281(1): Rayleigh Jeans Density of State and Reduction to One Photon Theory.

The Rayleigh Jeans density of state is based on the wave equation:

$$\Box \phi = 0 \quad -(1)$$

and does not consider photon mass and the $\underline{B}^{(3)}$ field. Eq. (1) is an ECE wave equation without photon mass. The Rayleigh Jeans density of states is the solution:

$$\phi = \exp\left(i\left(\omega t - \underline{\kappa}\cdot\underline{r}\right)\right) \quad -(2)$$

where:

$$\underline{\kappa} = \kappa_X \underline{i} + \kappa_Y \underline{j} + \kappa_Z \underline{k} \quad -(3)$$

so

$$\omega^2 = c^2\left(\kappa_X^2 + \kappa_Y^2 + \kappa_Z^2\right) \quad -(4)$$

with:

$$\kappa_X = n_1 \frac{\pi}{L}, \quad \kappa_Y = n_2 \frac{\pi}{L}, \quad \kappa_Z = n_3 \frac{\pi}{L} \quad -(5)$$

where

$$V = L^3 \quad -(6)$$

is the volume of radiation in a cube of side L. As pointed out by Jeans this is the octant of a sphere. Therefore in the Rayleigh Jeans density of states:

$$\omega^2 = \frac{c^2\pi^2}{L^2}\left(n_1^2 + n_2^2 + n_3^2\right) \quad -(7)$$

where:

$$n^2 = n_1^2 + n_2^2 + n_3^2 \quad -(8)$$

The number of photons in the octant of the sphere is:

$$N = \frac{1}{8}\left(\frac{4}{3}\pi n^3\right)$$
$$= \frac{1}{6\pi^2}\frac{\omega^3}{c^3} L^3 \quad -(9)$$

So the number of photons in the volume V is:

$$\frac{N}{V} = \frac{1}{6\pi^2}\left(\frac{\omega}{c}\right)^3 = \frac{1}{6\pi^2}\left(k_x^2 + k_y^2 + k_z^2\right)^{3/2} \quad -(10)$$

The total energy of the photons is:

$$U = \frac{E_0}{6\pi^2}\left(\frac{\omega}{c}\right)^3 L^3 \quad -(11)$$

If there is only one photon:

$$U = E_0 = \hbar\omega \quad -(12)$$

This result is obtained when the volume of radiation is:

$$\boxed{V = L^3 = 6\pi^2\left(\frac{c}{\omega}\right)^3} \quad -(13)$$

If the average energy of the Planck oscillator is denoted:

$$\langle \hbar\omega \rangle = \hbar\omega \frac{x}{1-x} \quad -(14)$$

then

$$U = \frac{\hbar\omega}{6\pi^2}\left(\frac{x}{1-x}\right)\left(\frac{\omega}{c}\right)^3 V \quad -(15)$$

and the energy density is:

$$\frac{U}{V} = \frac{\hbar\omega}{6\pi^2}\left(\frac{x}{1-x}\right)\left(\frac{\omega}{c}\right)^3 \quad -(16)$$

In the interval dU:

$$\frac{dU}{V} = \frac{\hbar\omega}{6\pi^2c^3}\left(\frac{x}{1-x}\right)\left((\omega+d\omega)^3-\omega^3\right)$$

$$\sim \frac{\hbar\omega^3 d\omega}{2\pi^2c^3}\left(\frac{x}{1-x}\right) \quad -(17)$$

In this interval:

$$\frac{dN}{V} = \frac{1}{6c^3\pi^2}\left((\omega+d\omega)^3-\omega^3\right)$$

$$\sim \frac{1}{2}\frac{\omega^2}{c^3\pi^2}\,d\omega \quad -(18)$$

It is usually considered that there are two states of polarization, so:

$$\frac{dN}{V} = \frac{\omega^2}{c^3\pi^2}\,d\omega \quad -(19)$$

This is the usual Rayleigh Jeans density of states

4) This depends on the assumption of zero photon mass and no $\underline{B}^{(3)}$ field. The energy density in this case is:

$$\frac{U}{V} = \int \frac{\hbar \omega^3}{\pi^2 c^3} \left(\frac{x}{1-x} \right) d\omega \quad -(20)$$

and the intensity in watts per square metre is:

$$I = \frac{\hbar}{\pi^2 c^2} \int \omega^3 \left(\frac{x}{1-x} \right) d\omega \quad -(21)$$

where

$$x = \exp\left(-\frac{\hbar \omega}{kT} \right) \quad -(22)$$

The usual Stefan Boltzmann law is obtained when

$$I = \frac{\hbar}{\pi^2 c^2} \int_0^\infty \omega^3 \left(\frac{x}{1-x} \right) d\omega$$

$$= \left(\frac{\pi^2 k^4}{60 c^2 \hbar^3} \right) T^4 \quad -(23)$$

The reduction to the one photon result:

$$E = \hbar \omega \quad -(24)$$

occurs when:

$$V = 6\pi^2 \left(\frac{1-x}{x} \right) \left(\frac{c}{\omega} \right)^3 \quad -(25)$$

when

$$u = \hbar \omega \quad -(26)$$

in eq. (15)