289 (3): Theory of Reflection and Refraction by Scattering of a Photon with Mass.

As in UFT 160 consider the general Compton scattering of a photon of mass from a static mass. The conservation of energy is:

$$\hbar\omega + \hbar\omega_0 = \hbar\omega_1 + \hbar\omega_2 \quad -(1)$$

and the conservation of momentum is:

$$\hbar\underline{\kappa} = \hbar\underline{\kappa}_1 + \hbar\underline{\kappa}_2 \quad -(2)$$

Here, $\hbar\omega$ is the energy of an incoming photon of mass m. In UFT 160 this photon collided with a static electron in a metal foil, the Compton effect modified for photon mass. The rest energy of the electron was:

$$\hbar\omega_0 = m_0 c^2 \quad -(3)$$

where m_0 is the electron mass. In Compton scattering in UFT 160, ω_1 denotes the scattered frequency of the photon and ω_2 the scattered frequency of the electron. This theory can be adapted for reflection and refraction in various ways.

First, the theory of UFT 160 is reviewed briefly. It eliminated the frequency ω_2 between eqs. (1) and (2) to give an expression for the Compton frequency of the electron, $mc^2/\hbar$, in terms of ω and ω_1. From eq. (2):

2)

$$\kappa_2^2 = \kappa^2 + \kappa_1^2 - 2\kappa\kappa_1\cos\theta \quad - (4)$$

where θ is the angle between $\underline{\kappa}$ and $\underline{\kappa}_1$, the incoming and scattered wave vectors of the photon, considered to be a photon with mass.

The de Broglie equations were used as follows:

$$\hbar\underline{\kappa} = \gamma m\underline{v} \quad - (5)$$

$$\hbar\underline{\kappa}_1 = \gamma_1 m_1 \underline{v}_1 \quad - (6)$$

$$\hbar\underline{\kappa}_2 = \gamma_2 m_2 \underline{v}_2 \quad - (7)$$

The de Broglie equations for energy are:

$$\hbar\omega = \gamma m c^2 \quad - (8)$$

$$\hbar\omega_1 = \gamma_1 m_1 c^2 \quad - (9)$$

$$\hbar\omega_2 = \gamma_2 m_2 c^2 \quad - (10)$$

Denoting magnitudes by:

$$\hbar\kappa = \gamma m v \quad - (11)$$

$$\hbar\kappa_1 = \gamma_1 m_1 v_1 \quad - (12)$$

$$\hbar\kappa_2 = \gamma_2 m_2 v_2 \quad - (13)$$

it follows from dividing eq. (11) by eq. (8) that:

$$\frac{\kappa}{\omega} = \frac{v}{c^2} \quad - (14)$$

so:

$$\kappa = \frac{\omega v}{c^2} \quad - (15)$$

Similarly:

$$\kappa_1 = \frac{\omega_1 v_1}{c^2}, \quad \kappa_2 = \frac{\omega_2 v_2}{c^2} \quad - (16)$$

3) Therefore:

$$\omega_2^2 v_2^2 = \omega^2 v^2 + \omega_1^2 v_1^2 - 2\omega\omega_1 v v_1 \cos\theta \quad - (17)$$

and

$$m_0 = \frac{\hbar}{c^2}(\omega_1 + \omega_2 - \omega) \quad - (18)$$

The velocities are eliminated as in Note 160(3), Eqs. (35) to (37):

$$v^2 = 1 - \left(\frac{mc^2}{\hbar\omega}\right)^2 \quad - (19)$$

$$v_1^2 = 1 - \left(\frac{mc^2}{\hbar\omega}\right)^2 \quad - (20)$$

$$v_2^2 = 1 - \left(\frac{m_0 c^2}{\hbar\omega_2}\right)^2 \quad - (21)$$

where m is the mass of the photon and m_0 is the mass of the electron.

Denoting:

$$x_1 = \frac{mc^2}{\hbar}, \quad x_2 = \frac{m_0 c^2}{\hbar} \quad - (22)$$

it is found that:

$$x_2 = \frac{\omega\omega_1}{\omega - \omega_1} - \left(\frac{x_1^2 + (\omega^2 - x_1^2)^{1/2}(\omega_1^2 - x_1^2)^{1/2}\cos\theta}{\omega - \omega_1}\right) \quad - (23)$$

If the photon mass is zero:

$$x_1 = 0 \quad - (24)$$

†) Eq. (23) reduces to the usual Compton formula:

$$\frac{1}{\omega_1} - \frac{1}{\omega} = \frac{\hbar}{m_0 c^2}\left(1 - \cos\theta\right) \quad -(25)$$

In terms of wavelength, eq. (25) is:

$$\lambda_1 - \lambda = \frac{h}{m_0 c}\left(1 - \cos\theta\right) \quad -(26)$$

Here:

$$\lambda_0 = \frac{h}{m_0 c} \quad -(27)$$

is the Compton wavelength, and

$$\omega_0 = \frac{m_0 c^2}{\hbar} \quad -(28)$$

is the Compton frequency.

So:

$$\frac{1}{\omega_1} - \frac{1}{\omega} = \frac{1}{\omega_0}\left(1 - \cos\theta\right) \quad -(29)$$

and

$$\lambda_1 - \lambda = \lambda_0\left(1 - \cos\theta\right) \quad -(30)$$

<u>Evans / Morris Shifts</u>

This theory can be adapted for the Evans / Morris shifts by reinterpreting Eqs. (1) and (2).

It is assumed that an incident photon of energy $\hbar\omega$ and momentum $\hbar\underline{\kappa}$ collides with

5) the Evans / Morris mass m_0 at the interface between the incident beam and a refracted beam of energy $\hbar\omega_1$ and momentum $\hbar \underline{\kappa}_1$. The initial energy of m_0 is $\hbar\omega_0$ and it has no initial momentum. Its final energy is $\hbar\omega_2$ and its final momentum is $\hbar \underline{\kappa}_2$.

In the massless photon theory the change of wavelength on refraction is given by eq. (30), and the change of frequency on refraction is given by eq. (29).

In the massive photon theory the changes in wavelength and frequency are given by eq. (28):

The Evans / Morris wavelength is given by:

$$\lambda_0 = \frac{\lambda_1 - \lambda}{1 - \cos\theta} \quad - (31)$$

The Evans / Morris frequency is given by:

$$\frac{1}{\omega_0} = \left(\frac{1}{\omega_1} - \frac{1}{\omega}\right) \Big/ \left(1 - \cos\theta\right) \quad - (32)$$

They can be determined experimentally and it will be interesting to find whether they are constants as in the Compton effect, or whether they are variable.

Another possible interpretation of eqs. (1)

) and (2) is that two photons $\hbar\omega$ and $\hbar\omega_0$ annihilate to give $\hbar\omega_1$ and $\hbar\omega_2$. The refracted frequency is ω_1 and the reflected frequency is ω_2. The incident frequency is ω. The Evans / Morris frequency is ω_0, and is a property of the material in which refraction occurs.

Then eqs. (1) and (2) can be solved for ω_1 ~~for~~ this note, or the the refracted frequency, and in the next note the reflected frequency ω_2.