245(1): Compatibility of the Heavy Photon with the Precision of the Coulomb Law.

Consider the ECE Proca equation in S.I. units:

$$\left(\square + \left(\frac{mc}{\hbar} \right)^2 \right) A^a_\mu = 0 \quad -(1)$$

For each state of polarization a. This is the Proca equation of the mid thirties, usually written in shorthand notation as:

$$(\square + m^2) A_\mu = 0 \quad -(2)$$

In S.I. units:

$$A_\mu = (\phi , - \underline{A}c) \quad -(3)$$

In seeking a static solution of eq. (2), the time dependent derivative of $\square$ is eliminated and:

$$\square \rightarrow - \nabla^2 \quad -(4)$$

Only the scalar potential is considered so eq. (2) reduces to

$$\nabla^2 \phi = m^2 \phi \quad -(5)$$

It is asserted in the standard literature that the Yukawa potential is a solution of eq. (5):

$$\phi = \frac{A}{r} e^{-mr} \quad -(6)$$

However, by direct differentiation:

$$\frac{d\phi}{dr} = -\frac{A}{r} \left(\frac{1}{r} + m \right) e^{-mr} \quad -(7)$$

and:

a)

$$\nabla^2 \phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{d\phi}{dr}\right)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}(r\phi)\right)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(\phi + r\frac{\partial\phi}{\partial r}\right)$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} \qquad -(8)$$

So:

$$\frac{\partial^2\phi}{\partial r^2} = Ae^{-mr}\left[\left(\frac{m^2}{r} + \frac{2m}{r^2} + \frac{2}{r^3}\right) - \frac{2}{r^2}\left(\frac{1}{r} + m\right)\right] \qquad -(9).$$

$$= \frac{Am^2}{r} e^{-mr}$$

Q.E.D.

However the Yukawa potential in nuclear physics is well known to be unphysical. It is incompatible with the heavy photon inferred from the Compton effect, because the heavy photon would imply deviations from the Coulomb law that are not observed. The correct interpretation is based on the fact that the Yukawa

3) potential is in fact the Fourier transform of the propagator of the Klein Gordon equation:

$$\frac{1}{p^\mu p_\mu + m^2} = \frac{1}{(2\pi)^4} \int \frac{e^{ikx} \, d^4k}{k^2 + m^2} \quad — (10)$$

$$= \frac{1}{4\pi} \delta(x_0) \frac{e^{-m(x)}}{|x|}$$

Implementing the method of note 157(9) the Proca equation (1) is rewritten for each sense of polarization as:

$$\Box A_\mu = \mu_0 J_\mu = -\left(\frac{mc}{\hbar}\right)^2 A_\mu \quad — (11)$$

where J_μ is the vacuum four-current. So:

$$\rho = -\epsilon_0 \left(\frac{mc}{\hbar}\right)^2 \phi \quad — (12)$$

$$\underline{J} = -\frac{1}{\mu_0} \left(\frac{mc}{\hbar}\right)^2 \underline{A} \quad — (13)$$

The solution of eq. (11) for all photon masses m consists of the Lienard Wiechert potentials for each sense of polarization:

$$\phi = \frac{1}{4\pi\epsilon_0} \left(\frac{e}{\left(1 - \underline{n} \cdot \frac{\underline{v}}{c}\right) |\underline{r} - \underline{r}'|} \right)_{t_r} \quad — (14)$$

$$\underline{A} = \frac{\mu_0 c}{4\pi} \left(\frac{e \underline{v} / c}{(1 - \underline{n} \cdot \underline{v}/c) |\underline{r} - \underline{r}'|} \right)_{t_r} \quad — (15)$$

4) where the retarded time is given by:

$$t_r = t - \frac{1}{c}|\underline{r} - \underline{r}'|, \quad -(16)$$

$$c = \frac{|\underline{r} - \underline{r}'|}{t - t_r} \quad -(17)$$

Therefore the static potential of the Proca equation is given by eq. (14) and for static charges:

$$\underline{v} = \underline{0} \quad -(18)$$

so the vacuum charge density is:

$$\rho = -\epsilon_0 \left(\frac{mc}{\hbar}\right)^2 \cdot \frac{1}{4\pi\epsilon_0} \left(\frac{e}{|\underline{r} - \underline{r}'|}\right)_{t_r} \quad -(19)$$

$$= -\left(\frac{mc}{\hbar}\right)^2 \cdot \frac{1}{4\pi} \left(\frac{e}{|\underline{r} - \underline{r}'|}\right)_{t_r}$$

which is the Coulomb law for any photon mass.

This means that the heavy photon is compatible with the high experimental precision of the Coulomb law, QED.