289(9): Precise Geometrical Conditions for the Two Photon Model

We first consider the conservation of momentum in the Compton effect:

$$\underline{\kappa} = \underline{\kappa_1} + \underline{\kappa_2} \quad -(1)$$

Eq. (1) means:

$$\kappa_x = \kappa_{1x} + \kappa_{2x} \quad -(2)$$

$$\kappa_y = \kappa_{1y} + \kappa_{2y} \quad -(3)$$

and

$$\kappa^2 = \kappa_1^2 + \kappa_2^2 + 2\kappa_1 \kappa_2 \cos\theta \quad -(4)$$

i.e.

$$\kappa_x^2 + \kappa_y^2 = \kappa_{1x}^2 + \kappa_{1y}^2 + \kappa_{2x}^2 + \kappa_{2y}^2 + 2\left(\kappa_{1x}^2 + \kappa_{1y}^2\right)^{1/2}\left(\kappa_{2x}^2 + \kappa_{2y}^2\right)^{1/2}\cos\theta \quad -(5)$$

Therefore:

$$\cos\theta = \frac{\kappa_{1x}\kappa_{2y} + \kappa_{1y}\kappa_{2y}}{\left(\kappa_{1x}^2 + \kappa_{1y}^2\right)^{1/2}\left(\kappa_{2x}^2 + \kappa_{2y}^2\right)^{1/2}} \quad -(6)$$

In the Compton effect:

$$\lambda_1 - \lambda = \lambda_0\left(1 - \cos\theta\right) \quad -(7)$$

where

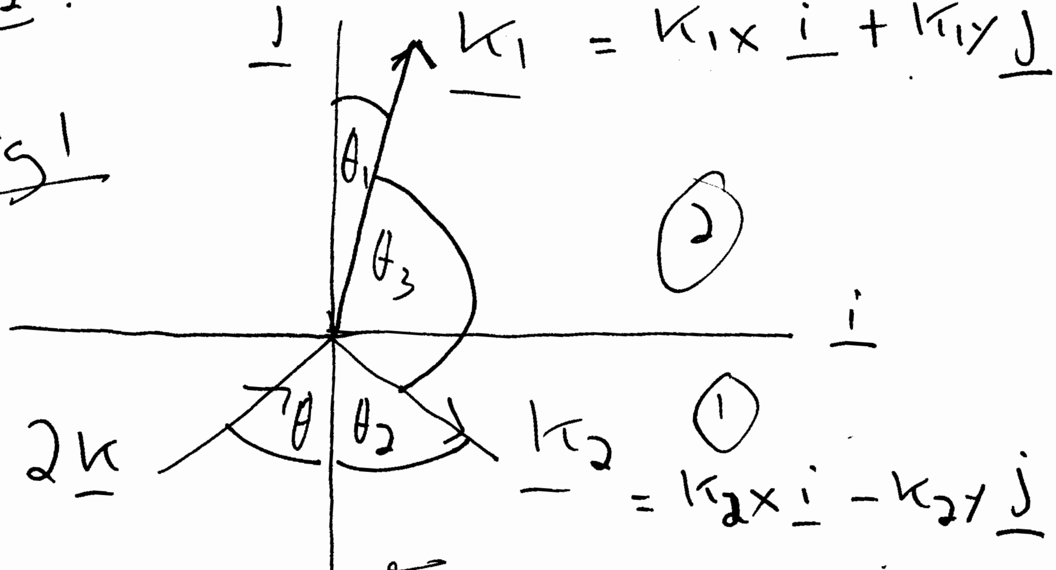
$$\lambda_1 > \lambda \quad -(8)$$

so

$$0 < \cos\theta < 1 \quad -(9)$$

2) In the two photon model a beam of total momentum $2\underline{k}$ is reflected into a beam of momentum $\underline{k_1}$ and refracted into a beam of momentum $\underline{k_2}$.

Fig 1



By conservation of momentum:

$$\underline{k} + \underline{k} = \underline{k_1} + \underline{k_2} \quad -(10)$$

So

$$2k_x = k_{1x} + k_{2x} \quad -(11)$$

$$2k_y = k_{1y} - k_{2y} \quad -(12)$$

$$4k^2 = k_1^2 + k_2^2 + 2k_1k_2\cos\theta_3 \quad -(13)$$

where

$$\theta_3 = \pi - (\theta_1 + \theta_2) \quad -(14)$$

$$= \pi - (\theta + \theta_1) \quad -(15)$$

So:

$$4k_x^2 = k_{1x}^2 + k_{2x}^2 + 2k_{1x}k_{2x} \quad -(16)$$

$$4k_y^2 = k_{1y}^2 + k_{2y}^2 - 2k_{1y}k_{2y} \quad -(17)$$

by squaring eqs. (11) and (12)

3) Eq. (13) is:

$$4\left(\kappa_x^2+\kappa_y^2\right)=\kappa_{1x}^2+\kappa_{1y}^2+\kappa_{2x}^2+\kappa_{2y}^2 + 2\left(\kappa_{1x}^2+\kappa_{1y}^2\right)^{1/2}\left(\kappa_{2x}^2+\kappa_{2y}^2\right)^{1/2}\cos\theta_3 \quad -(18)$$

Squaring eqs. (11) and (12):

$$4\kappa_x^2=\kappa_{1x}^2+\kappa_{2x}^2+2\kappa_{1x}\kappa_{2x} \quad -(19)$$

$$4\kappa_y^2=\kappa_{1y}^2+\kappa_{2y}^2-2\kappa_{1y}\kappa_{2y} \quad -(20)$$

so

$$4\left(\kappa_x^2+\kappa_y^2\right)=\kappa_{1x}^2+\kappa_{2x}^2+\kappa_{1y}^2+\kappa_{2y}^2 + 2\left(\kappa_{1x}\kappa_{2x}-\kappa_{1y}\kappa_{2y}\right) \quad -(21)$$

Fm eqs (18) and (21):

$$\cos\theta_3=\frac{\kappa_{1x}\kappa_{2x}-\kappa_{1y}\kappa_{2y}}{\left(\kappa_{1x}^2+\kappa_{1y}^2\right)^{1/2}\left(\kappa_{2x}^2+\kappa_{2y}^2\right)^{1/2}} \quad -(22)$$

$$=\cos\left(\pi-(\theta+\theta_1)\right)=-\cos\left(\theta+\theta_1\right)$$

so

$$\boxed{\cos\left(\theta+\theta_1\right)=\frac{\kappa_{1y}\kappa_{2y}-\kappa_{1x}\kappa_{2x}}{\left(\kappa_{1x}^2+\kappa_{1y}^2\right)^{1/2}\left(\kappa_{2x}^2+\kappa_{2y}^2\right)^{1/2}}} \quad -(23)$$

4) Therefore the two photon model is plausible provided that eq. (23) is true.

1) If

$$\kappa_{1Y} \kappa_{2Y} \geq \kappa_{1X} \kappa_{2X} \quad -(24)$$

then:

$$0 < \cos(\theta + \theta_1) < 1 \quad -(25)$$

and

$$0 < (\theta + \theta_1) < \pi/2 \quad -(26)$$

2) If

$$\kappa_{1X} \kappa_{2X} \geq \kappa_{1Y} \kappa_{2Y} \quad -(27)$$

then

$$-1 \leq \cos(\theta + \theta_1) < 0 \quad -(28)$$

and

$$\frac{\pi}{2} \leq (\theta + \theta_1) \leq 3\pi/2 \quad -(29)$$

It is important to note that these results are independent of the incident $2\underline{\kappa}$.

So eq. (16) does not lead to any self contradiction or difficulty.

PS By Snell's Laws:

$$\theta = \theta_2 \quad -(30)$$

$$n \sin\theta = n_1 \sin\theta_1 \quad -(31)$$

and θ_3 is determined experimentally for these laws as usual.