245(4): Derivation of Vacuum Charge and Current from the Linear ECE Theory

In the linear ECE theory:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(1)$$

$$F^a_{\mu\nu} = A^{(0)} \left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \right) \quad -(2)$$

$$(\square + R) A^a_\mu = 0 \quad -(3)$$

$$\partial^\mu F^a_{\mu\nu} = \square A^a_\nu = -R A^a_\nu = \mu_0 j^a_\nu \quad -(4)$$

$$\partial^\mu \tilde{F}^a_{\mu\nu} = 0 \quad -(5)$$

For each sense of polarization, as in Table 2 of UFT 167:

$$F_{\mu\nu} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{bmatrix}; \quad \tilde{F}_{\mu\nu} = \begin{bmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & E_z/c & -E_y/c \\ -B_y & -E_z/c & 0 & E_x/c \\ -B_z & E_y/c & -E_x/c & 0 \end{bmatrix} \quad -(6)$$

Inhomogeneous Vacuum Equations

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho(\text{vac})}{\epsilon_0} = -R\phi(\text{vac}) \quad -(7)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J}(\text{vac}) = -R\underline{A}(\text{vac}) \quad -(8)$$

Homogeneous Equations

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(9)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(10)$$

2)

The vacuum four current is:

$$j_\mu(\text{vac}) = (c\rho(\text{vac}), -\underline{J}(\text{vac})) \quad -(11)$$

The vacuum potential is:

$$A_\mu(\text{vac}) = \left(\frac{\phi(\text{vac})}{c}, -\underline{A}(\text{vac})\right) \quad -(12)$$

Units in S.I.

$$\mu_0 = J s^2 C^{-2} m^{-1}$$
$$\epsilon_0 = J^{-1} C^2 m^{-1}$$
$$E = J C^{-1} m^{-1}$$
$$B = J s C^{-1} m^{-2}$$
$$A = J s C^{-1} m^{-1}$$
$$\rho = C m^{-3}$$
$$\phi = J C^{-1}$$

The solution of eq. (7) is:

$$\phi(\text{vac}) = \frac{1}{\epsilon_0} \int \frac{\rho(\text{vac})\, d^3\underline{x}'}{|\underline{x} - \underline{x}'|} \quad -(13)$$

The vacuum electric field is:

$$\underline{E}(\text{vac}) = -\underline{\nabla}\, \phi(\text{vac}) \quad -(14)$$

From eqs. (7) and (13):

$$\phi(\text{vac}) = -\frac{\rho(\text{vac})}{\epsilon_0 R} = \frac{1}{\epsilon_0} \int \frac{\rho(\text{vac})\, d^3\underline{x}'}{|\underline{x} - \underline{x}'|} \quad -(15)$$

So:

$$\int \frac{\rho(\text{vac})}{|\underline{x}-\underline{x}'|} d^3\underline{x}' = -\frac{\rho}{R} \quad -(16)$$

where

$$R = -q_a^{\nu}\partial^{\mu}\left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}\right) \quad -(17)$$

so

$$\boxed{\int \frac{\rho(\underline{x}')\,d^3x'}{|\underline{x}-\underline{x}'|} = \frac{\rho(\text{vac})}{q_a^{\nu}\partial^{\mu}\left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right)}} \quad -(18)$$

The photon rest mass m_0 is interpreted as:

$$\left(\frac{m_0 c}{\hbar}\right)^2 = q_a^{\nu}\partial^{\mu}\left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}\right) \quad -(19)$$

so

$$\int \frac{\rho(\text{vac})\,d^3x'}{|\underline{x}-\underline{x}'|} = \left(\frac{\hbar}{m_0 c}\right)^2 \rho(\text{vac}) \quad -(20)$$

From eqs. (13) and (20):

$$\boxed{\phi(\text{vac}) = \frac{1}{\epsilon_0}\left(\frac{\hbar}{m_0 c}\right)^2 \rho(\text{vac})} \quad -(21)$$

Units check

$$\phi = JC^{-1} = JC^{-2}m\,m^2\,Cm^{-3} \quad \checkmark$$

From eq. (21):

$$\left(\frac{m_0 c}{\hbar}\right)^2 = \frac{1}{\epsilon_0}\frac{\rho(\text{vac})}{\phi(\text{vac})} \quad -(22)$$

$$\boxed{m_0^2 = \left(\frac{\hbar}{c}\right)^2 \frac{1}{\epsilon_0}\frac{\rho(\text{vac})}{\phi(\text{vac})}} \quad -(23)$$

Note carefully that the Yukawa potential the solution (13) for is rejected in favour of the vacuum potential. So the form of the Coulomb law is not affected by photon mass. The latter is defined by eq. (23) in terms of the vacuum scalar potential $\phi(\text{vac})$ and the vacuum charge density $\rho(\text{vac})$.