```
(%i1) kill(all);
(%o0) done
```

1 Equations for frequency shift

```
(%i1) omega: omega0*exp(-A*L/3);
```

(%o1) $omega0\ \%e^{-\frac{A\ L}{3}}$

```
(%i2) A: (N/V)*mu[fi]^2/(6*epsilon[0]*v*h[bar]);
```

(%o2) $\frac{\mu_{fi}^2\ N}{6\ \varepsilon_0\ h_{bar}\ v\ V}$

2 Transition matrix elements, N/V, omega0

```
(%i3) mu[1]: (2^(21/2)*3^(7/2)*a[0]*e)/(15625*sqrt(6)*Z) /*2s --> 3p*/;
      mu[2]: (16*6^(7/2)*a[0]*e)/(15625*Z) /*2p --> 3s*/;
      mu[3]: (8192*3^(9/2)*a[0]*e)/(5^(13/2)*sqrt(6)*sqrt(30)*Z) /*2p --
      mu[4]: (4096*3^(9/2)*sqrt(15)*a[0]*e)/(78125*sqrt(6)*sqrt(30)*Z) /
```

(%o3) $\frac{2^{21/2}\ 3^{7/2}\ a_0\ e}{15625\ \sqrt{6}\ Z}$

(%o4) $\frac{16\ 6^{7/2}\ a_0\ e}{15625\ Z}$

(%o5) $\frac{8192\ 3^{9/2}\ a_0\ e}{5^{13/2}\ \sqrt{6}\ \sqrt{30}\ Z}$

(%o6) $\frac{4096\ 3^{9/2}\ \sqrt{15}\ a_0\ e}{78125\ \sqrt{6}\ \sqrt{30}\ Z}$

Incident frequency (from 656.3 nm)

```
(%i7) omega0: 2*%pi*2.9979e8/656.3e-9, numer;
```

(%o7) $2.870083990917824\ 5\ 10^{15}$

Avogadro number

```
(%i8) N: 6.022141e23;
```

(%o8) $6.022141000000001\ 10^{23}$

Mole volume

```
(%i9) V: 22.4135e-3 /* m^3 */;
(%o9) 0.0224135
```

3 Numerical evaluation

SI units

```
(%i10) str1: [h[bar]=6.62618e-34, a[0]=5.29177e-11, e=1.602177e-19,
       epsilon[0]=8.8541878e-12, v=2.9979e8, Z=1];
```

(%o10) $[h_{bar}=6.6261800000000001\,10^{-34}, a_0=5.29177\,10^{-11}, e=1.6021770000000001\,10^{-19}, \varepsilon_0=8.8541877999999994\,10^{-12}, v=2.9979\,10^{8}, Z=1]$

```
(%i11) A1: ev(A, str1, numer);
```

(%o11) $2.5460145963651467\,10^{60}\,\mu_{fi}^2$

Transition matrix elements

```
(%i12) for i:1 thru 4 do (mu: ev(mu[i], str1, numer), print(mu));
```

$1.5002206794470551\,10^{-29}$

$4.593468957770953\,10^{-30}$

$2.0787667514942342\,10^{-29}$

$1.8002648153364615\,10^{-29}$

(%o12) *done*

```
(%i13) f1: ev(A1, [mu[fi]=ev(mu[1], str1)]);
       f2: ev(A1, [mu[fi]=ev(mu[2], str1)]);
       f3: ev(A1, [mu[fi]=ev(mu[3], str1)]);
       f4: ev(A1, [mu[fi]=ev(mu[4], str1)]);
```

(%o13) 573.0218525090949

(%o14) 53.72079867272765

(%o15) 1100.201956817462

(%o16) 825.1514676130968

```
(%i17) o1: ev(omega, [A=f1]);
       o2: ev(omega, [A=f2]);
       o3: ev(omega, [A=f3]);
       o4: ev(omega, [A=f4]);
```

(%o17) $2.8700839909178245\,10^{15}\,\%e^{-191.0072841696983\,L}$

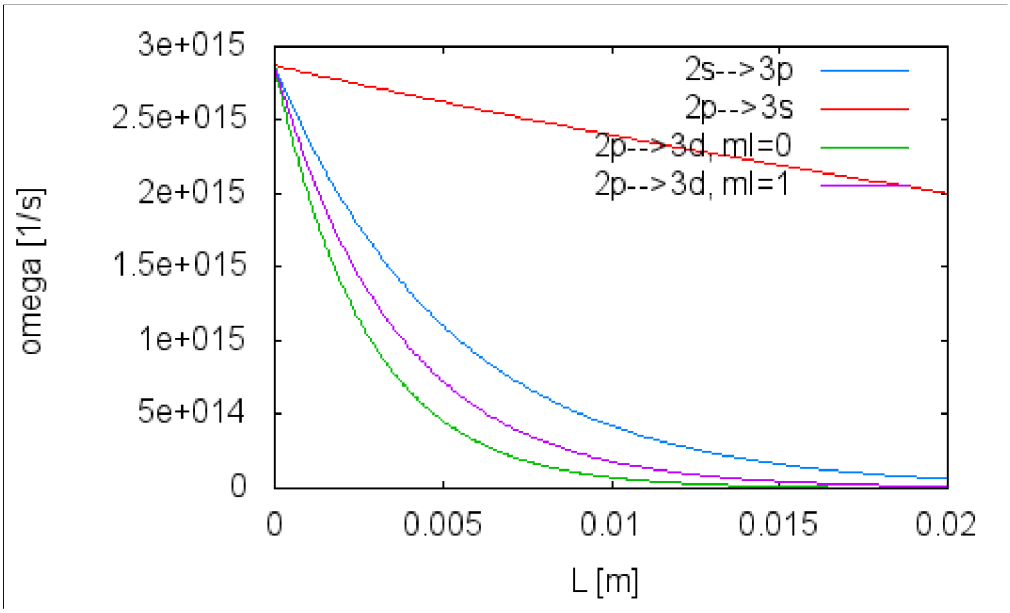
(%o18) $2.8700839909178245\,10^{15}\,\%e^{-17.90693289090922\,L}$

(%o19) $2.8700839909178245\,10^{15}\,\%e^{-366.7339856058207\,L}$

(%o20) $2.8700839909178245\,10^{15}\,\%e^{-275.0504892043656\,L}$

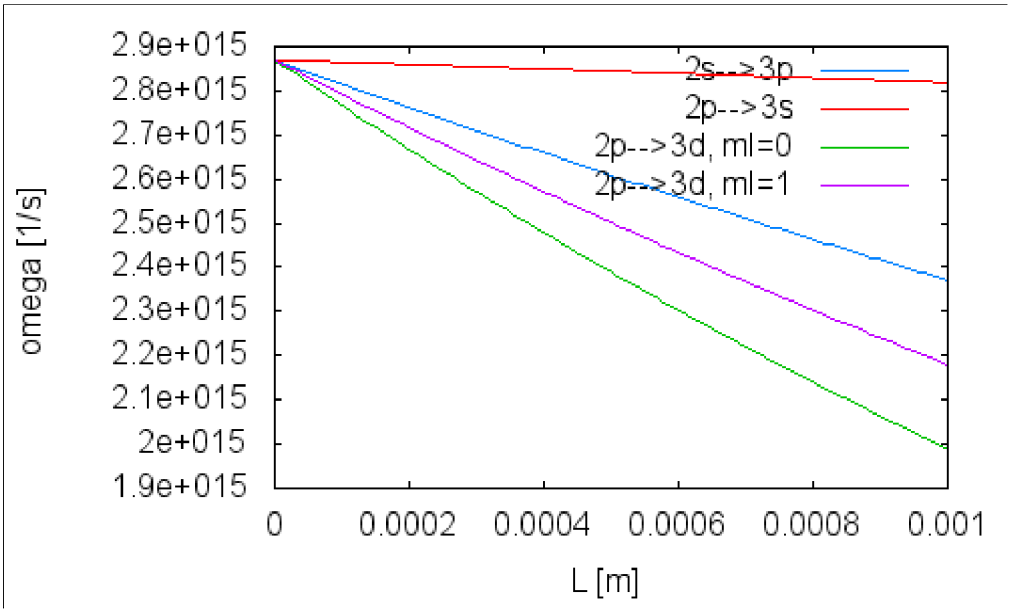
```
(%i21) wxplot2d([o1,o2,o3,o4], [L,0,2.e-2],
       [legend, "2s-->3p", "2p-->3s", "2p-->3d, ml=0", "2p-->3d, ml=1"],
       [xlabel, "L [m]"], [ylabel, "omega [1/s]"])$
```

(%t21)




```
(%i22) wxplot2d([o1,o2,o3,o4], [L,0,1.e-3],
       [legend, "2s-->3p", "2p-->3s", "2p-->3d, ml=0", "2p-->3d, ml=1"],
       [xlabel, "L [m]"], [ylabel, "omega [1/s]"])$
```

(%t22)



Atomic units

```
(%i23) str2: [h[bar]=1, a[0]=1, e=1, epsilon[0]=1/(4*%pi), v=137.036, Z=1
```

(%o23) $[h_{bar}=1, a_0=1, e=1, \varepsilon_0=\frac{1}{4\,\pi}, v=137.036, Z=1]$

```
(%i24) V: V/(5.29177*10^-11)^3;
```

(%o24) $1.5125397604097477\,10^{29}$

```
(%i25) A2: ev(A, str2, numer);
```

(%o25) $6.0851047708502077\,10^{-8}\,\mu_{fi}^{2}$

```
(%i26) for i:1 thru 4 do (mu: ev(mu[i], str2, numer), print(mu));
1.769472000000003
0.54178793926775
2.451852325256409
2.123366399999999
(%o26) done
```