245 (6): Spacetime Energy and Photon Mass

The Proca equations in general are:

$$\underline{\nabla} \times \underline{E} + R\phi = \frac{\rho}{\epsilon_0} \quad - (1)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = 0 \quad - (2)$$

$$\underline{\nabla} \times \underline{B} + R\underline{A} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad - (3)$$

$$\underline{\nabla} \cdot \underline{B} = 0 \quad - (4)$$

where

$$R = \left(\frac{mc}{\hbar}\right)^2 \quad - (5)$$

From previous notes to UFT245:

$$j^\mu = -\frac{R}{\mu_0} A^\mu \quad - (6)$$

where

$$j^\mu = \left(c\rho , \underline{J} \right) \quad - (7)$$

$$A^\mu = \left(\frac{\phi}{c} , \underline{A} \right) \quad - (8)$$

so

$$\rho = - R \epsilon_0 \phi \quad - (9)$$

$$\underline{J} = - \frac{R}{\mu_0} \underline{A} \quad - (10)$$

Conservation of charge current density means:

$$\partial_\mu j^\mu = 0 \quad - (11)$$

2) From eqs. (6) and (11):

$$\partial_\mu A^\mu = 0 \quad -(12)$$

In the standard model eq. (12) is known as the Lorenz condition, and is arbitrary. But the existence of photon mass means that it is the direct consequence of conservation of charge current density.

From the radiative effects it is known experimentally that spacetime contains a charge current density (anomalous g of the electron, Casimir effect, etc. and Lamb shift). So it follows that if there is photon mass, spacetime contains a four potential $A^\mu(\text{vac})$:

$$A^\mu(\text{vac}) = \left(\frac{\phi(\text{vac})}{c}, \underline{A}(\text{vac}) \right) \quad -(13)$$

It follows that a circuit can pick up electric and magnetic fields from spacetime if there is photon mass.

In this case eq. (3) becomes:

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} + R\,\underline{A}(\text{vac}) = \underline{0} \quad -(14)$$

and eq. (1) becomes:

3)
$$\underline{\nabla} \cdot \underline{E} + R\phi(\text{vac}) = 0 \quad -(15)$$

Multiply eq. (14) by $\cdot\underline{E}$:

$$\underline{E} \cdot (\underline{\nabla} \times \underline{B}) - \frac{1}{c^2}\underline{E} \cdot \frac{\partial \underline{E}}{\partial t} + R\,\underline{E} \cdot \underline{A}(\text{vac}) = 0 \quad -(16)$$

Use
$$\underline{E} \cdot (\underline{\nabla} \times \underline{B}) = -\underline{\nabla} \cdot (\underline{E} \times \underline{B}) + \underline{B} \cdot \underline{\nabla} \cdot \underline{E} \quad -(17)$$

and eq. (2) to find

$$\boxed{\frac{\partial W}{\partial t} + \underline{\nabla} \cdot \underline{S} = \frac{R}{\mu_0}\underline{E} \cdot \underline{A}(\text{vac}) \quad -(18).}$$

which is the Poynting Theorem in the presence of photon mass. The energy density in $J\,m^{-3}$ is:

$$W = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) \quad -(19)$$

and the Poynting vector is:

$$\underline{S} = \frac{1}{\mu_0}\underline{E} \times \underline{B} \quad -(20)$$

Eq. (19) is the electromagnetic energy available from spacetime given the photon mass.

Recall from previous notes that the

4) Proca equation was derived from the tetrad postulate with:

$$f_{\mu\nu}^{a} = \partial_{\mu} A_{\nu}^{a} \quad -(21)$$

$$f_{\nu\mu}^{a} = \partial_{\nu} A_{\mu}^{a} \quad -(22)$$

so either:

$$\underline{E} = -\underline{\nabla}\phi \quad -(23)$$

or

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} \quad -(24)$$

Antisymmetry is defined by:

$$F_{\mu\nu}^{a} = -F_{\nu\mu}^{a} \quad -(25).$$

$$= f_{\mu\nu}^{a} - f_{\nu\mu}^{a} + \omega_{\mu b}^{a} A_{\nu}^{b} - \omega_{\nu b}^{a} A_{\mu}^{b}$$

If eq. (24) is used then:

$$\frac{\partial W}{\partial t} + \underline{\nabla} \cdot \underline{S} = -\frac{R}{\mu_0} \underline{A}(\text{vac}) \cdot \frac{\partial \underline{A}(\text{vac})}{\partial t}$$

$$= -\frac{1}{2}\frac{R}{\mu_0}\frac{\partial}{\partial t}\left(A^2(\text{vac})\right) \quad -(26)$$

From eq. (10):

$$\underline{A}(\text{vac}) = -\frac{\mu_0}{R}\underline{J}(\text{vac}) \quad -(27)$$

5) So:

$$\boxed{\frac{\partial W}{\partial t} + \underline{\nabla} \cdot \underline{S} = -\frac{1}{2} R\mu_0 \frac{\partial}{\partial t}\left(\frac{J^2(\text{vac})}{R}\right)} \quad -(28)$$

Units Checks

In eq. (18), $R = m^{-2}$, $E = JC^{-1}m^{-1}$,
$A = JsC^{-1}m^{-1}$, $\mu_0 = Js^2C^{-2}m^{-1}$, so:

$$\frac{R}{\mu_0} \underline{E} \cdot \underline{A}(\text{vac}) = m^{-2} \frac{JsC^{-1}m^{-1}JC^{-1}m^{-1}}{Js^2C^{-2}m^{-1}}$$

$$= Js^{-1}m^{-3}$$

In eq. (28):
$R\mu_0 \frac{\partial}{\partial t}\left(\frac{J^2(\text{vac})}{R}\right)$ is evaluated with $J = Cm^{-2}s^{-1}$,

$$\text{So} \quad RHS = m^{-2} \frac{Js^2C^{-2}m^{-1}C^2m^{-4}s^{-2}}{sm^{-2}}$$

$$= Js^{-1}m^{-3}$$

If it is assumed that R is independent of t then:

$$\boxed{\frac{\partial W}{\partial t} + \underline{\nabla} \cdot \underline{S} = -\frac{1}{2} \mu_0 \frac{\partial}{\partial t}\left(J^2(\text{vac})\right)} \quad -(29)$$

which is conservation of energy. Energy density and Poynting vector are produced from spacetime.