260(4): Spherical Solutions of the Helmholtz Equation

Reference: Google "Helmholtz equation spherical harmonics"

Consider the ECE wave equation:

$$(\Box + \kappa_0^2)\, q_\mu^a = 0 \quad -(1)$$

where q_μ^a is the tetrad:

$$q_\mu^a = (q_0^0, -\underline{q}^a) \quad -(2)$$

Define:

$$\phi = q_0^0 \quad -(3)$$

then

$$(\Box + \kappa_0^2)\,\phi = 0. \quad -(4)$$

Assume that:

$$\phi = \phi_0(r, \theta, \phi)\, e^{i\omega t} \quad -(5)$$

then:

$$(\nabla^2 + \kappa^2)\,\phi_0 = 0 \quad -(6)$$

where

$$\frac{\omega}{c} = (\kappa^2 + \kappa_0^2)^{1/2} \quad -(7)$$

The simplest spherical solution of ϕ_0 is:

$$\phi_0 = j(r)\, Y(\theta, \phi) \quad -(8)$$

where

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial Y}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2} = -\,l(l+1)Y \quad -(9)$$

and:

2) $$\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}+\left(k^2-\frac{l(l+1)}{r^2}\right)\right)j(r)=0 \quad -(10)$$

Here Y are the spherical harmonics and j the Bessel functions. In general in three dimensions:

$$\phi_0(r,\theta,\phi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left(a_{lm}\,j_l(kr)+b_{lm}\,y_l(kr)\right)Y_l^m(\theta,\phi) \quad -(11)$$

where $j_l(kr)$ and $y_l(kr)$ are the spherical Bessel functions and $Y_l^m(\theta,\phi)$ the spherical harmonics.

In two dimensions:

$$\phi_0(r,\theta)=\sum_{k}\sum_{n=0}^{\infty}J_n(kr)\left(A_{kn}\cos(n\theta)+B_{kn}\sin(n\theta)\right) \quad -(12)$$

where $J_n(kr)$ is the Bessel function.

Eq. (12) may be written as:

$$\phi_0(r,\theta)=\sum_{k}\sum_{l=0}^{\infty}a_{kl}\,j_l(kr)\,P_l(\cos\theta) \quad -(13)$$

where $P_l(\cos\theta)$ are the Legendre polynomials.

These have the useful property:

$$e^{ikz}=e^{ikr\cos\theta}=\sum_{l=0}^{\infty}i^l(2l+1)\,j_l(kr)\,P_l(\cos\theta) \quad -(14)$$

3)

If $$\kappa = 0 \quad - (15)$$

Eq. (6) becomes the Laplace equation:

$$\nabla^2 \phi_0 = 0 \quad - (16)$$

with solution:

$$\phi_0(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{l+1} \right) P_l(\cos\theta) \quad - (17)$$

using the ECE hypothesis this is the scalar potential in regions where there is no charge. Coulomb's law is obtained when:

$$l = 0. \quad - (18)$$

As

$$r \to \infty \quad - (19)$$

then

$$\phi_0(r, \theta) \to \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta) \quad - (20)$$

which is the multipole expansion.

The Helmholtz wave equation can be used in acoustics, hydrodynamics, electrodynamics, aerodynamics, cosmology, diffusion and telegraph theory, can become the Klein Gordon, Schroedinger and Yukawa equations, and can be used in molecular dynamics.

4) The Helmholtz Theorem states that any vector $\underline{V}$ that obeys the vector Helmholtz equation:

$$(\nabla^2 + \kappa^2)\underline{V} = \underline{0} \quad -(21)$$

can be expressed as the sum of longitudinal and transverse solutions:

$$\underline{V} = \underline{V}_\ell + \underline{V}_t \quad -(22)$$

where:

$$\underline{V}_\ell = -\underline{\nabla}\phi \quad -(23)$$

and

$$\underline{V}_t = \underline{\nabla} \times \underline{A} \quad -(24)$$

As discussed in recent papers the structure of Cartan geometry may be expressed in the absence of a magnetic monopole as:

$$\underline{\nabla} \times \underline{q}^a = \kappa \underline{q}^a \quad -(25)$$

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \kappa \underline{\omega}^a{}_b \quad -(26)$$

$$\underline{\nabla} \times \underset{\sim}{T}^a = \kappa \underset{\sim}{T}^a \quad -(27)$$

$$\underline{\nabla} \times \underline{R}^a{}_b(\text{spin}) = \kappa \underline{R}^a{}_b(\text{spin}) \quad -(28)$$

These equations imply that basic geometry obeys Helmholtz wave equations in the absence of a magnetic monopole:

5) $$(\nabla^2+\kappa^2)\,\underline{q}^a = \underline{0} \qquad -(29)$$

$$(\nabla^2+\kappa^2)\,\underline{\omega}^a{}_b = \underline{0} \qquad -(30)$$

$$(\nabla^2+\kappa^2)\,\underline{T}^a = \underline{0} \qquad -(31)$$

$$(\nabla^2+\kappa^2)\,\underline{R}^a{}_b(\text{spin}) = \underline{0} \qquad -(32)$$

with:

$$\underline{\nabla}\cdot\underline{q}^a = 0 \qquad -(33)$$

$$\underline{\nabla}\cdot\underline{\omega}^a{}_b = 0 \qquad -(34)$$

$$\underline{\nabla}\cdot\underline{T}^a = 0 \qquad -(35)$$

$$\underline{\nabla}\cdot\underline{R}^a{}_b(\text{spin}) = 0 \qquad -(36)$$

Each scalar component of these three dimensional vectors will have solutions of the type reviewed for ϕ_0.

Application to the Structure of the Electron and Proton.

In the absence of a magnetic monopole all of the above equations will apply to the internal structure of elementary particles such as the electron and proton. The standard model has completely failed to describe the structure of the electron. No electric dipole moment has been found for the electron, contrary to supersymmetry theory. On a qualitative level and in a first attempt at an explanation, the structure of the electron could be explained by the multipole expansion (20), so there are results, such as:

$$\underline{V}^a = \underline{V}^a(0) \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta) \quad -(37)$$

and also for $\underline{\omega}^a{}_b$, $\underset{\sim}{T}^a$ and $\underline{R}^a{}_b$ (spin).

The electron in this representation would be described by

$$l = 0 \quad -(38)$$

i.e. as a sphere. This is exactly what is observed experimentally. The electron has no dipole, quadrupole, octopole or hexadecapole moment. A more complete solution would be made up of results such as eqns. (11), (12), ... The spherical harmonics have zonal, sectorial and tesseral solutions. For the electron the zero order spherical harmonic would apply.

The proton on the other hand is described in the standard model as a hadron made up

7) of three valence quarks, two up quarks and one down quark. These results are based on deep inelastic electron proton scattering. The standard theory is full of unsupportable assumptions such as approximate symmetry, virtual particles (gluons), renormalization and many adjustables.

In the ECE geometric theory of the proton, it would be described by zonal, sectorial and tesseral solutions of the spherical harmonics, the simplest solution would be eq. (8), a product of a Bessel function and spherical harmonic. So the vector potential inside the proton would be:

$$\underline{A}^a = \underline{A}^a(0)\, j(r)\, Y(\theta, \phi) \quad -(39)$$

This solution can be adjusted to give regions of high potential, representing the three "quarks" as in the standard model.

<u>Graphics and Animation</u>

It would be very interesting to graph and animate these solutions ...