Note 245(3): Development of ECE Theory Based on the Tetrad Postulate

The tetrad postulate can be expressed as:

$$\partial_\mu q^a_\nu = \Omega^a_{\mu\nu} \quad -(1)$$

where:

$$\Omega^a_{\mu\nu} = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}. \quad -(2)$$

Differentiating eq. (1):

$$\Box q^a_\nu = \partial^\mu \partial_\mu q^a_\nu = \partial^\mu \Omega^a_{\mu\nu}. \quad -(3)$$

The original ECE postulate is:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(4)$$

and this is supplemented by:

$$F^a_{\mu\nu} = A^{(0)} \Omega^a_{\mu\nu} \quad -(5)$$

to give:

$$\partial^\mu F^a_{\mu\nu} = \Box A_\nu \quad -(6)$$

The ECE wave equation is obtained by:

$$\partial^\mu \partial_\mu q^a_\nu = \Box q^a_\nu = \partial^\mu \Omega^a_{\mu\nu} = -R q^a_\nu \quad -(7)$$

so

$$(\Box + R) A^a_\mu = 0 \quad -(8)$$

and

$$\partial^\mu F^a_{\mu\nu} + R A^a_\nu = 0 \quad -(9)$$

therefore:

2)
$$\partial^\mu F^a_{\mu\nu} = -RA^a_\nu = \Box A_\nu \quad -(10)$$

where
$$R = -V^\nu_a \partial^\mu \Omega^a_{\mu\nu} \quad -(11)$$

The Proca equation is obtained with:
$$R = \left(\frac{mc}{\hbar}\right)^2 \quad -(12)$$

The electromagnetic field is defined directly as:
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu . \quad -(13)$$

The first Cartan structure equation is:
$$T^a_{\mu\nu} = \Gamma^a_{\mu\nu} - \Gamma^a_{\nu\mu} \quad -(14)$$

So
$$\Gamma^a_{\mu\nu} = -\Gamma^a_{\nu\mu} \quad -(15)$$

The electromagnetic field is antisymmetric so:
$$F^a_{\mu\nu} = -F^a_{\nu\mu} \quad -(16)$$

and
$$\omega^a_{\mu\nu} = -\omega^a_{\nu\mu} . \quad -(17)$$

The following equations are written for each polarization index a, with:
$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right) - (18)$$

) I_L the case:

$$\partial^1 F_{10} + \partial^2 F_{20} + \partial^3 F_{30} = \Box A_0 \quad -(19)$$

we define:

$$F_{10} = \frac{E_1}{c} = -\frac{E_x}{c} \quad \text{etc.} \quad -(20)$$

so

$$\underline{\nabla} \cdot \underline{E} = \Box A_0 \quad -(21)$$

If it is assumed that A_0 has no time dependence

then

$$\underline{\nabla} \cdot \underline{E} = -\nabla^2 \phi \quad -(22)$$

where

$$\phi = A_0 \quad -(23)$$

By experiment:

$$\boxed{\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\epsilon_0}} \quad -(24)$$

so we obtain the Poisson equation:

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0} \quad -(25)$$

From eqns. (9) and (19): $-(26)$

$$\partial^1 F_{10} + \partial^2 F_{20} + \partial^3 F_{30} = -R A_0 = -R \frac{\phi}{c}$$

where

$$A_\mu = \left(\frac{\phi}{c}, -\underline{A} \right) -(27)$$

$$j_\mu = \left(c\rho, -\underline{J} \right) -(28)$$

so

$$\boxed{\underline{\nabla} \cdot \underline{E} = -R \phi} \quad -(29)$$

4)

So:

$$\boxed{\rho / \epsilon_0 = -R\phi} \quad -(30)$$

which is eq. (12) of note 245(1). In this theory there is a direct relation between scalar potential and charge density, originating directly in the tetrad postulate.

For each index a, eq. (29) is a generalization of the Proca equation to general relativity, and so the Proca equation can always be written as:

$$\boxed{\underline{\nabla} \cdot \underline{E} = -R\phi = \rho / \epsilon_0} \quad -(31)$$

It follows that photon mass does not result in any deviation from the Coulomb law.

Now consider the case:

$$\left.\begin{aligned} \partial^0 F_{01} + \partial^2 F_{21} + \partial^3 F_{31} &= \Box A_1 \\ \partial^0 F_{02} + \partial^1 F_{12} + \partial^3 F_{32} &= \Box A_2 \\ \partial^0 F_{03} + \partial^1 F_{13} + \partial^2 F_{23} &= \Box A_3 \end{aligned}\right\} \quad -(32)$$

and define the field matrix:

5)

$$F_{\mu\nu} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{bmatrix} \quad -(33)$$

then

$$\boxed{\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} = \square \underline{A}} \quad -(34)$$

which is the Ampère Maxwell law with additional input:

$$\boxed{\square \underline{A} = \mu_0 \underline{J}} \quad -(35)$$

In this theory the vector potential is directly proportional to current density.

In general:

$$\boxed{\partial^\mu F_{\mu\nu} = \square A_\nu = \mu_0 j_\nu} \quad -(36)$$

From the ECE wave equation (8) for each a:

$$\square A_\nu = \mu_0 j_\nu = -R A_\nu \quad -(37)$$

Using the definitions (27) and (28):

$$\mu_0 c \rho = -R \frac{\phi}{c} \quad -(38)$$

$$\mu_0 \underline{J} = -R \underline{A} \quad -(39)$$

6) In S.I. units:

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad -(40)$$

So

$$\boxed{\begin{aligned} \frac{\rho}{\epsilon_0} &= -R\phi \\ \mu_0 \underline{J} &= -R\underline{A} \end{aligned}} \quad \begin{aligned} &-(41) \\ &-(42) \end{aligned}$$

and

These are equations (12) and (13) of note 245(1).

In the special case of the Proca equation:

$$R = \left(\frac{m_0 c}{\hbar}\right)^2 \quad -(43)$$

where m_0 is the photon ~~mass~~ rest mass.

In this theory the existence of the photon rest mass does not change the inhomogeneous equations of electrodynamics structurally, they are always:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(44)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(45)$$

It can be seen that eqs. (41) and (42) transform eqs. (44) and (45) into the Proca equations:

7)

$$\underline{\nabla} \cdot \underline{E} = -R\phi \quad -(46)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = -R\underline{A} \quad -(47)$$

Clearly, eqs. (46) and (47) have the same solutions as eqns (44) and (45), and this is the direct result of the tetrad postulate.

The use of the Yukawa potential is therefore an error, because the Yukawa potential is not used in eqs. (44) and (45).

Finally the definition (13) can be written as:

$$F^a_{\mu\nu} = \frac{1}{2}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) \quad -(48)$$

It follows from eq. (48) that:

$$\partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho\mu} + \partial_\mu F_{\nu\rho} = 0 \quad -(49)$$

i.e.

$$\boxed{\partial^\mu \tilde{F}_{\mu\nu} = 0} \quad -(50)$$

i.e.

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(51)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(52)$$

which are the homogeneous field equations.