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[ (%i1) kill(all);
  (%o0) done

[ (%i1) E12: 2*c^2*kappa[0]*kappa[1]*cos(theta[3]) = 2*omega[0]*omega[1]-x
  (%o1)  $2 \kappa_0 \kappa_1 \cos(\theta_3) c^2 = 2 \omega_0 \omega_1 - c^2 x$ 

[ (%i2) E13: E12^2;
  (%o2)  $4 \kappa_0^2 \kappa_1^2 \cos(\theta_3)^2 c^4 = (2 \omega_0 \omega_1 - c^2 x)^2$ 

[ (%i3) k0: sqrt((omega[0]/c)^2-x);
  (%o3)  $\sqrt{\frac{\omega_0^2}{c^2} - x}$ 

[ (%i4) E16a: ratsubst(k0, kappa[0], E13);
  (%o4)  $4 \kappa_1^2 \cos(\theta_3)^2 c^4 \left( \frac{\omega_0^2}{c^2} - x \right) = c^4 x^2 - 4 \omega_0 \omega_1 c^2 x + 4 \omega_0^2 \omega_1^2$ 

[ (%i5) E16b: ratsubst(sqrt((omega[1]/c)^2-x), kappa[1], E16a);
  (%o5)  $4 \omega_0^2 \cos(\theta_3)^2 c^2 \left( \frac{\omega_1^2}{c^2} - x \right) - 4 \cos(\theta_3)^2 c^4 \left( \frac{\omega_1^2}{c^2} - x \right) x = c^4 x^2 - 4 \omega_0 \omega_1 c^2 x + 4 \omega_0^2 \omega_1^2$ 

[ (%i6) E16: factor(E16b);
  (%o6)  $4 \cos(\theta_3)^2 (c^2 x - \omega_0^2) (c^2 x - \omega_1^2) = (c^2 x - 2 \omega_0 \omega_1)^2$ 

[ (%i7) E20: expand(E16);
  (%o7)  $4 \cos(\theta_3)^2 c^4 x^2 - 4 \omega_1^2 \cos(\theta_3)^2 c^2 x - 4 \omega_0^2 \cos(\theta_3)^2 c^2 x + 4 \omega_0^2 \omega_1^2 \cos(\theta_3)^2$ 
 $= c^4 x^2 - 4 \omega_0 \omega_1 c^2 x + 4 \omega_0^2 \omega_1^2$ 

[ (%i8) E23: solve(E20, x);
  (%o8)  $[ x = - ( 2 \cos(\theta_3) ) \sqrt{(\omega_1^4 - 2 \omega_0^2 \omega_1^2 + \omega_0^4) \cos(\theta_3)^2 - 2 \omega_0 \omega_1^3 + 5 \omega_0^2 \omega_1^2 - 2 \omega_0^3 \omega_1} +$ 
 $( - 2 \omega_1^2 - 2 \omega_0^2 ) \cos(\theta_3)^2 + 2 \omega_0 \omega_1 } / ( ( 4 \cos(\theta_3)^2 - 1 ) c^2 ) , x = ( 2 \cos(\theta_3) )$ 
 $\sqrt{(\omega_1^4 - 2 \omega_0^2 \omega_1^2 + \omega_0^4) \cos(\theta_3)^2 - 2 \omega_0 \omega_1^3 + 5 \omega_0^2 \omega_1^2 - 2 \omega_0^3 \omega_1} + ( 2 \omega_1^2 + 2 \omega_0^2 ) \cos(\theta_3)^2 - 2 \omega_0$ 
 $\omega_1 ) / ( ( 4 \cos(\theta_3)^2 - 1 ) c^2 ) ]$ 

[ (%i9) assume(c>0, h[bar]>0);
  (%o9)  $[ c > 0 , h_{bar} > 0 ]$ 

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(%i10) m: (sqrt(E23*h[bar]^2/c^2));
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$$(\%o10) \left[ \frac{h_{bar} \sqrt{x}}{c} = \left( \sqrt{\frac{\left( \omega_1^4 - 2 \omega_0^2 \omega_1^2 + \omega_0^4 \right) \cos(\theta_3)^2 - 2 \omega_0 \omega_1^3 + 5 \omega_0^2 \omega_1^2 - 2 \omega_0^3 \omega_1 + \left( -2 \omega_1^2 - 2 \omega_0^2 \right) \cos(\theta_3)^2 + 2 \omega_0 \omega_1}{\left( 4 \cos(\theta_3)^2 - 1 \right)}} h_{bar} \right) / c^2, \frac{h_{bar} \sqrt{x}}{c} = \left( \sqrt{\frac{\left( \omega_1^4 - 2 \omega_0^2 \omega_1^2 + \omega_0^4 \right) \cos(\theta_3)^2 - 2 \omega_0 \omega_1^3 + 5 \omega_0^2 \omega_1^2 - 2 \omega_0^3 \omega_1 + \left( 2 \omega_1^2 + 2 \omega_0^2 \right) \cos(\theta_3)^2 - 2 \omega_0 \omega_1}{\left( 4 \cos(\theta_3)^2 - 1 \right)}} h_{bar} \right) / c^2 \right]$$