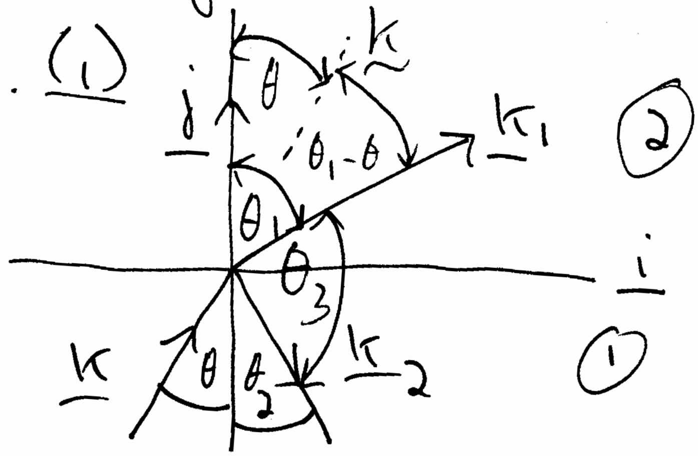
289(6): Two Photon Theory of Reflection and Refraction

In this theory a beam consisting of two photons is reflected and refracted. One photon is reflected and one photon is refracted.

Fig. (1)




The equations of conservation of energy and momentum are:

$$\hbar\omega + \hbar\omega = \hbar\omega_1 + \hbar\omega_2 \quad -(1)$$

and

$$\hbar\underline{k} + \hbar\underline{k} = \hbar\underline{k}_1 + \hbar\underline{k}_2 \quad -(2)$$

Therefore:

$$2\omega = \omega_1 + \omega_2 \quad -(3)$$

and

$$2\underline{k} = \underline{k}_1 + \underline{k}_2 \quad -(4)$$

From eq. (4)

$$4k^2 = k_1^2 + k_2^2 + 2k_1k_2\cos\theta_3 \quad -(5)$$

where

$$\theta_3 = \pi - (\theta_1 + \theta_2) \quad -(6)$$

is the angle between $\underline{k}$ and $\underline{k}_1$.

) Assume that medium (1) is air and medium (2) is glass. Then:

$$K = \frac{\omega}{c}, \quad K_2 = \frac{\omega_2}{c}, \quad K_1 = \frac{\omega_1}{v} \quad - (7).$$

It has been assumed that the phase velocity in air is c and the phase velocity in glass is v. The refractive index of medium (2) is:

$$n = \frac{c}{v} \quad - (8)$$

Therefore eq. (5) becomes:

$$4\frac{\omega^2}{c^2} = \frac{\omega_1^2}{v^2} + \frac{\omega_2^2}{c^2} + \frac{2\omega_1\omega_2}{cv}\cos(\theta_3) \quad - (9)$$

Here:

$$\omega_2^2 = (2\omega - \omega_1)^2 \quad - (10)$$

from eq. (3). So:

$$4\omega^2 = n^2\omega_1^2 + (2\omega - \omega_1)^2 + 2n\omega_1\omega_2\cos(\theta_3)$$

$$= 4\omega^2 - 2\omega\omega_1 + \omega_1^2 + n^2\omega_1^2 + 2n\omega_1(2\omega - \omega_1)\cos(\theta_3)$$

$$- (11)$$

So:

$$\omega_1^2(1+n^2) + 2\omega_1\left(n(2\omega-\omega_1)\cos(\theta_3) - \omega\right) = 0$$

$$- (12)$$

Therefore:

$$\omega_1 = \frac{2}{1+n^2}\left(\omega - (2\omega - \omega_1)\, n \cos(\theta_3)\right) \quad -(13)$$

So:

$$(1+n^2)\,\omega_1 = 2\omega - 2n(2\omega - \omega_1)\cos(\theta_3) \quad -(14)$$

i.e.

$$\omega_1\left(1+n^2 - 2n\cos(\theta_3)\right) = 2\omega\left(1 - 2n\cos(\theta_3)\right)$$

and

$$\omega_1 = \frac{2\omega\left(1 - 2n\cos(\theta_3)\right)}{\left(n^2 + 1 - 2n\cos(\theta_3)\right)} \quad -(15)$$

Eq. (15) is:

$$\omega_1 = \frac{2\omega}{1 + \frac{n^2}{y}} \quad -(16)$$

where

$$y = 1 - 2n\cos(\theta_3) \quad -(17)$$

In order for $\omega_1$ to be positive:

$$2n\cos(\theta_3) \leq 1 \quad -(18)$$

and

$$\theta_3 \leq \cos^{-1}\left(\frac{1}{2n}\right) \quad -(19)$$

If the refractive index of the glass is:

$$n = 1.5 \quad -(20)$$

Then:

$$\theta_3 \leq 70.53^\circ \quad -(21)$$

$$\theta_1 + \theta = \theta_1 + \theta_2 \leq 109.47^\circ$$

If

$$\theta_3' = 70.53^\circ \quad -(22)$$

then:

$$\omega_1 = 0 \quad -(23)$$

and

$$\omega_2 = 2\omega \quad -(24)$$

In this case the reflected light appears to be blue shifted and the refracted light is shifted to zero frequency - the maximum red shift.

If the refractive index of medium (2) is the same as that of medium (1), for example air, with $n = 1$, then:

$$\theta_1' = \theta \quad -(25)$$

and the formula (18) is no longer valid. In this two photon model the refracted light is red shifted and the reflected light is blue shifted.

The frequency of the reflected light is found from eqs (16) and (3):

5)

$$\omega_2 = 2\omega - \omega_1$$

$$= 2\omega\left(1 - \frac{1}{1+\frac{n^2}{y}}\right)$$

$$= 2\omega\frac{n^2}{y}\left(1+\frac{n^2}{y}\right)^{-1} \quad -(26)$$

$$= 2\omega\frac{n^2}{y+n^2}$$

<u>Results</u>

$$\omega_1 = 2\omega\left(\frac{y}{y+n^2}\right) \quad -(27)$$

$$\omega_2 = 2\omega\left(\frac{n^2}{y+n^2}\right) \quad -(28)$$

where

$$y = 1 - 2n\cos\left(\theta_3 - \right) \quad -(29)$$

and

$$\theta_3 \leq \cos^{-1}\left(\frac{1}{2n}\right) \quad -(30)$$

<u>Check with Snell's Law</u>

This is $\sin\theta = 1.5\sin\theta_1$ $-(31)$

If $\theta = 0°$, $\theta_1 = 0°$ $-(32)$

If $\theta = 90°$, $\theta_1 = 48.19°$ $-(33)$

so $\theta_1 - \theta \leq 48.19°$ $-(34)$

... (31) QED