5.4 REFUTATION OF INDETERMINACY: QUANTUM HAMILTON AND FORCE EQUATIONS

The methods used to derive the fermion equation can be used as in UFT 175 to UFT 177 on www.aias.us can be used to derive the Schroedinger equation from differential geometry. The fundamental axioms of quantum mechanics can be derived from geometry and relativity. These methods can be used to infer the existence of the quantized equivalents of the Hamilton equations of motion, which Hamilton derived in about 1833 without the use of the lagrangian dynamics. It is very well known that the Hamilton equations use position (x) and momentum (p) as conjugate variables in a well defined classical sense {1 - 10} and so x and p are "specified simultaneously" in the dense Copenhagen jargon of the twentieth century. Therefore, by quantum classical equivalence, x and p are specified simultaneously in the quantum Hamilton equations, thus refuting the Copenhagen interpretation of quantum mechanics based on the commutator of operators of position and momentum . The quantum Hamilton equations were derived for the first time in UFT 175 in 2011, and are described in this section. They show that x and p are specified simultaneously in quantum mechanics, a clear illustration of the confusion caused by the Copenhagen interpretation.

The anti commutator $\{\hat{x}, \hat{p}\}$ is used in this section to derive further refutations of Copenhagen, in that $\{\hat{x}, \hat{p}\}$ acting on a wavefunctions that are exact solutions of Schroedinger's equation produces expectation values that are zero for the harmonic oscillator, and non zero for atomic H. The anti commutator $\{\hat{x}, \hat{p}\}$ is shown to be proportional to $[x^2, p^2]$, whose expectation values for the harmonic oscillator are all zero, while for atomic H they are all non-zero. For the particle on a ring, combinations can be zero, while individual commutators of this type are non-zero. For linear motion self inconsistencies in the Copenhagen interpretation are revealed, and for the particle on a sphere the commutator is again non-zero. The hand calculations in fifteen additional notes accompanying UFT 175 are checked with computer algebra, as are all calculations in UFT theory to which computer algebra may be applied. Tables were produced in UFT 175 of the relevant expectation values. The Copenhagen interpretation is completely refuted because in that interpretation it makes no sense for the expectation value of a commutator of operators to be both zero and non-zero for the same pair of operators. One of the operators would be absolutely unknowable and the other precisely knowable if the expectation value were non zero, and both precisely knowable if it were zero. These two interpretations refer respectively to non zero and zero commutator expectation values, and both interpretations cannot be true for the same pair of operators. Prior to the work in UFT 175 in 2011, commutators of a given pair of operators were thought to be zero or non zero, never both zero and non zero, so a clear refutation of Copenhagen was never realized. In ECE theory, Copenhagen and it unscientific, anti Baconian, jargon are not used, and expectation values are straightforward consequences of the fundamental operators introduced by Schroedinger. The latter immediately rejected Copenhagen, as did Einstein and de Broglie.

The Schroedinger equation is derived in ECE from the tetrad postulate of Cartan geometry, which is reformulated as the ECE wave equation:

$$(\Box + R)\, q^a_\nu = 0 \qquad -(232)$$

where:

$$R := q_a^\nu \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} \right) \qquad -(233)$$

as discussed earlier in this book. The fermion equation in its wave format is the limit:

$$R \rightarrow \left(\frac{mc}{\hbar}\right)^2 \quad -(234)$$

and for the free particle reduces to:

$$-\frac{\hbar^2}{2m}\nabla^2\phi = (E - mc^2)\phi. \quad -(235)$$

This equation reduces to the Schroedinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\phi = E_{NR}\phi \quad -(236)$$

where:

$$E_{NR} = E - mc^2. \quad -(237)$$

In the presence of potential energy the Schroedinger equation becomes:

$$-\frac{\hbar^2}{2m}\nabla^2\phi = (E_{NR} + V)\phi. \quad -(238)$$

In this derivation, the fundamental axiom of quantum mechanics follows from the wave equation (232) and from the necessity that the classical equivalent of the hamiltonian operator H is the hamiltonian in classical dynamics, the sum of the kinetic and potential energies:

$$H = E_{NR} + V. \quad -(239)$$

So in ECE physics, quantum mechanics can be derived from general relativity in a straightforward way that can be tested against experimental data at each stage. For example earlier in this chapter the method resulted in many new types of spin orbit spectroscopies.

The two quantum Hamilton equations are derived respectively using the well known position and momentum representations of quantum mechanics. In the position representation the Schroedinger axiom is:

$$\hat{p}\psi = -i\hbar\frac{\partial\psi}{\partial x}, \quad \left(\hat{p}\psi\right)^* = i\hbar\frac{\partial\psi}{\partial x} \quad -(240)$$

from which it follows that:

$$[\hat{x}, \hat{p}]\psi = i\hbar\psi. \quad -(241)$$

So the expectation value of the commutator is:

$$\langle[\hat{x}, \hat{p}]\rangle = i\hbar. \quad -(242)$$

In the position representation the expectation value, <x>, of x is x. It follows that:

$$\frac{d}{dx}\langle\hat{x}\rangle = -\frac{i}{\hbar}\langle[\hat{x}, \hat{p}]\rangle = 1. \quad -(243)$$

Note that this tautology can be derived as follows from the equation:

$$\frac{d}{dx}\langle\hat{x}\rangle = \frac{d}{dx}\int\psi^*\hat{x}\psi\, d\tau \quad -(244)$$

which can be proven as follows. First use the Leibnitz Theorem to find that:

$$\frac{d}{dx}\int\psi^*\hat{x}\psi\, d\tau = \left(\int\frac{d\psi^*}{dx}\hat{x}\psi\, d\tau + \int\psi^*\hat{x}\frac{d\psi}{dx}\, d\tau\right). \quad -(245)$$

In quantum mechanics the operators are hermitian operators defined as follows:

$$\int\psi_m^*\hat{A}\psi_n\, d\tau = \left(\int\psi_n^*\hat{A}\psi_m\, d\tau\right)^* = \left(\int\hat{A}^*\psi_m^*\psi_n\, d\tau\right) \quad -(246)$$

Therefore it follows that that Eq. (245) is:

$$\frac{d}{dx}\langle \hat{x} \rangle = 1 = -\frac{i}{\hbar}\int \psi^*(\hat{p}\hat{x} - \hat{x}\hat{p})\psi \, d\tau \quad -(247)$$

which is Eq. (243), Q. E. D.

The first quantum Hamilton equation is obtained by generalizing x to any hermitian operator A of quantum mechanics:

$$\hat{x} \rightarrow \hat{A} \quad -(248)$$

so one format of the first quantum Hamilton equation is:

$$\frac{d}{dx}\langle \hat{A} \rangle = \frac{i}{\hbar}\langle [\hat{p}, \hat{A}] \rangle. \quad -(249)$$

In the special case:

$$\hat{A} = \hat{H} \quad -(250)$$

then:

$$\frac{d}{dx}\langle \hat{H} \rangle = \frac{i}{\hbar}\langle [\hat{p}, \hat{H}] \rangle. \quad -(251)$$

However, it is known that:

$$\frac{d}{dt}\langle \hat{p} \rangle = \frac{i}{\hbar}\langle [\hat{H}, \hat{p}] \rangle \quad -(252)$$

so from Eqs. (251) and (252) the quantum Hamilton equation is:

$$\frac{d}{dx}\langle \hat{H} \rangle = -\frac{d}{dt}\langle \hat{p} \rangle. \quad -(253)$$

The expectation values in this equation are:

$$H = \langle \hat{H} \rangle , \quad p = \langle \hat{p} \rangle \quad -(254)$$

so the first Hamilton equation of motion of 1833 follows, Q. E. D.:

$$\frac{dH}{dx} = - \frac{dp}{dt} . \quad -(255)$$

The second quantum Hamilton equation follows from the momentum

representation:

$$\hat{x} \phi = - \frac{\hbar}{i} \frac{\partial \phi}{\partial p} , \quad \hat{p} \phi = p \phi \quad -(256)$$

from which the following tautology follows:

$$\frac{d}{dp} \langle \hat{p} \rangle = \frac{\hbar}{i} [\langle \hat{x}, \hat{p} \rangle] = 1 \quad -(257)$$

This tautology can be obtained from the equation:

$$\frac{d}{dp} \langle \hat{p} \rangle = \frac{d}{dp} \int \phi^* \hat{p} \phi \, d\tau . \quad -(258)$$

Now generalize p to any operator A:

$$\hat{p} \rightarrow \hat{A} \quad -(259)$$

and the second quantum Hamilton equation in one format is:

$$\frac{d}{dp} \langle \hat{A} \rangle = - \frac{i}{\hbar} \langle [\hat{x}, \hat{A}] \rangle . \quad -(260)$$

In the special case:

$$\hat{A} = \hat{H} \quad -(261)$$

the second quantum Hamilton equation is:

$$\frac{d}{dp}\langle \hat{H} \rangle = -\frac{i}{\hbar}\langle [\hat{x}, \hat{H}] \rangle \quad -(262)$$

However it is known that:

$$\langle [\hat{x}, \hat{H}] \rangle = -\frac{\hbar}{i}\frac{d\langle \hat{x} \rangle}{dt} \quad -(263)$$

so the second quantum Hamilton equation is:

$$\frac{d\langle \hat{H} \rangle}{dp} = \frac{d}{dt}\langle \hat{x} \rangle \quad -(264)$$

which reduces to its classical counterpart, the second quantum Hamilton equation of classical dynamics, Q. E. D.:

$$\frac{dH}{dp} = \frac{dx}{dt} \quad -(265)$$

Note carefully that both the quantum Hamilton equations derive directly from the familiar commutator (242) of quantum mechanics. Conversely the Hamilton equations of 1833 imply the commutator (242) given only the Schroedinger postulate in position and momentum representation respectively. In the Hamilton equations of classical dynamics, x and p are simultaneously observable, so they are also simultaneously observable in the quantized Hamilton equations of motion and in quantum mechanics in general. This argument refutes Copenhagen straightforwardly, and the arbitrary assertion that x and p are not simultaneously observable.

The anti commutator method of refuting Copenhagen was also developed in UFT 175 on www.aias.us and is based on the definition of the anti commutator:

$$\{\hat{x}, \hat{p}\}\phi = -i\hbar\left(x\frac{\partial\phi}{\partial x} + \frac{\partial}{\partial x}(x\phi)\right) = -i\hbar\left(\phi + 2x\frac{d\phi}{dx}\right) \quad -(266)$$

In the position representation the anti commutator is:

$$\{\hat{x}, \hat{p}\}\phi = -i\hbar\left(x\frac{\partial\phi}{\partial x} + \frac{\partial}{\partial x}(x\phi)\right) = -i\hbar\left(\phi + 2x\frac{\partial\phi}{\partial x}\right). \quad -(267)$$

Similarly the commutator of $\hat{p}^2$ and $\hat{x}^2$ is defined as:

$$[\hat{x}^2, \hat{p}^2]\phi = \left([\hat{x}^2, \hat{p}]\hat{p} + \hat{p}([\hat{x}^2, \hat{p}])\right)\phi. \quad -(268)$$

Now use the quantum Hamilton equations to find that:

$$[\hat{p}, \hat{x}^2]\phi = -2i\hbar x\phi \quad -(269)$$

$$[\hat{x}^2, \hat{p}]\phi = 2i\hbar x\phi \quad -(270)$$

It follows that:

$$[\hat{x}^2, \hat{p}^2]\phi = 2i\hbar(\hat{p}\hat{x} + \hat{x}\hat{p})\phi \quad -(271)$$

so the following useful equation has been proven in one dimension:

$$[\hat{x}^2, \hat{p}^2]\phi = 2i\hbar\{\hat{x}, \hat{p}\}\phi. \quad -(272)$$

In three dimensions the Schroedinger axiom in position representation is:

$$\hat{p}\phi = -i\hbar\underline{\nabla}\phi \quad -(273)$$

and in three dimensions the relevant commutator is:

$$[\underset{\sim}{r}, \underline{p}]\phi = -i\hbar\left(\underset{\sim}{r}\cdot\underline{\nabla}\phi - \underline{\nabla}\cdot(\underline{r}\phi)\right) \quad -(274)$$

where in Cartesian coordinates:

$$r^2 = X^2 + Y^2 + Z^2 \quad -(275)$$

Therefore:

$$[\underset{\sim}{r}, \underline{p}]\phi = -i\hbar\left(\underset{\sim}{r}\cdot\underline{\nabla}\phi - \phi\,\underline{\nabla}\cdot\underline{r} - \underset{\sim}{r}\cdot\underline{\nabla}\phi\right) \quad -(276)$$

where:

$$\underline{\nabla}\cdot(\underline{r}\phi) = \phi\,\underline{\nabla}\cdot\underline{r} + \underset{\sim}{r}\cdot\underline{\nabla}\phi \quad -(277)$$

in which:

$$\underline{\nabla}\cdot\underline{r} = 3 \quad -(278)$$

So:

$$[\hat{\underset{\sim}{r}}, \hat{\underline{p}}]\phi = 3i\hbar\phi. \quad -(279)$$

In three dimensions:

$$[\hat{r}^2, \hat{p}^2]\phi = \left([\hat{r}^2, \hat{\underline{p}}]\cdot\hat{\underline{p}} + \hat{\underline{p}}\cdot\left([\hat{r}^2, \hat{\underline{p}}]\right)\right)\phi \quad -(280)$$

where:

$$\begin{aligned}[\hat{r}^2, \hat{\underline{p}}]\phi &= r^2\hat{p}\phi - \hat{p}(r^2\phi)\\ &= i\hbar\nabla r^2\phi \quad -(281)\end{aligned}$$

and where:

$$\underline{\nabla} r^2 = \frac{\partial r^2}{\partial X}\underline{i} + \frac{\partial r^2}{\partial Y}\underline{j} + \frac{\partial r^2}{\partial Z}\underline{k} \quad -(282)$$

with:

$$r^2 = X^2 + Y^2 + Z^2. \quad -(283)$$

So:

$$\underline{\nabla} r^2 = 2\underline{r} \quad -(284)$$

and the three dimensional equivalent of Eq. (272) is:

$$[\hat{r}^2, \hat{p}^2]\phi = 2i\hbar\{\underline{r}, \underline{p}\}\phi. \quad -(285)$$

The anti commuttator in this equation is:

$$(\hat{\underline{r}} \cdot \hat{\underline{p}} + \hat{\underline{p}} \cdot \hat{\underline{r}})\phi = \hat{\underline{r}} \cdot \hat{\underline{p}}\phi + \hat{\underline{p}} \cdot (\hat{\underline{r}}\phi) \quad -(286)$$

$$= -i\hbar(2\underline{r} \cdot \underline{\nabla}\phi + 3\phi)$$

where:

$$\underline{r} \cdot \underline{\nabla}\phi = X\frac{\partial\phi}{\partial X} + Y\frac{\partial\phi}{\partial Y} + Z\frac{\partial\phi}{\partial Z} \quad -(287)$$

so in Cartesian coordinates:

$$\{\hat{\underline{r}}, \hat{\underline{p}}\}\phi = -i\hbar\left(2\left(X\frac{\partial\phi}{\partial X} + Y\frac{\partial\phi}{\partial Y} + Z\frac{\partial\phi}{\partial Z}\right) + 3\phi\right). \quad -(288)$$

When considering the H atom the relevant anti commutator is:

$$\{\hat{r}, \hat{p}_r\}\phi = -i\hbar\left\{r, \frac{\partial}{\partial r}\right\}\phi. \quad -(289)$$

With these definitions some expectation values:

$$\langle[\hat{r}^2, \hat{p}_r^2]\rangle = 2i\hbar\langle\{\hat{\underline{r}}, \hat{\underline{p}}\}\rangle \quad -(290)$$

are worked out for exact solutions of the Schroedinger equation in the fifteen calculational notes accompanying UFT 175 on www.aias.us. All expectation values were checked by computer algebra and tabulated. The result is a definitive refutation of Copenhagen because expectation values can be zero or non-zero depending on which solution of Schroedinger's equation is used, as discussed already. So this method reduces Copenhagen to absurdity, Q. E. D., a reductio ad absurdum refutation of the Copenhagen interpretation of quantum mechanics.

The force equation of quantum mechanics was first inferred in 2011 in UFT 176 and UFT 177 on www.aias.us and have been very influential. It was derived from the two quantum Hamilton equations:

$$i\hbar \frac{d}{dq} \langle \hat{H} \rangle = \langle [\hat{H}, \hat{p}] \rangle \quad -(291)$$

and

$$i\hbar \frac{d}{dp} \langle \hat{H} \rangle = -\langle [\hat{H}, \hat{q}] \rangle \quad -(292)$$

applied to canonical operators $\hat{p}$ and $\hat{q}$. By using the well known {1 - 10}:

$$\frac{d}{dq} \langle \hat{H} \rangle = \left\langle \frac{d\hat{H}}{dq} \right\rangle, \quad \frac{d}{dp} \langle \hat{H} \rangle = \left\langle \frac{d\hat{H}}{dp} \right\rangle \quad -(293)$$

these equations can be put in to operator format as follows:

$$i\hbar \frac{d\hat{H}}{dq} \psi = [\hat{H}, \hat{p}] \psi \quad -(294)$$

and

$$i\hbar \frac{d\hat{H}}{dp} \psi = -[\hat{H}, \hat{q}] \psi \quad -(295)$$

where ϕ is the wave function. If the hamiltonian is defined as:

$$H = \frac{p^2}{2m} + V(x) \quad - (296)$$

then:

$$\frac{dH}{dx} = \frac{dV}{dx} \quad - (297)$$

because in the Hamilton dynamics x and p are independent, canonical variables. Therefore Eq. (293) is satisfied automatically. Using the result:

$$[\hat{H}, \hat{p}]\phi = i\hbar \frac{dV}{dx}\phi = -i\hbar F \phi \quad - (298)$$

where F is force, Eq. (291) gives the force equation of quantum mechanics:

$$-\left(\frac{d\hat{H}}{dx}\right)\phi = F\phi \quad - (299)$$

where the eigenoperator is defined by:

$$\frac{d\hat{H}}{dx} := -\hbar^2 \frac{\partial^3}{\partial x^3} + \frac{dV(x)}{dx}. \quad - (300)$$

In the classical limit, the corresponding principle of quantum mechanics means that Eq. (299) becomes one of the Hamilton equations:

$$F = \frac{dp}{dt} = -\frac{dH}{dx}. \quad - (301)$$

In the momentum representation Eq. (295) gives a second fundamental equation of quantum mechanics:

$$\left(\frac{d\hat{H}}{dp}\right)\phi = v\phi \quad -(302)$$

where the eigenvalues are those of quantized velocity. Here:

$$\frac{dH}{dp} = \frac{p}{m} \quad -(303)$$

and:

$$\left(\frac{d\hat{H}}{dp}\right)\phi = v\phi. \quad -(304)$$

Eq. (302) corresponds in the classical limit to the second Hamilton equation:

$$v = \frac{dx}{dt} = \frac{dH}{dp}. \quad -(305)$$

The general, or canonical, formulation of Eqs. (299) and (302) is as follows:

$$-\left(\frac{d\hat{H}}{dq}\right)\phi = F\phi \quad -(306)$$

and

$$\left(\frac{d\hat{H}}{dp}\right)\phi = v\phi \quad -(307)$$

which reduce to the canonical Hamilton equations:

$$-\frac{dH}{dq} = \frac{dp}{dt} \quad -(308)$$

and

$$\frac{dH}{dp} = \frac{dq}{dt} \quad -(309)$$

The rotational equivalent of Eq. (294) is:

$$i\hbar\left(\frac{d\hat{H}}{d\phi}\right)\phi = [\hat{H}, \hat{J}_Z]\phi \quad -(310)$$

in which the canonical variables are:

$$q = \phi, \; \hat{p} = \hat{J}_Z . \quad -(311)$$

For rotational problems in the quantum mechanics of atoms and molecules, H commutes with $\hat{J}_Z$ so

$$[\hat{H}, \hat{J}_Z] = 0 \quad -(312)$$

in which case:

$$\left(\frac{d\hat{H}}{d\phi}\right)\phi = 0 . \quad -(313)$$

In order for $d\hat{H}/d\phi$ to be non-zero there must be a ϕ dependent potential energy in the hamiltonian:

$$H = \frac{J^2}{2I} + V(\phi) \quad -(314)$$

so the hamiltonian operator must be:

$$\hat{H} = -\frac{\hbar^2}{2I}\hat{\Lambda}^2 + V(\phi) \quad -(315)$$

where $\hat{\Lambda}$ is the lagrangian operator. In this case:

$$\frac{d\hat{H}}{d\phi} = -\frac{\hbar^2}{2I}\hat{\Lambda}^2 + \frac{dV}{d\phi} \quad -(316)$$

and Eq (310) gives the torque equation of quantum mechanics:

$$-\left(\frac{d\hat{H}}{d\phi}\right)\phi = T_q\phi = -\left(\frac{dV}{d\phi}\right)\phi \quad -(317)$$

where T_q are eigenvalues of torque.

There also exist higher order quantum Hamilton equations as discussed in UFT 176, and quantum Hamilton equations for rotation in a plane.

Finally as shown in detail in the influential UFT 177 on www.aias.us the force equation of quantum mechanics can be derived from the quantum Hamilton equations and is:

$$(\hat{H} - E)\frac{d\phi}{dx} = F\phi \quad -(318)$$

where the force is defined by:

$$\frac{d}{dx}\langle\hat{H}\rangle = \frac{dH}{dx} = \frac{dV}{dx} = -F = -\frac{dp}{dt} \quad -(319)$$

In the force equation the hamiltonian operator acts on the derivative of the Schroedinger wave function or in general on the derivative of a quantum mechanical wave function obtained in any way, for example in computational quantum chemistry, and this is a new method of general utility as developed in UT 175.