245(7): Resonance Structures, Photon Mass and Energy from Spacetime

Consider the tetrad postulate:

$$\partial_\mu q^a_\nu = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \quad -(1)$$

It follows that:

$$\partial^\mu \partial_\mu q^a_\nu = \Box q^a_\nu = \partial^\mu\left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}\right) \quad -(2)$$

Assume that R can be defined by:

$$\partial^\mu\left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}\right) := -R\, q^a_\nu \quad -(3)$$

This is always true if:

$$R = q^\nu_a \partial^\mu\left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right) \quad -(4)$$

Therefore:

$$\partial^\mu \Omega^a_{\mu\nu} + R\, q^a_\nu = 0 \quad -(5)$$

where

$$\Omega^a_{\mu\nu} := \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} \quad -(6)$$

With the postulates:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(7)$$

and

$$F^a_{\mu\nu} = A^{(0)} \Omega^a_{\mu\nu} \quad -(8)$$

the fundamental equations of electromagnetism are obtained:

$$\boxed{\begin{aligned}(\Box + R)\, A^a_\mu &= 0 \quad &-(9)\\ \partial^\mu F^a_{\mu\nu} + R A^a_\nu &= 0 \quad &-(10)\end{aligned}}$$

2) In these equations:

$$j_\mu^a = -\frac{R}{\mu_0} A_\mu^a \quad -(11)$$

Eq. (11) transforms eqs. (9) and (10) into the d'Alembert wave equation in the presence of charge current density:

$$\square A_\mu^a = \mu_0 j_\mu^a \quad -(12)$$

and the inhomogeneous field equation:

$$\partial^\mu F_{\mu\nu}^a = \mu_0 j_\mu^a \quad -(13)$$

The geometrical structure also applies to the vacuum, so there exists a vacuum charge current density:

$$j_\mu^a(\text{vac}) = -\frac{R_0}{\mu_0} A_\mu^a(\text{vac}) \quad -(14)$$

as <u>evidenced experimentally</u> by the radiative <u>corrections</u>.

Eqs. (12) and (13) describe an isolated circuit for example. When the circuit interacts with the vacuum:

$$j_\mu^a \rightarrow j_\mu^a + j_\mu^a(\text{vac}) \quad -(15)$$

so:

$$\boxed{\begin{aligned} \square A_\mu^a &= \mu_0 \left(j_\mu^a + j_\mu^a(\text{vac}) \right) \quad -(16) \\ \partial^\mu F_{\mu\nu}^a &= \mu_0 \left(j_\mu^a + j_\mu^a(\text{vac}) \right) \quad -(17) \end{aligned}}$$

<u>The</u> iso<u>lated</u> cir<u>cuit</u> <u>may</u> <u>obtain</u> po<u>tential</u> from the va<u>cuum</u>.

Therefore:

$$\left(\Box A_\mu^a - \mu_0 j_\mu^a\right)_{circuit} = \mu_0 j_\mu^a (vacuum) \quad -(18)$$

$$\left(\partial^\mu F_{\mu\nu}^a - \mu_0 j_\mu^a\right)_{circuit} = \mu_0 j_\mu^a (vacuum) \quad -(18a)$$

These equations are true for each sense of polarization a, and for each sense of polarization:

$$j_\mu = (c\rho, -\underline{J}) \quad -(19)$$

$$A_\mu = \left(\frac{\phi}{c}, -\underline{A}\right) \quad -(20)$$

The Coulomb law is modified to

$$\underline{\nabla} \cdot \underline{E} = \frac{1}{\epsilon_0}\left(\rho(circuit) + \rho(vacuum)\right) \quad -(21)$$

i.e

$$\boxed{\left(\underline{\nabla} \cdot \underline{E} + R\phi\right)_{circuit} = \frac{\rho}{\epsilon_0}(vacuum)} \quad -(22)$$

and the d'Alembert equation becomes:

$$\boxed{\left((\Box + R) A_\mu\right)_{circuit} = \mu_0 j_\mu (vacuum)} \quad -(23)$$

For the scalar potential:

$$(\Box + R)\phi = \mu_0 c^2 \rho(vac)$$

$$= \frac{\rho(vac)}{\epsilon_0} = -R_0 \phi(vac)$$

4)

The d'Alembertian is:

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(25)$$

Consider the time dependent part of ϕ, then:

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} + R\phi = \frac{\rho(t)}{\epsilon_0} (vac)} \quad -(26)$$

The most fundamental unit of mass in the circuit is the electron of mass m_0. Define:

$$R = \left(\frac{mc}{\hbar}\right)^2 = \frac{\omega_0^2}{c^2} \quad -(27)$$

$$= V^a_{\;a}\, \partial^\mu (\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu})$$

Assume for the sake of argument that:

$$m = m_0 \quad -(28)$$

then ω is the rest angular frequency of the electron.

Thus:

$$\boxed{\frac{\partial^2 \phi}{\partial t^2} + \omega_0^2 \phi = \frac{c^2 \rho(t)}{\epsilon_0} (vac)} \quad -(29)$$

Units Check

$\phi = JC^{-1}$, $\rho = Cm^{-3}$, $\epsilon_0 = J^{-1}C^2m^{-1}$

LHS $= JC^{-1}s^{-2}$; RHS $= m^2s^{-2}Cm^{-3}JC^{-2}m$

$= JC^{-1}s^{-2}$

5) Finally define:

$$\frac{c^2 \rho(t)(\text{vac})}{\epsilon_0} := A \cos \omega t \quad -(30)$$

to obtain the Euler-Bernoulli equation:

$$\frac{d^2 \phi}{dt^2} + \omega_0^2 \phi = A \cos \omega t \quad -(31)$$

whose solution is

$$\phi(t) = \frac{A}{(\omega_0^2 - \omega^2)^{1/2}} \cos \omega t \quad -(32)$$

Resonance occurs when:

$$\omega = \omega_0 \quad -(33)$$

at which condition:

$$\boxed{\phi \to \infty} \quad -(34)$$

From eqs. (24) and (30):

$$\frac{c^2 \rho(t)(\text{vac})}{\epsilon_0} = -c^2 R_0 \phi(\text{vac}) \quad -(35)$$

where

$$R_0 = R(\text{vac}) = \left(\frac{n(\text{vac})\, c}{\hbar}\right)^2 \quad -(36)$$

$$= \frac{\omega^2(\text{vac})}{c^2}$$

$$= \left(q^{\nu}_{a}\, \partial^{\mu} \left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu}\right)\right)(\text{vac})$$

From eq. (23) of note 245(4):

$$n^2(vac) = -\left(\frac{\hbar}{c}\right)^2 \frac{1}{\epsilon_0} \frac{\rho(vac)}{\phi(vac)} \quad -(37)$$

which is consistent with eq. (35) Q.E.D.

From eq. (37):

$$R_0 = \left(\frac{n(vac)c}{\hbar}\right)^2 = -\frac{1}{\epsilon_0}\frac{\rho(vac)}{\phi(vac)} \quad -(38)$$

so eq. (31) becomes:

$$\frac{\partial^2 \phi}{\partial t^2} + \omega_0^2 \phi = -\omega^2(vac)\,\phi(vac) := A\cos\omega t \quad -(39)$$

where

$$\omega(vac) = \frac{n(vac)\,c^2}{\hbar} \quad -(40)$$

It is possible to amplify the vacuum potential so that the circuit potential becomes infinite at resonance. This kind of resonance has been investigated theoretically by Eckardt and Lindstrom in a series of recent papers.