notation which can be built up from the notation of Eq.(16.1). An important example of Eq.(16.5) is the metric transformation law:

$$g_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\nu}}{\partial x^{\nu'}}g_{\mu\nu} \tag{16.6}$$

Eq (16.6) is the fundamental axiom of relativity theory [1]– [21], a tensor transforms generally and covariantly, producing a new tensor. In Eq. (16.6) the tensor is the fundamental field, implying that the field is covariant to an observer moving arbitrarily with respect to another observer. In the Evans unified field theory this axiom is applied to all radiated and matter fields [1]– [21] self consistently using Cartan geometry.

The covariant and exterior derivatives [21] of a vector transform covariantly in relativity theory, whereas the ordinary partial derivative does not. For example, the covariant derivative transforms covariantly as:

$$D_{\mu'}V^{\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\nu'}}{\partial x^{\nu}}D_{\mu}V^{\nu} \tag{16.7}$$

provided that the Christoffel symbol transforms as:

$$\Gamma^{\nu'}{}_{\mu'\lambda'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial x^{\nu'}}{\partial x^{\nu}}\Gamma^{\nu}{}_{\mu\lambda} - \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial^2 x^{\nu'}}{\partial x^{\mu}\partial x^{\lambda}}. \tag{16.8}$$

The Christoffel symbol itself does not transform as a tensor, as is well known. As a final example the torsion tensor [1]– [21] transforms covariantly as a three index tensor:

$$T^{\lambda'}{}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\nu}}{\partial x^{\nu'}}\frac{\partial x^{\lambda'}}{\partial x^{\lambda}}T^{\lambda}{}_{\mu\nu}\,. \tag{16.9}$$

In the unified field theory [1]– [20] the Palatini variation of general relativity is used, a variation in which the fundamental field is the tetrad, a vector-valued one-form of Cartan differential geometry [21]. The general transformation law for forms is:

$$T^{a'\mu'}{}_{b'\nu'} = \Lambda^{a'}{}_{a}\frac{\partial x^{\mu'}}{\partial x^{\mu}}\Lambda_{b'}{}^{b}\frac{\partial x^{\mu}}{\partial x^{\nu'}}T^{a\mu}{}_{b\nu} \tag{16.10}$$

where $\Lambda^{a'}{}_{a}$ is a Lorentz transform defined in the tangent spacetime by [21]:

$$\eta_{a'b'} = \Lambda_{a'}{}^{a}\Lambda_{b'}{}^{b}\eta_{ab} \tag{16.11}$$

Here $\Lambda_{a'}{}^{a}$ and $\Lambda_{b'}{}^{b}$ are inverse Lorentz transforms. From Eq.(16.10) a vector valued one-form $X^{a}{}_{\mu}$ transforms as:

$$X^{a'}{}_{\mu'} = \Lambda^{a'}{}_{a}\frac{\partial x^{\mu}}{\partial x^{\mu'}}X^{a}{}_{\mu} \tag{16.12}$$

v where the Lorentz transform ${\Lambda^{a'}}_a$ in the tangent spacetime is defined by [21]:

$$x^{a'} = {\Lambda^{a'}}_a x^a. \tag{16.13}$$

If μ is fixed then:

$$ {A^{a'}}_\mu = {\Lambda^{a'}}_\mu {A^a}_\mu \tag{16.14}$$

and if a is fixed:

$$ {A^{a'}}_\mu = \left(\frac{\partial x^\mu}{\partial x^{\mu'}}\right) {A^a}_\mu. \tag{16.15}$$

The torsion form in Cartan differential geometry is a vector valued two-form defined by:

$$ {T^a}_{\mu\nu} = \partial_\mu {q^a}_\nu - \partial_\nu {q^a}_\mu + {\omega^a}_{\mu b} {q^b}_\nu - {\omega^a}_{\nu b} {q^b}_\mu \tag{16.16}$$

where ${q^a}_\mu$ is the tetrad and where ${\omega^a}_{\mu b}$ is the spin connection. The torsion form transforms as a tensor:

$$ {T^{a'}}_{\mu'\nu'} = {\Lambda^{a'}}_a \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} {T^a}_{\mu\nu}\,. \tag{16.17}$$

In the unified field theory [1]– [20] the electromagnetic field tensor is also a vector valued two-form defined by:

$$ {F^a}_{\mu\nu} = A^{(0)} {T^a}_{\mu\nu} \tag{16.18}$$

where $A^{(0)}$ is the vector potential magnitude. The generally covariant Lorentz force law is therefore expressed most generally as:

$$ {F^{a'}}_{\mu'\nu'} = {\Lambda^{a'}}_a \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} {F^a}_{\mu\nu}\,. \tag{16.19}$$

The mutual effect of gravitation and electromagnetism within this law is contained within Eq.(16.19). For fixed a:

$$ {F^a}_{\mu'\nu'} = {\Lambda^{a'}}_a \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} {F^a}_{\mu\nu} \tag{16.20}$$

and so for fixed a the generally covariant Lorentz force law is described by the metric transformation law (16.6). In order to calculate the effect of gravitation on the Lorentz force law we need know only the metric transformation law for a given metric, defined by:

$$ g_{\mu\nu} = {q^a}_\mu {q^b}_\nu \eta_{ab}. \tag{16.21}$$

Here η_{ab} is the Minkowski metric [21] of the tangent spacetime.

16.3 Special Relativistic Limit

In the special relativistic limit of Eq.(16.20) we obtain the Lorentz transformation:

$$ {F^a}_{\mu'\nu'} = {\Lambda^\mu}_{\mu'} {\Lambda^\nu}_{\nu'} {F^a}_{\mu\nu}\,. \tag{16.22}$$

for each index a. The latter is a polarization index. Since a appears on both sides of Eq.(16.22) it may be omitted for ease of notation. The conventional Lorentz transform of the electromagnetic field [22] is therefore obtained:

$$F_{\mu'\nu'} = \Lambda^{\mu}{}_{\mu'} \Lambda^{\nu}{}_{\nu'} F_{\mu\nu}. \tag{16.23}$$

In vector notation it is well known [22] that Eq.(16.23) is:

$$\mathbf{E}' = \gamma \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) + \cdots \tag{16.24}$$

$$\mathbf{B}' = \gamma \left(\mathbf{B} - \frac{\mathbf{v}}{c} \times \mathbf{E} \right) + \cdots \tag{16.25}$$

where $\mathbf{E}$ denotes electric field strength in volt m^{-1} and $\mathbf{B}$ is magnetic flux density. Here

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \tag{16.26}$$

originateS in the Lorentz transform matrix of special relativity. The Lorentz force law in S.I. units is:

$$\frac{d\mathbf{p}}{dt} = e \left(\mathbf{E} + \frac{v}{c} \times \mathbf{B} \right) \tag{16.27}$$

where $\mathbf{p}$ is linear momentum and e is electric charge, and the Lorentz force law holds at non-relativistic velocities, where

$$\gamma \sim 1 \tag{16.28}$$

From Eq.(16.23) the magnetic induction due to the Lorentz transformation at non-relativistic velocities is [22]:

$$\mathbf{B} = \frac{e}{c} \frac{\mathbf{v} \times \mathbf{r}}{r^3} \tag{16.29}$$

which is the Ampère Biot Savart law. It is seen that the Lorentz force law is built up from a sum of $\mathbf{E}$ and $\frac{\mathbf{v}}{c} \times \mathbf{B}$ in Eq.(16.24) in the non-relativistic limit.

The correct laws of electrodynamics are therefore obtained from the Evans unified field theory and from the generally covariant transformation (16.19) of the electromagnetic field tensor. The effect of gravitation on these well known laws of electrodynamics may therefore be calculated for a given metric.

Finally, in quantum electrodynamics [1]– [20] the tetrad is the fundamental field and the tetrad transforms according to Eq.(16.12).

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Chapter 17

The Role Of Gravitational Torsion In General Relativity: The S Tensor

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Abstract

The conventional definition of the Riemann tensor is shown to be incomplete because the torsional component is missing. The commutator of covariant derivatives acting on the four vector is shown to produce a tensor that is conventionally antisymmetric in its first two indices (the conventional curvature or Riemann tensor) stemming from the use of the Christoffel connection. More generally both the Riemann and torsional tensors are asymmetric in their first two indices because there is no torsion free condition in general. The complete tensor is the sum of these two tensors and is named the S tensor, and the generalized Einstein Hilbert field equation deduced for the S tensor. In this way spin or torsion is introduced into general relativity in a novel and fundamental manner, and the ramifications of this modification work through into all areas of dynamics.
Key words: Riemann tensor, torsion, commutator of covariant derivatives, round trip with covariant derivatives, general relativity.

17.1 Introduction

The theory of relativity [1] is based conventionally on Riemann geometry and the use of the Christoffel connection [2], which is symmetric in its lower two indices. The Einstein Hilbert field equation is deduced from the second Bianchi identity with the torsion free condition stemming from the Christoffel connection. In consequence all the information given from considerations of gravitational torsion is lost. Recently [3]– [25] it has been realized that the electromagnetic field tensor is spacetime torsion within a C negative vector potential magnitude $A^{(0)}$. This is electromagnetic torsion as distinct from the novel gravitational torsion considered in this paper. Therefore torsion is fundamentally important in relativity theory and cannot be neglected. The role of torsion is seen most clearly through the Cartan structure equations and the Bianchi identities of Cartan geometry.

In Section 17.2 the commutator of covariant derivatives acting on the four vector V^μ in the four dimensions of spacetime is shown to produce in general a sum of two tensors, a sum that premultiplies the vector itself. In addition there are four other terms which premultiply the four derivative of the vector. One of the terms that premultiply the four vector itself has the same structure as the conventional Riemann tensor, but in general the connections within this tensor are asymmetric in their lower two indices, and are not torsion free and are not Christoffel connections in general. The second tensor premultiplying the vector itself is novel to this work, and is named the torsional tensor. The symmetries of the various connections within the torsional tensor are determined by the commutator itself. The sum of these two tensors is named the S tensor in order to distinguish it from the conventional Riemann tensor. The S tensor is therefore defined as the sum of the two tensors that premultiply the vector itself. The S tensor is always needed for a complete description of gravitation in a spacetime with both curvature and torsion present - the Evans spacetime of unified field theory [3]– [25].

In Section 17.3 the generalization of the Einstein Hilbert field equation is deduced for the S tensor, showing the presence of novel terms due to gravitational torsion. In general gravitational torsion affects cosmological observations, but gravitational torsion is neglected in conventional general relativity. The latter appears to be very accurate for the solar system [26] but in other contexts appears to be very inaccurate [27]. Therefore the presence of gravitational torsion is indicated experimentally by data which cannot be explained with the conventional Riemann tensor. This is unsurprising in retrospect because the Riemann tensor is always predicated on the assumption that the connection is the Christoffel connection. This assumption is equivalent to assuming that there is no torsion in the universe, and there is no a priori reason why torsion should be absent, in unified field theory, torsion is the fundamental electromagnetic field itself.

17.2 Derivation Of The S Tensor

The S tensor is derived straightforwardly by operating on the four-vector V^ρ with the commutator of covariant derivatives. This is how the Riemann tensor is derived conventionally [2], but with the torsion free condition always assumed. There are four terms missing from the derivation by Carroll [2], a derivation which is corrected as follows to produce the S tensor.

Consider the commutator of covariant derivatives D_μ, acting on the four vector V^ρ in Evans spacetime:

$$[D_\mu, D_\nu]\, V^\rho = (D_\mu D_\nu - D_\nu D_\mu)\, V^\rho \tag{17.1}$$

The covariant derivative is defined by:

$$D_\nu V^\rho = \partial_\nu V^\rho + \Gamma^\rho{}_{\nu\sigma} V^\sigma \tag{17.2}$$

and in general the connection $\Gamma^\rho{}_{\nu\sigma}$ is asymmetric in its lower two indices, indicating the simultaneous presence of curving and spinning:

$$\Gamma^\rho{}_{\nu\sigma} \neq \Gamma^\rho{}_{\sigma\nu}. \tag{17.3}$$

The Christoffel connection is symmetric in its lower two indices:

$$\Gamma^\rho{}_{\nu\sigma} \neq \Gamma^\rho{}_{\sigma\nu} \tag{17.4}$$

indicating the absence of spinning or torsion, but the presence of curving. From Eqs. (17.1) and (17.2):

$$\begin{aligned} [D_\mu, D_\nu]\, V^\rho = {} & \partial_\mu \left(\partial_\nu V^\rho + \Gamma^\rho{}_{\nu\sigma} V^\sigma\right) \\ & - \Gamma^\lambda{}_{\mu\nu} \left(\partial_\lambda V^\rho + \Gamma^\rho{}_{\lambda\sigma} V^\sigma\right) \\ & + \Gamma^\rho{}_{\mu\sigma} \left(\partial_\nu V^\sigma + \Gamma^\sigma{}_{\nu\lambda} V^\lambda\right) - (\mu \leftrightarrow \nu). \end{aligned} \tag{17.5}$$

Now use the Leibnitz Theorem to obtain:

$$\begin{aligned} [D_\mu, D_\nu]\, V^\rho = {} & \partial_\mu \partial_\nu V^\rho + \left(\partial_\mu \Gamma^\rho{}_{\mu\sigma}\right) V^\rho + \Gamma^\rho{}_{\mu\sigma} \partial_\mu V^\sigma \\ & - \Gamma^\lambda{}_{\mu\nu} \partial_\lambda V^\rho - \Gamma^\lambda{}_{\mu\nu} \Gamma^\rho{}_{\lambda\sigma} V^\sigma \\ & + \Gamma^\rho{}_{\mu\sigma} \partial_\nu V^\sigma + \Gamma^\rho{}_{\mu\sigma} \Gamma^\sigma{}_{\nu\lambda} V^\lambda \\ & - \partial_\nu \partial_\mu V^\rho - \left(\partial_\nu \Gamma^\rho{}_{\mu\sigma}\right) V^\sigma - \Gamma^\rho{}_{\mu\sigma} \partial_\nu V^\sigma \\ & + \Gamma^\lambda{}_{\nu\mu} \partial_\lambda V^\rho + \Gamma^\lambda{}_{\nu\mu} \Gamma^\rho{}_{\lambda\sigma} V^\sigma \\ & - \Gamma^\rho{}_{\nu\sigma} \partial_\mu V^\sigma - \Gamma^\rho{}_{\nu\sigma} \Gamma^\sigma{}_{\nu\lambda} V^\lambda \end{aligned}. \tag{17.6}$$

Finally rearrange terms and dummy indices to obtain:

$$\begin{aligned} [D_\mu, D_\nu]\, V^\rho = {} & \Big(\partial_\mu \Gamma^\rho{}_{\nu\sigma} - \partial_\nu \Gamma^\rho{}_{\mu\sigma} + \Gamma^\rho{}_{\mu\lambda} \Gamma^\lambda{}_{\nu\sigma} - \Gamma^\rho{}_{\mu\lambda} \Gamma^\lambda{}_{\mu\sigma} \\ & + \left(\Gamma^\lambda{}_{\nu\mu} - \Gamma^\lambda{}_{\mu\nu}\right) \Gamma^\rho{}_{\lambda\sigma}\Big) V^\sigma \\ & - \left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}\right) \partial_\lambda V^\rho \\ & + \Gamma^\rho{}_{\mu\sigma} \partial_\nu V^\sigma - \Gamma^\rho{}_{\nu\sigma} \partial_\mu V^\sigma \\ & := \left(R^\rho{}_{\sigma\mu\nu} - \Gamma^\rho{}_{\lambda\sigma} T^\lambda{}_{\mu\nu}\right) V^\sigma - T^\lambda{}_{\mu\nu}\, \partial_\lambda V^\rho \\ & + \Gamma^\rho{}_{\mu\sigma} \partial_\nu V^\sigma - \Gamma^\rho{}_{\nu\sigma} \partial_\mu V^\sigma. \end{aligned} \tag{17.7}$$

The S tensor is defined as the sum:

$$R^{\rho}{}_{\sigma\mu\nu} := R^{\rho}{}_{\sigma\mu\nu} - \Gamma^{\rho}{}_{\lambda\sigma} T^{\lambda}{}_{\mu\nu} \tag{17.8}$$

Of the general Riemann tensor $R^{\rho}{}_{\sigma\mu\nu}$ (denoted henceforth as the R tensor) and the general torsional tensor $R^{\rho}{}_{\sigma\mu\nu}$ (denoted henceforth as the T tensor):

$$T^{\rho}{}_{\sigma\mu\nu} := -\Gamma^{\rho}{}_{\lambda\sigma} T^{\lambda}{}_{\mu\nu}\,. \tag{17.9}$$

Note carefully that the T tensor is different from the conventional torsion tensor used in Cartan geometry [2]:

$$T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}. \tag{17.10}$$

The T tensor is one of the terms that premultiplies V^{σ} in Eq.(17.7) while the conventional torsion tensor (17.10) premultiplies $\partial_{\lambda} V^{\rho}$ in Eq. (17.6). There are two other terms present in Eq.(17.7) which are incorrectly omitted by Carroll [2]. These terms, which premultiply $\partial_{\nu} V^{\sigma}$ and $\partial_{\mu} V^{\sigma}$, also have a fundamental physical significance in relativity theory but will be considered in future work. The S, T and R tensors are by definition all antisymmetric in their last two indices μ and ν, but in general are asymmetric in their first two indices ρ and σ. The conventional Riemann tensor is antisymmetric in its first two indices ρ and σ because of the torsion free condition used in deriving it [2] . The same torsion free condition means that the conventional Ricci and metric tensors [2] are symmetric. More generally they are asymmetric [3]– [25] and in general there is no unique Ricci type tensor definable from the S tensor by index contraction. Therefore the conventional Einstein Hilbert field equation is a special case of many possible field equations of relativity and unified field theory [3]– [25].

Due to the antisymmetry in μ and ν the S tensor obeys the identities:

$$S_{\rho\sigma\mu\nu} + S_{\rho\mu\nu\sigma} + S_{\rho\nu\sigma\mu} := 0 \tag{17.11}$$

and

$$D_{\lambda} S_{\rho\sigma\mu\nu} + D_{\rho} S_{\sigma\lambda\mu\nu} + D_{\sigma} S_{\lambda\rho\mu\nu} := 0 \tag{17.12}$$

which are generalizations of the first and second Bianchi identities obeyed by the conventional Riemann tensor. The Bianchi identities are examples of the Jacobi identity:

$$\begin{aligned} &[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = \\ &ABC - BCA - ACB + CBA \\ &+ BCA - CAB - BAC + ACB \\ &+ CAB - ABC - CBA + BAC \\ &:= 0. \end{aligned} \tag{17.13}$$

The second Bianchi identity is most generally a relation between covariant derivatives:

$$[[D_{\lambda}, D_{\rho}], D_{\sigma}] + [[D_{\rho}, D_{\sigma}], D_{\lambda}] + [[D_{\sigma}, D_{\lambda}], D_{\rho}] := 0 \tag{17.14}$$

and this is true for any connection. As first shown by Feynman (17.29) the Jacobi identity can be derived by a round trip of covariant derivatives around a cube. In the condensed notation of differential geometry [2]– [25] the identities (17.11) and (17.12) become:

$$S^a{}_b \wedge q^b := 0 \tag{17.15}$$

$$D \wedge S^a{}_b := 0 \tag{17.16}$$

where q^b is the tetrad form and where $D\wedge$ is the covariant exterior derivative of differential geometry.

17.3 Field Equation For The S Tensor

The well known historical route to the Einstein Hilbert field equation is adhered to in this section, but the end result is more general, because it considers non-zero torsion. The first step is to define the S tensor with lowered indices:

$$S_{\rho\sigma\mu\nu} = g_{\rho\lambda} S^\lambda{}_{\sigma\mu\nu} \tag{17.17}$$

No assumptions are made concerning the symmetry of the metric tensor $g_{\rho\lambda}$. In general it is a tensor with symmetric and asymmetric components. This can be seen using differential geometry, in which the symmetric metric is the dot product of two tetrads:

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{ab} \tag{17.18}$$

where η_{ab} is the Minkowski metric. The asymmetric metric is the wedge product of two tetrads:

$$g^c{}_{\mu\nu} = -g^c{}_{\nu\mu} = q^a{}_\mu \wedge q^b{}_\nu \tag{17.19}$$

and for each index c is an antisymmetric tensor of the base manifold, Q.E.D. The most general metric is the outer or tensor product of two tetrads:

$$q^{ab}{}_{\mu\nu} = q^a{}_\mu q^b{}_\nu . \tag{17.20}$$

and for index do ab is an asymmetric tensor of the base manifold, Q.E.D. Therefore the symmetric metric is a special case (the symmetric part) of the most general metric formed form the tensor or outer product of two tetrads. Since tetrads are always mixed index tensors [2]– [25], a dot, wedge and tensor product of two tetrads may always be defined, and so the asymmetric metric may always be defined in the n dimensional manifold using the principles of standard differential geometry. The asymmetric metric $g_{\mu\nu}$ in Riemann geometry is thus A defined for a given index ab of the tangent space to the n dimensional base manifold at point P. This tangent space always exists but was not considered in Riemann geometry (which predated differential geometry by many years). This appears to be the root cause of the incorrect assertion sometimes made that the metric must always be a symmetric tensor. Therefore, as in Eq. (17.17) it is always possible to define the S tensor with lowered indices using a metric of any symmetry. It is understood that Eq. (17.17) applies in the base manifold for each ab index of the tangent spacetime in general.

Now make a double index contraction on the identity (17.12):

$$g^{\nu\sigma} g^{\mu\lambda} \left(D_\lambda S_{\rho\sigma\mu\nu} + D_\rho S_{\sigma\lambda\mu\nu} + D_\sigma S_{\lambda\rho\mu\nu} \right) := 0 \tag{17.21}$$

and define:

$$D^\mu S_{\rho\mu} := -\left(g^{\mu\lambda} D_\lambda\right) \left(g^{\nu\sigma} S_{\rho\sigma\mu\nu}\right) \tag{17.22}$$

$$D^\nu S_{\rho\nu} := -\left(g^{\nu\sigma} D_\sigma\right) \left(g^{\mu\lambda} S_{\sigma\lambda\mu\nu}\right) \tag{17.23}$$

$$D_\rho S := D_\rho \left(g^{\nu\sigma} g^{\mu\lambda} S_{\sigma\lambda\mu\nu}\right). \tag{17.24}$$

The sign difference convention comes from the antisymmetry of the S tensor in μ and ν. This convention, used by Einstein in 1915, is defined as follows. If indices are in the same order in the metric and in the tensor multiplied by the metric, then the resulting sign is positive. If indices are in the opposite order in the tensor to the index order in the metric, then the sign is negative. Adhering to this convention then:

$$D^\mu S_{\rho\mu} - D_\rho S + D^\nu S_{\rho\nu} := 0 \tag{17.25}$$

i.e.

$$D^\mu S_{\rho\mu} - \frac{1}{2} D_\rho S := 0 \tag{17.26}$$

or

$$S_{\rho\nu} = \frac{1}{4} S g_{\rho\nu}. \tag{17.27}$$

Finally use:

$$D_\rho = g_{\rho\mu} D^\mu \tag{17.28}$$

to obtain:

$$D^\mu \left(S_{\rho\mu} - \frac{1}{2} S g_{\rho\mu} \right) := 0. \tag{17.29}$$

The field equation is obtained by the equation:

$$D^\mu \left(S_{\rho\mu} - \frac{1}{2} S g_{\rho\mu} \right) = k D^\mu T_{\rho\mu} \tag{17.30}$$

where k is the Einstein constant, and $T_{\rho\mu}$ is a more general canonical energy-momentum tensor than used by Einstein and Hilbert. Here $T_{\rho\mu}$ contains angular or torsional energy momentum as well as energy momentum defined by curvature as in the original Einstein Hilbert field equation. Therefore the field equation of the S tensor is:

$$S_{\rho\mu} - \frac{1}{2} S g_{\rho\mu} = k T_{\rho\mu}. \tag{17.31}$$

17.4 Discussion

By carefully considering the convention for defining the Ricci tensor from the Riemann tensor, it is possible to factorise Eq.(17.31) into the Einstein Hilbert field equation and a new field equation for torsion. The Ricci tensor is defined (17.30) conventionally by contracting indices of the Riemann tensor with the symmetric metric:

$$R_{\kappa\rho} = g^{\mu\lambda} R_{\mu\kappa\rho\lambda}. \tag{17.32}$$

The order of the indices is the same in the metric and the Riemann tensor. The conventional Ricci tensor is a symmetric tensor because of the following property of the Riemann tensor:

$$R_{\rho\sigma\mu\nu} = R_{\mu\nu\rho\sigma} = R_{\nu\mu\sigma\rho}. \tag{17.33}$$

Note that Eq. (17.33) is true if and only if the Christoffel connection is symmetric, i.e. if there is no torsion present. More generally the Ricci tensor is asymmetric in the presence of torsion. In the absence or in the presence of torsion the following property is true of the R tensor:

$$R_{\rho\sigma\mu\nu} = -R_{\rho\sigma\nu\mu} \tag{17.34}$$

but this becomes the Riemann tensor if and only if torsion is absent. Therefore the following property is always true of the Ricci tensor contracted from the R tensor:

$$R_{\rho\mu} = g^{\sigma\nu} R_{\rho\sigma\nu\mu} = g^{\nu\sigma} R_{\rho\sigma\nu\mu} = -g^{\nu\sigma} R_{\rho\sigma\mu\nu}. \tag{17.35}$$

In the absence of torsion the Riemann tensor is antisymmetric in its first two indices as well as in its last two indices. This implies the property:

$$R_{\mu\nu\rho\sigma} = R_{\nu\mu\sigma\rho}. \tag{17.36}$$

The scalar curvature formed by double contraction of the Riemann tensor is therefore always positive in the absence of torsion:

$$R = g^{\sigma\nu} g^{\lambda\mu} R_{\sigma\lambda\mu\nu}. \tag{17.37}$$

These are the conventions and properties that lead to the 1915 Einstein Hilbert field equation.

In the presence of torsion however, the Riemann tensor is no longer antisymmetric in its first two indices, and the Ricci tensor is no longer a symmetric tensor. The Einstein Hilbert field equation also depends on the use of a symmetric metric in the index contraction that leads from the second Bianchi identity of Riemann geometry. Every asymmetric tensor is the sum of an antisymmetric and symmetric component, so it is always possible to write:

$$S_{\mu\nu} = S^{(S)}_{\mu\nu} + S^{(A)}_{\mu\nu} \tag{17.38}$$

$$T_{\mu\nu} = T^{(S)}_{\mu\nu} + T^{(A)}_{\mu\nu}. \tag{17.39}$$

If the symmetric part of the metric is used in the index contraction procedure then:

$$g_{\mu\nu} = g^{(S)}_{\mu\nu}. \tag{17.40}$$

The conventional and symmetric Ricci tensor used in the 1915 Einstein Hilbert field equation is defined by:

$$R_{\mu\nu} = R_{\nu\mu} = S^{(S)}_{\mu\nu}. \tag{17.41}$$

Under these conditions, Eq. (17.31) splits into a symmetric part, which is the 1915 Einstein Hilbert field equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = kT_{\mu\nu} \tag{17.42}$$

and an antisymmetric part:

$$S^{(A)}_{\mu\nu} = kT^{(A)}_{\mu\nu} \tag{17.43}$$

which is a new torsional field equation, containing new physics, notably considerations of angular energy / momentum from ${T_{\mu\nu}}^{(A)}$.

The ${S_{\mu\nu}}^{(A)}$ tensor is defined from:

$$S^{(A)}_{\rho\sigma\mu\nu} = -g_{\rho\tau} \Gamma^{\tau}{}_{\lambda\sigma} T^{\lambda}{}_{\mu\nu} \tag{17.44}$$

so that

$$S^{(A)}_{\rho\mu} = g^{\nu\sigma} g_{\rho\kappa} \Gamma^{\tau}{}_{\lambda\sigma} \left(\Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu} \right). \tag{17.45}$$

The ${S_{\rho\mu}}^{(A)}$ tensor is therefore proportional to the torsion form used in Cartan differential geometry:

$$S^{(A)}_{\rho\mu} = g^{\mu\sigma} g_{\rho\tau} \Gamma^{\tau}{}_{\lambda\sigma} q^{\lambda}{}_{a} T^{a}{}_{\mu\nu} \tag{17.46}$$

Eddington type experiments test only the Einstein Hilbert field equation of 1915, and do not consider torsion at all. In this paper we have deduced a torsional equation (17.43) but in so doing have restricted consideration to the symmetric metric. More general considerations require the use of the tetrad and the Palatini variation of general relativity. The Einstein Hilbert field equation is replaced by the more fundamental:

$$R = -kT \tag{17.47}$$

as described in detail in refs. [3] to [25]. In this way the mutual influence of gravitation and electromagnetism may be investigated using the asymmetric connection in differential geometry using the tetrad as the fundamental field rather than the metric as in the Einstein Hilbert variation of general relativity.

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Chapter 18

Explanation Of The Faraday Disc Generator In The Evans Unified Field Theory

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Abstract

The Evans unified field theory is used to give a straightforward explanation of the Faraday disc generator in general relativity using Cartan geometry instead of Riemann geometry. The electromagnetic field tensor is the torsion tensor, which in differential geometry becomes the torsion form. The Faraday law of induction is shown to hold in any frame of reference, and a torsion is set up mechanically in the Faraday disc generator. This mechanically induced spacetime torsion is the cause of the electric induction observed in the Faraday disc when the magnet is stationary. The unified field theory explains observed electric induction by a solenoid in a wire loop. Magnetic lines of force are contained within the solenoid

and electric induction occurs through the spin connection of general relativity. In special relativity there is no spin connection and no explanation for electric induction by a solenoid. There are several effects now known to be explicable by the Evans unified field theory of general relativity but not by the Maxwell-Heaviside field theory of special relativity.

Key words: Evans unified field theory, Faraday disc generator, electric induction by a solenoid.

18.1 Introduction

Recently [1]– [25] a unified field theory has been developed and based rigorously on the principles of Einsteinian general relativity. The original field theory of Einstein and Hilbert, developed independently [26]– [28] in 1915 used Riemann geometry and was applicable only to central forces in gravitation. In 1922 Cartan [29] suggested that the electromagnetic field might be his newly inferred torsion form, but despite the well known correspondence between Cartan and Einstein a unified field theory based on Cartan geometry did not emerge. This might have been due to the fact that the understanding of non-linear optics [1]– [25] necessary for a unified field theory was not available to Cartan and Einstein. If the torsion form of Cartan [26] is to be the electromagnetic field then there must exist the fundamental Evans spin field $\mathbf{B}^{(3)}$ observed in the inverse Faraday effect [1]– [25] and inferred by Evans in 1992 [30]. The inverse Faraday effect (now routinely observable) was not inferred until the mid fifties by Piekara and Kielich [1]– [25] and was not observed experimentally until the mid sixties [31]. The Evans spin field is an intrinsic part of the Cartan torsion tensor multiplied by a scalar valued potential $A^{(0)}$, and the Evans spin field is now known to be responsible for the inverse Faraday effect, which is the magnetization of matter by circularly polarized electromagnetic radiation at any frequency. The spin field is generated by the term $\omega^a{}_b \wedge A^b$, where $\omega^a{}_b$ is the spin connection of Cartan and where A^b is the potential one-form [1]– [25] of the Evans field theory. It is shown in Section 18.2 that for electromagnetism the spin connection is always dual to the Cartan tetrad q^a, which defines the potential one form as follows:

$$A^a = A^{(0)} q^a. \tag{18.1}$$

This inference leads to the well established expression [1]– [25] for the Evans spin field in vector notation:

$$\mathbf{B}^{(3)*} = -i\frac{\kappa}{A^{(0}}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{18.2}$$

using the complex circular basis $((1),(2),(3))$. Here κ is the wavenumber:

$$\kappa = \frac{\omega}{c} \tag{18.3}$$

where ω is the angular frequency of the radiation and where c is the speed of light. The spin field is therefore an ineluctable result of the fact that in general relativity and the Evans field theory, electromagnetism is spinning spacetime in which the spin connection must be non-zero. In the Maxwell-Heaviside field theory the frame is passive and there is no spin connection and no inverse Faraday effect. General relativity is preferred because a theory of physics must always be objective to all observers, and general relativity is indicated by the experimental data. Without empiricism (extraneous to the Maxwell-Heaviside field theory) there is no explanation for the inverse Faraday effect in special relativity.

In Section 18.3 it is shown that the Faraday law of induction in Cartan geometry is the same in all frames of reference because the spin connection for rotational motion is always dual to the tetrad form. In consequence the Riemann form [1]– [26] is always dual to the torsion form for rotational motion and the Faraday law of induction in consequence is always:

$$(d \wedge F^a)_1 = (d \wedge F^a)_2 = 0 \tag{18.4}$$

where the subscripts denote frames 1 and 2. In the presence of mechanically induced torsion (such as that in the Faraday disc), the total electromagnetic two-form is the sum:

$$F^a = F^a{}_1 + F^a{}_2 \tag{18.5}$$

where $F^a{}_1$ is the intrinsic (frame 1) component and where $F^a{}_2$ is the component induced by mechanical torsion (frame 2 component). The complete Faraday law of induction is therefore part of the expression:

$$d \wedge F^a = 0. \tag{18.6}$$

Thus mechanical torsion as in the Faraday disc generator [32]– [36] induces an extra electromagnetic two-form. The latter also obeys the Faraday law of induction in frame 2, so:

$$(d \wedge F^a)_2 = 0. \tag{18.7}$$

In special relativity the electromagnetic field is thought of as an entity separate from spacetime superimposed on a passive or static frame, so in special relativity mechanical torsion does not result in electric induction, contrary to the experimental data given by the Faraday disc generator. As for the inverse Faraday effect, general relativity and the Evans unified field theory are preferred.

In Section 18.4 the unified field theory is used to give a third example of the fact that in classical electrodynamics, general relativity is preferred to special relativity. This is electric induction by a solenoid that encloses the magnetic field . If a wire loop is placed outside the solenoid electric induction is observed [32]– [36] contrary to Maxwell-Heaviside field theory (i.e. special relativistic electrodynamics). In the region occupied by the wire loop:

$$\mathbf{B} = \mathbf{0} \tag{18.8}$$

where $\mathbf{B}$ is the magnetic flux density enclosed inside the solenoid. Therefore in the Maxwell-Heaviside field theory there should be no electric induction. The

Faraday induction law of that theory is:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \tag{18.9}$$

and so if $\mathbf{B} = \mathbf{0}$ there is no electric induction:

$$\nabla \times \mathbf{E} = \mathbf{0}. \tag{18.10}$$

This result is contrary to the data [32]– [36] . In Section 18.4 it is shown that the observed induction is due to the spin connection term $\omega^a{}_b \wedge A^b$, as for the inverse Faraday effect, and also the Aharonov Bohm effects [1]– [25]. The electromagnetic form in the Evans field theory of Einsteinan general relativity is defined by the first Cartan structure equation and is:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{18.11}$$

where $d\wedge$ is the exterior derivative. In the Maxwell-Heaviside field theory of special relativity the electromagnetic form is defined by:

$$F = d \wedge A \tag{18.12}$$

and the spin connection term is missing because the frame is passive or static. The confined magnetic flux density inside the solenoid is defined by the $d \wedge A$ term in Eq.(18.11), but outside the solenoid there exists the term $\omega^a{}_b \wedge A^b$, which produces the experimentally observed [32]– [36] electric induction through the equation:

$$d \wedge \left(\omega^a{}_b \wedge A^b\right) = 0. \tag{18.13}$$

18.2 Rotational Dynamics and Cartan Geometry

The complete expression for the homogeneous electromagnetic field equation in the Evans field theory is [1]– [25]

$$d \wedge F^a = \mu_0 j^a \tag{18.14}$$

where the homogeneous current three-form is defined by:

$$j^a = \frac{A^{(0)}}{\mu_0}\left(R^a{}_b \wedge q^b + T^b \wedge \omega^a{}_b\right). \tag{18.15}$$

Here μ_0 is the S. I. vacuum permeability. This expression is one in two variables only, the spin connection and the tetrad, because the Riemann and torsion forms are always defined in terms of them by the two Cartan structure equations [26] as follows:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{18.16}$$

$$R^a{}_b = D \wedge \omega^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b. \tag{18.17}$$

If the spin connection is dual to the tetrad:

$$\omega^a{}_b = \kappa \epsilon^a{}_{bc} q^c \tag{18.18}$$

it follows from Eqs.(18.16) and (18.17) that the Riemann form is dual to the torsion form:

$$R^a{}_b = \kappa \epsilon^a{}_{bc} T^c. \tag{18.19}$$

Here $\epsilon^a{}_{bc}$ is the rank three totally antisymmetric unit tensor in the tangent spacetime of Cartan geometry, a Minkowski spacetime with metric η_{ab} [1]– [26]. Thus:

$$\epsilon^a{}_{bc} = \eta^{ad} \epsilon_{dbc}. \tag{18.20}$$

From Eqs.(18.18) and (18.19) in Eq. (18.15):

$$j^a = \frac{A^{(0)}}{\mu_0} \kappa \epsilon^a{}_{bc} \left(T^c \wedge q^b + T^b \wedge q^c \right). \tag{18.21}$$

For $a = 1$ for example:

$$\begin{aligned} j^1 &= \frac{A^{(0)}}{\mu_0} \kappa \left(\epsilon^1{}_{23} T^3 \wedge q^2 + \epsilon^1{}_{32} T^2 \wedge q^3 + \epsilon^1{}_{23} T^2 \wedge q^3 + \epsilon^1{}_{32} T^3 \wedge q^2 \right) \\ &= 0 \end{aligned} \tag{18.22}$$

because:

$$\epsilon^1{}_{23} = -\epsilon^1{}_{32} \tag{18.23}$$

and similarly:

$$j^a = 0, \quad a = 0, 1, 2, 3. \tag{18.24}$$

Therefore the homogeneous current vanishes when the spin connection is dual to the tetrad.

For space indices (using Eq.(18.20)) :

$$\omega_{ij} = -\frac{\omega}{c} \epsilon_{ijk} q^k, \quad i = 1, 2, 3. \tag{18.25}$$

The tetrad components are cartesian vector components [1]– [25] within a phase factor and so $e^{i\phi}$ are antisymmetric tensor components or spin generator components dual to axial vector components. This is always true for any kind of rotational motion [1]– [26], [37] so it is concluded that the spin connection for rotational motion is always dual to the tetrad for rotational motion and that the Riemann form for rotational motion is always dual to the torsion form for rotational motion. This means that Eq.(18.14) simplifies to:

$$d \wedge F^a = 0 \tag{18.26}$$

if the torsion giving rise to electrodynamics is generated by a pure rotational dynamics.

This is the case when electrodynamics is assumed to be free from any type of gravitational influence (translational or central dynamics). If gravitation influences electromagnetism (i.e. if translation or curving influences rotation or spinning) then the duality relations (18.18) and (18.19) are no longer true in general and the homogeneous current may be non-zero in general. A violation of the Faraday law of induction may therefore be observable if and when very intense gravitation influences electromagnetism. Whether such an influence exists in nature must be determined experimentally - perhaps using cosmology but perhaps such effects occur at a one electron level in suitable circuits and so may be observed in the laboratory. The electron induces considerable spacetime curvature.

The well known 1915 theory of Einstein and Hilbert corresponds to:

$$R^a{}_b \wedge q^b = 0 \tag{18.27}$$

$$T^a = 0 \tag{18.28}$$

because in the 1915 theory there is only curvature described by the Riemann tensor and the Christoffel connection:

$$\Gamma^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\nu\mu}. \tag{18.29}$$

The torsion tensor therefore vanishes:

$$T^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\mu\nu} - \Gamma^\kappa{}_{\nu\mu} = 0. \tag{18.30}$$

Eq.(18.27) is also the result of using a Christoffel connection, which in Riemann normal coordinates [26] leads to:

$$R_{\alpha\beta\gamma\delta} = \frac{1}{2}\left(\partial_\beta\partial_\gamma g_{\alpha\delta} - \partial_\alpha\partial_\gamma g_{\beta\delta} - \partial_\beta\partial_\delta g_{\alpha\gamma} + \partial_\alpha\partial_\delta g_{\beta\gamma}\right) \tag{18.31}$$

where $g_{\alpha\beta}$ is the symmetric metric tensor [26]. It follows from Eq.(18.31) that:

$$\begin{aligned} R_{\alpha\beta\gamma\delta}+&R_{\alpha\gamma\delta\beta} + R_{\alpha\gamma\beta\delta} = 0 \\ =&\frac{1}{2}\left(\partial_\beta\partial_\gamma g_{\alpha\delta} - \partial_\alpha\partial_\gamma g_{\beta\delta} - \partial_\beta\partial_\delta g_{\alpha\gamma} + \partial_\alpha\partial_\gamma g_{\beta\gamma}\right. \\ &+ \partial_\gamma\partial_\delta g_{\alpha\beta} - \partial_\alpha\partial_\delta g_{\gamma\beta} - \partial_\gamma\partial_\beta g_{\alpha\delta} + \partial_\alpha\partial_\beta g_{\gamma\delta} \\ &\left.+ \partial_\delta\partial_\beta g_{\alpha\gamma} - \partial_\alpha\partial_\beta g_{\alpha\gamma} - \partial_\delta\partial_\gamma g_{\alpha\beta}\partial_\alpha\partial_\gamma g_{\delta\beta}\right) \end{aligned} \tag{18.32}$$

which is the equivalent of Eq.(18.27) in tensor notation rather than form notation. Eq (18.32) was discovered by Ricci and Levi-Civita but is sometimes known [26] as the first Bianchi identity. In fact it is not a general identity and is true if and only if the connection is a Christoffel connection and if and only if the metric is symmetric; in which case there is a particular relation [26] between the connection and the metric. These basic facts are difficult to find in textbooks and are given here for clarity of exposition. If the connection is asymmetric in its lower two indices then:

$$R^a{}_b \wedge q^b \neq 0 \tag{18.33}$$

$$T^a \neq 0 \tag{18.34}$$

in general, and as argued gravitation may influence electromagnetism.

The second Bianchi identity:

$$D \wedge R^a{}_b := 0 \tag{18.35}$$

in contrast is a true identity for any type of connection because it is the Jacobi identity for covariant derivatives:

$$[[D_\lambda, D_\rho], D_\sigma] + [[D_\rho, D_\sigma], D_\lambda] + [[D_\sigma, D_\lambda], D_\rho] := 0 \tag{18.36}$$

The Jacobi identity is true for any three operators A, B and C as follows:

$$\begin{aligned} [A,[B,C]] + [B,[C,A]] + [C,[A,B]] = & \\ & ABC - BCA - ACB + CBA \\ & + BCA - CAB - BAC + ACB \\ & + CAB - ABC - CBA + BAC \\ & := 0. \end{aligned} \tag{18.37}$$

18.3 Faraday Disk Generator

As discussed by Guala-Valverde et al. [32]– [36] using a series of reproducible experiments the well known Faraday disc generator is an example of Einsteinian general relativity. The original experiment by Faraday was reported in his diary on Dec 26th 1831 and consisted of a disc placed on top of a permanent magnet and separated from the magnet by paper. The assembly was suspended by a string and the complete assembly rotated. An e.m.f. was observed between the center of the disc and an edge of the disc. The e.m.f. vanished when the mechanical torsion (rotation) was absent. The effect also occurs when the magnet is rotated with respect to a stationary disc or vice versa. This experiment was part of a series of famous experiments carried out by Faraday in the year 1831, and the Faraday law of induction of the standard model (special relativistic electrodynamics) later emerged to describe the induction seen when a magnet is translated with respect to a stationary induction loop. The vector form of the law in the Maxwell-Heaviside theory was given by Heaviside.

In the Evans unified field theory [1]– [25] (generally relativistic electrodynamics) the Faraday law of induction is part of Eq.(18.26), the two homogeneous laws being:

$$\nabla \cdot \mathbf{B}^a = 0 \tag{18.38}$$

$$\nabla \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mathbf{0}. \tag{18.39}$$

The extra index a indicates a state of polarization, and the complex circular basis [1]– [25] may be used:

$$a = (0), (1), (2), (3). \tag{18.40}$$

Therefore a indicates transverse and longitudinal space-like states and a time-like state (0).

Although the equation (18.39) of general relativity looks similar to the familiar:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}. \tag{18.41}$$

of special relativity there is a major conceptual difference between the two. In Eq. (18.39) the phenomenon is due to the spinning of spacetime itself, in Eq.(18.41) it is due to the spinning of the field considered as an entity distinct from the passive frame. When the apparatus is spun mechanically in the Faraday disc generator there is a torsion form generated mechanically. This produces a mechanically generated electromagnetic form using the Evans Ansatz [1]– [25]:

$$F^a{}_{mech} = A^{(0)} T^a{}_{mech}. \tag{18.42}$$

The $A^{(0)}$ coefficient is a number is scalar, and originates in the magnet of the generator. If the magnet were taken away there would be no induction (spinning a metal disc about Z does not produce an e.m.f. between its center and rim, for this to occur a magnet is needed). The mechanically induced torsion is due to mechanical spin, and this spin occurs if the whole apparatus is spun, as in the original experiment of Faraday, if only the disc is spun, or if only the magnet is spun. The basic new concept at work here is that mechanical spin produces spacetime spin, and IS spacetime spin. This concept of general relativity does not occur in the Maxwell-Heaviside field theory of special relativity. The complete electromagnetic field present when there is mechanical spin is:

$$F^a{}_{tot} = F^a + F^a{}_{mech} \tag{18.43}$$

and obeys the homogeneous equation:

$$d \wedge F^a{}_{tot} = 0. \tag{18.44}$$

Without spin we have:

$$d \wedge F^a = 0. \tag{18.45}$$

The homogeneous equation is therefore obeyed in any frame of reference as follows:

$$d \wedge F^a{}_{tot} = d \wedge F^a = d \wedge F^a{}_{mech} = 0 \tag{18.45a}$$

and this is a direct result of Einsteinian relativity. In other words one cannot tell whether 1 is spinning with respect to 2 or vice versa - the physics is objective, the form of the equation is the same in any frame, to any observer. These concepts are missing completely from Maxwell-Heaviside (MH) theory. In consequence the MH theory runs into well known [32]– [36] problems when attempting to explain the Faraday disc generator. For example if the magnet is stationary, then:

$$\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{18.46}$$

and in consequence there is no induction expected in MH theory, contrary to the data of 26^{th} Dec., 1831. In the Evans field theory and general relativity the magnet can be stationary, but when the disc is spun, induction occurs through:

$$d \wedge F^a{}_{mech} = 0 \tag{18.47}$$

provided that there are two ingredients present, $A^{(0)}$ and mechanical torsion. If the magnet is aligned in Z and spun about Z with disc stationary, then again:

$$\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{18.48}$$

and no induction is expected in MH theory. In the Evans field theory induction occurs through Eq.(18.47). Finally, similar arguments apply when both disc and magnet are spun about Z.

18.4 Induction By Solenoid

If a solenoid is placed inside an induction loop or coil, then electric induction is observed [32]– [36] despite the fact that the magnetic flux density $\mathbf{B}$ is confined wholly inside the solenoid and does not reach the induction loop. In this situation:

$$\mathbf{B} = \mathbf{0} \tag{18.49}$$

and there is no induction possible in the MH theory. The explanation in the Evans field theory parallels that given earlier [1]– [25] for the well known Chambers experiment and Aharonov Bohm effects. In the Evans field theory the electromagnetic field is always defined by:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{18.50}$$

and obeys Eq.(18.26) if there is no influence of gravitation. Using the Evans Ansatz:

$$A^a = A^{(0)} q^a \tag{18.51}$$

$$F^a = A^{(0)} T^a \tag{18.52}$$

it is seen that Eq.(18.50) is a direct result of the first Cartan structure relation (18.16). Electric induction is therefore described in the Evans theory by:

$$d \wedge (d \wedge A^a) + d \wedge \left(\omega^a{}_b \wedge A^b\right) = 0. \tag{18.53}$$

There is local induction caused by the first term on the left hand side of Eq.(18.53) and a non-local induction caused by the second term involving the spin connection. The magnetic field component confined to the solenoid is defined as being part of:

$$F^a{}_{solenoid} = (d \wedge A^a)_{solenoid} \,. \tag{18.54}$$

Outside the solenoid there is a component defined as being part of:

$$F^a{}_{loop} = \left(\omega^a{}_b \wedge A^b\right)_{loop} \tag{18.55}$$

This component causes electric induction through the equation:

$$d \wedge F^a{}_{loop} - 0. \tag{18.56}$$

In electrodynamics free of gravitation we have seen from Section 18.2 that:

$$\omega^a{}_b = \kappa\epsilon^a{}_{bc}q^c = \frac{\kappa}{A^{(0)}}\epsilon^a{}_{bc}A^c \tag{18.57}$$

and so:

$$\frac{\kappa}{A^{(0)}}\left(d \wedge \left(\epsilon^a{}_{bc}A^c \wedge A^b\right)\right) = 0. \tag{18.58}$$

This equation can be interpreted as a first order effect:

$$d \wedge \left(\epsilon^a{}_{bc}q^c \wedge A^b\right) = 0 \tag{18.59}$$

or as a second order effect:

$$d \wedge \left(\epsilon^a{}_{bc}A^c \wedge A^b\right) = 0 \tag{18.60}$$

and is the same in structure as the first and second order Aharonov Bohm effects in the Evans unified field theory [1]– [25]. It is seen that if A^b is transverse in Eq.(18.59) there is a transverse electric field generated around the induction coil, i.e. in vector notation

$$\nabla \times \mathbf{E} \neq \mathbf{0}. \tag{18.61}$$

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Chapter 19

Experiments To Test The Evans Unified Field Theory And General Relativity In Classical Electrodynamics

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Abstract

It is argued that the Faraday experiment with rotating disc verifies the Evans unified field theory in classical electrodynamics, being an experiment in rotational relativity. Thus classical electrodynamics is a theory of general relativity, requiring the use of a spin connection as in the Evans unified field theory. Several other experiments are suggested with which to test general relativity in classical electrodynamics. These forge a basis for extending quantum electrodynamics to a theory of general relativity, and for incorporating gravitation into electrodynamics.

Key words: Evans unified field theory, general relativity, Faraday rotating disc experiment, new experiments in general relativity.

19.1 Introduction

The essence of relativity theory is that physics is the objective investigation of nature. This is a fundamental principle which prevents physics being subjective (all things to all observers) and incomplete. Relativity theory was forged by several investigators from about 1892 to 1915, major advances being made in 1905 by Einstein, and in 1915 by Einstein, and independently, Hilbert. From about 1925 to about 1955 Einstein, Cartan and others sought a truly objective theory of nature based on relativity applied to electrodynamics as well as to gravitation. As early as 1922, Cartan sensed that electrodynamics must be based on the torsion tensor in his newly inferred Cartan geometry. The solution to this type of unification was finally inferred from 2003 to present in the Evans unified field theory [1]– [28], based directly and straightforwardly on the well known structure [29]– [31] of Cartans differential geometry. The Evans field theory has been tested experimentally in several different ways, and has been shown to reduce to the correct mathematical structure of all the major equations of both classical and quantum physics. This paper is concerned with further testing of the theory in classical electrodynamics using available experiments and inferences of new experiments.

In Section 19.2 the new theory is applied in all theoretical detail to the Faraday rotating disc experiment of the nineteenth century. In Section 19.3 several new experiments are inferred from the new theory, to be tested at a later stage and the paper ends with a discussion of developments, notably on the interpretation of the wave-function in the Evans unified field theory, on the meaning of locality and non-locality and related topics which remain points of debate in contemporary physics. This discussion is intended to prepare the ground philosophically for the systematic extension of general relativity in classical electrodynamics to quantum electrodynamics.

19.2 Faraday Rotating Disc Experiment

On Dec 26^{th} 1831 Faraday noted in his diary the results of a new experiment in a famous series of experiments involving the interaction of electricity and magnetism. In this experiment a conducting disc was attached to a cylindrical bar magnet and separated from it by paper. The assembly was rotated and an electromotive force observed between the center of the disc and a rim of the disc. This experiment developed into the homopolar generator of contemporary engineering. Its attempted interpretation using special relativity (notably the Maxwell Heaviside field theory) has caused protracted confusion, the issue finally being settled in a series of reproducible experiments by Guala-Valverde at al. [32]– [37]. These investigators show clearly and simply that induction by a rotating disc is an example of general relativity: it is the relative angular frequency of rotation that counts. If the disc is spun with respect to a static magnet aligned in Z, the electromotive force is measured by a voltmeter at rest with respect to the spinning disc. Similarly the magnet can be spun about Z with respect to a static disc, or both magnet and disc can be spun about Z with respect to the voltmeter, as in the original experiment of Dec 26^{th} 1831. None of these experiments can be explained with special relativity because the latter does not deal with accelerations induced by rotation. In order to deal with rotational relativity [32]– [37] the Evans unified field theory is needed, the latter being a theory of general relativity. The latter theory in turn is covariant under any coordinate transformation, meaning that it can deal with accelerations in central and rotational dynamics. In gravitational general relativity the acceleration is central, reducing to Newtonian acceleration in the weak field limit. The Evans unified field theory introduces general relativity to electrodynamics, and does this via the torsion form of Cartan [1]– [28]. The unified gravito-electromagnetic field is then governed directly by the rules of Cartan geometry. The latter is rigorously equivalent to the most general type of Riemann geometry, but is much more elegant and clear.

The Faraday disc experiment is therefore governed by the field equations of the Evans theory. The electromagnetic potential is a vector valued one- form of differential geometry and is defined through the Evans Ansatz as:

$$A^a = A^{(0)} q^a \tag{19.1}$$

where $A^{(0)}$ is a scalar magnitude, a C negative number. Here q^a denotes the tetrad form [29]– [31]. The electromagnetic field is a vector valued two-form and is defined by the first Cartan structure equation:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{19.2}$$

where T^a is the torsion form and $\omega^a{}_b$ is the spin connection. Here $d\wedge$ is the exterior derivative and $D\wedge$ the covariant exterior derivative of Cartan geometry. The Ansatz (1) implies that:

$$F^a = A^{(0)} T^a \tag{19.3}$$

and so Eq.(19.2) becomes:

$$F^a = D \wedge A^a = d \wedge A^{(0)} + \omega^a{}_b \wedge A^b. \tag{19.4}$$

The homogeneous field equation of classical electrodynamics is defined by the following identity of Cartan geometry:

$$\begin{aligned} D \wedge T^a =& d \wedge T^a + \omega^a{}_b \wedge T^b \\ =& R^a{}_b \wedge q^b. \end{aligned} \tag{19.5}$$

Using the Ansatz we obtain:

$$d \wedge F^a = \mu_0 j^a = A^{(0)} \left(R^a{}_b \wedge q^b + T^b \wedge \omega^a{}_b \right) \tag{19.6}$$

which in general is an equation which shows how gravitation and electromagnetism are inter-related. For the general spin connection the homogeneous current j^a in Eq.(19.6) is non-zero, but for rotational motion [1]– [28] the spin connection is dual to the tetrad, and in consequence the homogeneous current vanishes. This is the case for classical electromagnetism free from gravitation. For all practical purposes this is sufficient for macroscopic experiments in electro-dynamics, but at the electronic level hybrid effects may occur which would result in a non-zero homogeneous current.

For our purposes in this paper the homogeneous current is taken to be zero, and in consequence the homogeneous field equation is:

$$d \wedge F^a = 0. \tag{19.7}$$

The Faraday disc and Rowland experiments are therefore described by Eqs.(19.4) and (19.7) - equations of Einsteinian general relativity as required for a correctly objective description of classical electrodynamics. The index a used in this paper is the index of the complex circular basis:

$$a = (1), (2), (3). \tag{19.8}$$

In the standard model and in the Maxwell Heaviside theory of special relativity the equivalents of Eqs.(19.4) and (19.7) are:

$$F = d \wedge A, \tag{19.9}$$

$$d \wedge F = 0. \tag{19.10}$$

It is seen that the spin connection is missing and that the index a is not given. The reason for this is that the electromagnetic field in the standard model is an entity superimposed on a passive frame. In Einsteinian general relativity both gravitation and electromagnetism (and indeed any field) must be space-time in four dimensions. In consequence the electromagnetic field must be spinning space-time, and the spin connection must be used.

There is no explanation for the Faraday rotating disc experiment in the standard model, because induction is described by the Faraday law of induction:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \tag{19.11}$$

where $\mathbf{B}$ is magnetic flux density (tesla) and $\mathbf{E}$ is electric field strength (volt/m). If $\mathbf{B}$ is aligned in Z and is static, the disc being spun, induction is observed experimentally but $\mathbf{B}$ does not change in Eq.(19.11). This means that there is no induction theoretically:

$$\nabla \times \mathbf{B} = \mathbf{0} \tag{19.12}$$

contrary to experimental data. If the magnet is spun about Z, then

$$\frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \tag{19.13}$$

and no induction occurs from Eq.(19.11). Induction is observed experimentally however. Similarly when both the disc and magnet are spun about Z, as in Faradays original experiment. In order to try to save the standard model the Lorentz force law is sometimes used in an attempt to explain the Faraday disc, but Guala-Valverde et al. [32]– [37] have shown that induction occurs even when the Lorentz force law does not apply.

In the Evans field theory the explanation of the Faraday disc experiment is as follows.

Using the complex circular basis [1]– [28], the magnetic flux density is defined by:

$$\mathbf{B}^{(1)*} = \nabla \times \mathbf{A}^{(1)*} - i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(2)} \times \mathbf{A}^{(3)} \tag{19.14}$$

$$\mathbf{B}^{(2)*} = \nabla \times \mathbf{A}^{(2)*} - i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(3)} \times \mathbf{A}^{(1)} \tag{19.15}$$

$$\mathbf{B}^{(3)*} = \nabla \times \mathbf{A}^{(3)*} - i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{19.16}$$

where

$$\kappa = \frac{\Omega}{c} \tag{19.17}$$

and where Ω is an angular frequency in radians / second. Here c is the vacuum speed of light, a universal constant of Einsteinian general relativity. When the disc is stationary the vector potential is defined by:

$$\mathbf{A}^{(1)} = A^{(0)}\mathbf{q}^{(1)}, \tag{19.18}$$

$$\mathbf{A}^{(2)} = A^{(0)}\mathbf{q}^{(2)}, \tag{19.19}$$

$$\mathbf{A}^{(3)} = A^{(0)}\mathbf{q}^{(3)}, \tag{19.20}$$

where the tetrads are [1]– [28]:

$$\mathbf{q}^{(1)} = \frac{1}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right), \tag{19.21}$$

$$\mathbf{q}^{(2)} = \frac{1}{\sqrt{2}}\left(\mathbf{i} + i\mathbf{j}\right), \tag{19.22}$$

$$\mathbf{q}^{(3)} = \mathbf{k}. \tag{19.23}$$

The tetrads form an $O(3)$ cyclically symmetric group:

$$\mathbf{q}^{(1)} \times \mathbf{q}^{(2)} = i\mathbf{q}^{(3)*}, \tag{19.24}$$

$$\mathbf{q}^{(2)} \times \mathbf{q}^{(3)} = i\mathbf{q}^{(1)*}, \tag{19.25}$$

$$\mathbf{q}^{(3)} \times \mathbf{q}^{(1)} = i\mathbf{q}^{(2)*}. \tag{19.26}$$

Thus in the absence of rotation about Z:

$$\nabla \times \mathbf{A}^{(1)} = \nabla \times \mathbf{A}^{(2)} = \mathbf{0}, \tag{19.27}$$

$$\mathbf{A}^{(3)} = A^{(0)}\mathbf{k}. \tag{19.28}$$

From Eq.(19.7) and using the complex circular basis we obtain:

$$\nabla \times \mathbf{E}^{(1)} + \frac{\partial \mathbf{B}^{(1)}}{\partial t} = \mathbf{0} \tag{19.29}$$

$$\nabla \times \mathbf{E}^{(2)} + \frac{\partial \mathbf{B}^{(2)}}{\partial t} = \mathbf{0} \tag{19.30}$$

$$\nabla \times \mathbf{E}^{(3)} + \frac{\partial \mathbf{B}^{(3)}}{\partial t} = \mathbf{0} \tag{19.31}$$

Therefore from Eqs.(19.14) to (19.16) and (19.29) to (19.31) the only field present is

$$\mathbf{B}^{(3)*} = \mathbf{B}^{(3)} = -iB^{(0)}\mathbf{q}^{(1)} \times \mathbf{q}^{(2)} = B_Z^{(3)}\mathbf{k} = B_Z\mathbf{k}, \tag{19.32}$$

which is the static magnetic field of the magnet.

When the disc is rotated at an angular frequency Ω:

$$\mathbf{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} - i\mathbf{j})\, e^{i\Omega t}, \tag{19.33}$$

$$\mathbf{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} + i\mathbf{j})\, e^{-i\Omega t}, \tag{19.34}$$

and from Eqs.(19.14) to (19.16) and (19.29) to (19.31) electric and magnetic fields are induced in the direction transverse to Z, i.e. in the XY plane of the spinning disc. However the longitudinal magnetic flux density component in Eq.(19.32) is unchanged by the rotation, as occurs experimentally. The (2) component of the transverse electric field spins around the rim of the disc and is defined form Eq.(19.4) as [1]– [28]:

$$\mathbf{E}^{(2)} = \mathbf{E}^{(1)*} = -\left(\frac{\partial}{\partial t} + i\Omega\right)\mathbf{A}^{(2)}. \tag{19.35}$$

Its real and physical part is:

$$Real(\mathbf{E}^{(1)}) = \frac{2}{\sqrt{2}} A^{(0)}\Omega\, (\mathbf{i}\sin\Omega t - \mathbf{j}\cos\Omega t)\,, \tag{19.36}$$

and it is proportional to the product of $a^{(0)}$ and Ω as observed experimentally. It sets up an electromotive force between the center of the disc and the rotating rim, and this is measured by a voltmeter in the laboratory frame, at rest with respect to the rotating disc. As demonstrated clearly by Guala-Valverde et al. [32]– [37], this is an example of rotational relativity.

The homogeneous law (7) retains its form in any frame of reference, as required by general relativity, and in consequence the rotating electric field induces a rotating magnetic field in the frame of the mechanically rotated disc:

$$\left(\nabla \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t}\right)_{mechanical} = \mathbf{0}. \tag{19.37}$$

This is therefore a simple and complete description of the Faraday disc experiment in general relativity. The origin of the effect is rotating or spinning space-time, induced by mechanically rotating the disc, and described by the rotating tetrads [1]– [28]:

$$\mathbf{q}^{(1)} = \mathbf{q}^{(2)*} = \frac{1}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right)e^{i\Omega t}. \tag{19.38}$$

In the same philosophy of Einsteinian general relativity, gravitation is curving space-time, again described by the tetrad appropriate to curving space-time. The philosophy is therefore self -consistent, and the results completely describe the experiment.

19.3 Suggested New Experiments

In a circularly polarized electromagnetic wave the phaseless Evans spin field is given by the spin connection of general relativity:

$$\mathbf{B}^{(3)*} = -iB^{(0)}\mathbf{q}^{(1)} \times \mathbf{q}^{(2)} \tag{19.39}$$

This field propagates with the radiation at c in free space. This field can be used in principle instead of the static magnetic field or conventional electromagnet of the homopolar generator to induce an electric field in an induction coil. The simplest design is mechanical rotation of the antenna sources of $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ at an angular frequency Ω in a clockwise or anticlockwise direction. This produces extra induction due to mechanical rotation as in Eqs.(19.33) and (19.34) leading to rotating electric and magnetic fields in the XY plane. Thus, spinning an electromagnetic field should produce induction over and above that observed in the absence of spin and careful design should produce a homopolar generator of this type without moving parts. For example phase, frequency or amplitude modulation could be used.

Consider a circularly polarized electromagnetic component propagating in the Z axis with phase $e^{i\phi}$ in the absence of rotation:

$$\mathbf{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right)e^{i\phi}. \tag{19.40}$$

The complex conjugate of this wave is:

$$\mathbf{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}}\left(\mathbf{i} + i\mathbf{j}\right)e^{-i\phi}. \tag{19.41}$$

In the absence of mechanical spin at angular frequency Ω the Evans spin field in free space is phaseless and propagates at c:

$$\mathbf{B}^{(3)*} = -i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = B_Z\mathbf{k}, \tag{19.42}$$

In the presence of mechanical spin about Z the components (19.40) and (19.41) become:

$$\mathbf{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} - i\mathbf{j})\, e^{i(\phi+\Omega t)} \tag{19.43}$$

and

$$\mathbf{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} + i\mathbf{j})\, e^{-i(\phi+\Omega t)}. \tag{19.44}$$

The conjugate product of Eq.(19.43) and (19.44) produces an unchanged, phase free, spin field

$$\mathbf{B}^{(3)} = B^{(0)}\mathbf{k}. \tag{19.45}$$

However from Eq.(19.37) mechanical spin induces an electric field in the XY plane given by Eq(19.36) and an accompanying magnetic field spinning in the XY plane. This is the counterpart of the Faraday disc generator with the bar magnet or electromagnet replaced with a spinning electromagnetic field. Finally if the rate of mechanical spin Ω were time dependent, it might be possible to use amplitude, phase or frequency modulation techniques to monitor the induced electric field in this type of homopolar generator.

One possible way of generating a phase dependent axial magnetic field is to mechanically rotate a left circularly polarized field in the left-wise direction. The wave is given by:

$$\mathbf{A}_L^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} - i\mathbf{j})\, e^{i\phi} \tag{19.46}$$

where

$$\phi = \omega t - \kappa Z \tag{19.47}$$

is the electromagnetic phase. The mechanical rotation at angular frequency Ω induces a change in frequency of the wave:

$$\omega \longrightarrow \omega + \Omega \tag{19.48}$$

and the rotation results in the extra potential:

$$\mathbf{A}_{L,mech}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} - i\mathbf{j})\, e^{i\Omega t}. \tag{19.49}$$

The complex conjugate of Eq.(19.46) is:

$$\mathbf{A}_L^{(2)} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} + i\mathbf{j})\, e^{-i\phi}. \tag{19.50}$$

A phase dependent axial magnetic field is set up through the conjugate product:

$$\begin{aligned} \mathbf{B}^{(3)*} &= -ig\mathbf{A}_{L,mech}^{(1)} \times \mathbf{A}_L^{(2)} \\ &= B^{(0)} e^{i\Omega t} e^{-i\phi}\mathbf{k}. \end{aligned} \tag{19.51}$$

The real part of this is:

$$Re(\mathbf{B}^{(3)*}) = B^{(0)} cos\,(\Omega t - \phi)\,\mathbf{k} \tag{19.52}$$

and consists of a slow modulation at frequency Ω superimposed on oscillation at frequency ω. This slow modulation could be detected in principle with a lock in amplifier, using amplitude or phase modulation techniques well known in Michelson interferometry [38].

If the static, uniform magnetic field of the Faraday disc generator were replaced by a static, uniform electric field in the Z axis, a rotating potential of type (19.49) would be set up if the disc were rotated about Z with respect to the static electric field in Z. The electric field should be well insulated from the rotating disc so that the latter would not become charged, causing possible artifacts to complicate the experimental data. An e.m.f. of the Faraday disc type would be expected between the center of the rotating disc and a rim. This effect again depends on there being two essential ingredients present, the scalar $A^{(0)}$ (this time from the electric field), and mechanical rotation at angular frequency Ω. The role of the magnetic field in the original Faraday disc experiment and of the electric field in this experiment is to supply $A^{(0)}$. Similarly a rotating electromagnetic field would supply $A^{(0)}$ through the root mean square of the oscillating potential. These are all examples of general relativity, mechanical rotation sets up a rotation of spacetime, inducing the tetrad (19.38) and the potential (19.49). The rotating electric fields in the disc back induce a Z axis magnetic field through the equation:

$$\mathbf{B}^{(3)*} = -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}. \tag{19.53}$$

Similarly a rotating disc made up of a dielectric spinning about a Z axis electric or magnetic field would carry the induced potential (19.49), but in this case there are no electric fields induced directly, only through polarization and magnetization effects.

In the papers by Guala-Valverde et al. [32]– [37] a description is given of induction by a long solenoid in a loop placed outside the solenoid. There is no magnetic field outside the solenoid, yet induction occurs experimentally. In this case there is a circling electric field $\mathbf{E}^{(1)} = \mathbf{E}^{(2)*}$ present in the solenoid. In general relativity this originates in the transverse tetrad (19.38). Inside the solenoid there is also a magnetic field $\mathbf{B}^3$ in the Z axis, originating in the longitudinal tetrad $\mathbf{q}^{(3)}$. Together with $A^{(0)}$, the cross product of the tetrads $\mathbf{q}^{(1)}$ and $\mathbf{q}^{(3)}$ are part of the non-local spin connection term (second term in Eq.(19.4)). This non-local term sets up the circulating magnetic fields as in Eqs(19.14) and (19.15) (second terms right hand side), and these non-local fields cause induction in an arbitrarily shaped wire as described by Guala-Valverde et al. [14]– [15]. There is no explanation for this effect in the standard model. In the latter only the local term (Eq. (19.9)) is defined and there is no induction possible from the spin connection term of general relativity (second term on the right hand side of Eq.(19.4)), the electric field of the winding of the solenoid is confined to the solenoid, and the magnetic field is confined to the Z axis inside the solenoid.

Similarly in the Aharanov Bohm effect the magnetic field is confined to the Z axis, whereas the potential causing the electron diffraction shift is non-local - another example of the spin connection at work in classical electrodynamics [1]–[28]. Finally if the solenoid were spun about its Z axis at angular frequency Ω, extra induction would be expected from general relativity. Careful experimental design is needed to observe this induction [32]– [37]. Alternatively the solenoid could generate the Z axis magnetic field in the Faraday disc experiment with solenoid static and disc spun, or vice-versa.. Other configurations of this type could be designed by the electrical engineer.

Therefore a considerable amount of experimental evidence is building up for the Evans unified field theory and rotational relativity in classical electrodynamics. This evidence provides a solid basis for a unified field theory based on Cartan geometry.

19.4 Discussion

In order to incorporate gravitational effects into classical electrodynamics a more complete description is needed of the charge current density. In the homogeneous equation this is defined as on the right hand side of Eq(19.6) and as argued already, vanishes for rotational motion because the spin connection is dual to the tetrad, and the Riemann form is dual to the torsion form. This type of duality is the same as the duality between a rank two antisymmetric tensor and an axial vector. In the presence of gravitation however the duality is no longer valid, because superimposed on the spinning is a curving. In the presence of gravitation therefore the homogeneous current j^a may not be zero indicating a violation of the Faraday law of induction due to gravitation. When the electromagnetic field interacts with matter gravitation is present because mass is present. The Evans field equation describing the interaction is the inhomogeneous field equation, the Hodge dual of the homogeneous field equation. In free space the inhomogeneous charge current density J^a is the Hodge dual of the homogeneous charge current density j^a:

$$J^a = \frac{A^{(0)}}{\mu_0}\left(\widetilde{R}^a{}_b \wedge q^b + \widetilde{T}^b \wedge \omega^a{}_b\right). \tag{19.54}$$

In the presence of field matter interaction the spin connection is changed from $\omega^a{}_b$ to in general. This change incorporates the ad hoc constitutive equations of non-linear optics in the standard model. In free space therefore the electromagnetic wave free of gravitational influence is given by:

$$d \wedge F^a = 0, \tag{19.55}$$

$$d \wedge \widetilde{F}^a = 0, \tag{19.56}$$

two simultaneous equations which must be solved for given initial and boundary conditions. In the presence of gravitation and field matter interaction the relevant equations become:

$$d \wedge F^a = \mu_0 j^a, \tag{19.57}$$

$$d \wedge \widetilde{F}^a = \mu_0 J^a. \tag{19.58}$$

If there is no torsion in field matter interaction then $\widetilde{R} \wedge q$ is central and:

$$d \wedge \widetilde{F}^a = A^{(0)} \left(\widetilde{R}^a{}_b \wedge q^b \right)_{central} \tag{19.59}$$

The Coulomb law in general relativity is part of Eq.(19.59) and describes the central force between two charges. Similarly the Newton inverse square law describes the central gravitational attraction between two masses. The Coulomb law in the laboratory is a very precise law (19.39) and this verifies Eq.(19.59) experimentally to high precision. However the most general form of the inhomogeneous field equation is:

$$d \wedge \widetilde{F}^a = A^{(0)} \left(\widetilde{R}^a{}_b \wedge q^b + \widetilde{T}^b \wedge \Omega^a{}_b \right) \tag{19.60}$$

and by comparison with Eq.(19.59) it is seen that extra contributions due to spacetime torsion may exist most generally. These interactions could conceivably occur in close vicinity to an electron, which curves spacetime considerably.

The discussion in this paper has been confined to the classical level, but it is known that Cartan geometry gives the mathematical structure of quantum mechanics through the standard tetrad postulate and the Evans Lemma [1]–[28] derived straightforwardly from the tetrad postulate. The wave-function of the Evans unified field theory is the tetrad for all radiated and matter fields. The tetrad is well known to be the fundamental field of the Palatini variation of general relativity. Therefore it is unsurprising that the tetrad should be the wave-function in generally covariant quantum mechanics. As discussed already, classical electrodynamics in general relativity has a local and non-local nature, well verified experimentally. It follows that the wave-function also has a local and non-local nature. The non-locality is a property of spacetime (the connection). The Evans unified field theory reduces to the Einstein Hilbert field theory of gravitation when the latter is decoupled from electromagnetism, so all that is known about gravitational general relativity can be applied to classical electrodynamics in a fully objective manner. As in all relativity theory an effect is preceded by a cause, so the Heisenberg Bohr complementarity is rejected. This is again in accord with recent data [1]– [28] which show that the complementarity idea is dubious at best.

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