289(4): Expression for ω_1 in terms of ω in Note 289(3)

The starting equation is eq. (23) of note 289(3):

$$x_2 = \frac{\omega\omega_1}{\omega-\omega_1} - \left(\frac{x_1^2 + (\omega^2 - x_1^2)^{1/2}(\omega_1^2 - x_1^2)^{1/2}\cos\theta}{\omega-\omega_1}\right) \quad -(1)$$

which can be expressed as:

$$x_1^2 + (\omega-\omega_1)x_2 - \omega\omega_1 + (\omega^2 - x_1^2)^{1/2}(\omega_1^2 - x_1^2)^{1/2}\cos\theta = 0 \quad -(2)$$

Therefore:

$$(\omega^2 - x_1^2)^{1/2}(\omega_1^2 - x_1^2)^{1/2}\cos\theta = \omega\omega_1 - (\omega-\omega_1)x_2 - x_1^2 \quad -(3)$$

Here m_2 is the electron mass and m_1 is the photon mass, and:

$$x_1 = \frac{m_1 c^2}{\hbar}, \quad x_2 = \frac{m_2 c^2}{\hbar} \quad -(4)$$

1) Eq. (3) can be solved by computer algebra to find ω_1 in terms of ω. To give an idea of the results a hand calculation is carried out as follows.

2) The Evans Morris mass is m_2 and is given by eq. (1). The Evans Morris mass

2) depends on the mass of the photon m_1 and the incident and scattered frequencies, ω and ω_1.

3) In a Compton effect experiment the photon mass can be found by solving eq. (1) for x_1 in terms of x_2 and ω and ω_1.

From eq. (2):

$$(\omega^2 - x_1^2)(\omega_1^2 - x_1^2)\cos^2\theta = \left(x_1^2 + (\omega - \omega_1)x_2 - \omega\omega_1\right)^2 \quad -(5)$$

Therefore:

$$\left(\omega^2\omega_1^2 - x_1^2(\omega^2 + \omega_1^2) + x_1^4\right)\cos^2\theta = \left(x_1^2 + \omega x_2 - \omega_1(\omega + x_2)\right)^2 \quad -(6)$$

Let

$$\left.\begin{aligned} A &= x_1^2 + \omega x_2 \\ B &= \omega + x_2 \\ C &= \omega^2 - x_1^2 \\ D &= x_1^2(x_1^2 - \omega^2) \end{aligned}\right] \quad -(7)$$

Then:

$$(\omega_1^2 C + D)\cos^2\theta = (A - \omega_1 B)^2 \quad -(8)$$

$$= A^2 - 2\omega_1 B + \omega_1^2 B^2$$

Therefore ω_1 can be found from the

3) quadratic:

$$(C\cos^2\theta - B^2)\omega_1^2 + 2\omega_1 B + D\cos^2\theta - A^2 = 0 \qquad -(9)$$

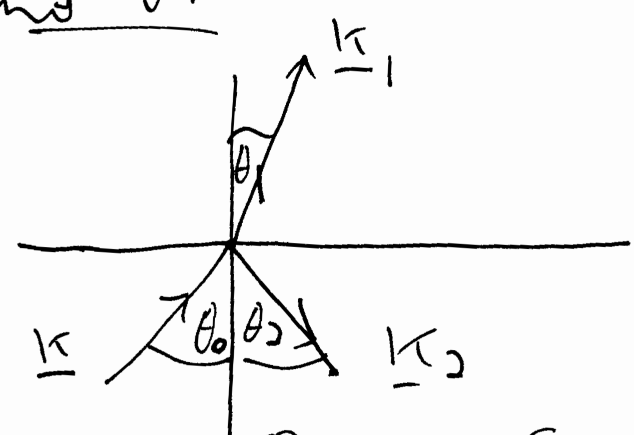
i.e.
$$a\omega_1^2 + b\omega_1 + c' = 0 \qquad -(10)$$

where:
$$\left.\begin{aligned} a &= C\cos^2\theta - B^2 \\ b &= 2B \\ c' &= D\cos^2\theta - A^2 \end{aligned}\right\} \qquad -(11)$$

i.e.
$$\omega_1 = \frac{1}{2a}\left(-b \pm (b^2 - 4ac)^{1/2}\right) \qquad -(12)$$

Therefore ω_1 can be found in terms of ω, x_1, x_2 and θ.

Fig. (1)



For refraction, the angle in these equations is
$$\theta = \pi - \theta_0 + \theta_1 \qquad -(13)$$
where the angle of incidence is:
$$\theta_0 = \theta_2 \qquad -(14)$$
For reflection
$$\theta = \theta_0 + \theta_2 = 2\theta_0 \qquad -(15)$$

4)

If the photon mass is assumed to be zero then:

$$x_2 = \frac{\omega\omega_1}{\omega-\omega_1} - \frac{\omega\omega_1 \cos\theta}{\omega-\omega_1} \quad -(16)$$

so

$$\left(\frac{\omega-\omega_1}{\omega\omega_1}\right) x_2 = 1 - \cos\theta \quad -(17)$$

i.e.

$$\frac{1}{\omega_1} - \frac{1}{\omega} = \frac{\hbar}{m_2 c^2}\left(1-\cos\theta\right) \quad -(18)$$

which is a formula for the Evans Morris mass similar to the Compton effect formula.

<u>Equal</u> <u>Mass</u> <u>Theory</u>

In this case the Evans Morris mass is assumed to be the photon mass, so:

$$x_1 = x_2 \quad -(19)$$

and for a given ω_1, ω and θ, the Evans Morris mass may be found from eq. (1) by solving it with computer algebra.