304(6) : Calculation of Photon Mass from the Power Absorption Coefficient.

Consider the phase of an electromagnetic wave:

$$E = E^{(0)} \exp\left(i\omega \left(t - \frac{Z}{v} \right) \right) \quad -(1)$$

where the velocity v is defined by:

$$\frac{1}{v} = \frac{1}{c} \left(n' - i n'' \right) \quad -(2)$$

Here n' and n'' are the real and imaginary parts of the refractive index. The imaginary part n'' is known as the extinction coefficient.

So:

$$E = E^{(0)} \exp(i\omega t) \exp\left(-i \frac{\omega}{c} Z n' \right) \exp\left(- \frac{\omega}{c} Z n'' \right) \quad -(3)$$

Define eq. (3) as:

$$E = E_1 \exp\left(- \frac{\omega Z}{c} n'' \right) \quad -(4)$$

The intensity ratio is:

$$\frac{E}{E_1} = \frac{I}{I_0} = \exp\left(-2 \frac{\omega n''}{c} Z \right) \quad -(5)$$

Therefore by the Beer Lambert law:

2)
$$\frac{I}{I_0} = \exp\left(-\alpha Z\right) = \exp\left(-\frac{2\omega n''}{c} Z\right) \quad -(6)$$

The power absorption coefficient is therefore:

$$\boxed{\alpha = \frac{2\omega}{c} n''} \quad -(7)$$

By definition:

$$\epsilon' - i\epsilon'' = (n' - in'')^2 \quad -(8)$$
$$= n'^2 - n''^2 - 2in'n''.$$

So:

$$\epsilon' = n'^2 - n''^2 \quad -(9)$$
$$\epsilon'' = 2n'n'' \quad -(10)$$

Fm eqs (7) and (10):

$$\boxed{\alpha = \frac{\omega \epsilon''}{n' c}} \quad -(11)$$

as in the early Omnia Opera papers.

Therefore:

$$n' = \frac{\omega \epsilon''}{\alpha c}, \quad -(12)$$
$$n'' = \frac{\alpha c}{2\omega} \quad -(13)$$

3) From eq. (2):

$$1/x = \frac{c}{v} = n' - in'' \quad -(14)$$

and

$$x = \frac{v}{c} = \frac{1}{n' - in''} = \frac{n' + in''}{n'^2 + n''^2} \quad -(15)$$

Therefore:

$$n' + in'' = (n'^2 + n''^2)x \quad -(16)$$

$$n' - in'' = \frac{1}{x} \quad -(17)$$

Adding:

$$2n' = (n'^2 + n''^2)x + \frac{1}{x} \quad -(18)$$

If

$$n'' = 0 \quad -(19)$$

then:

$$2n' = n'^2\left(\frac{v}{c}\right) + \frac{c}{v} \quad -(20)$$

i.e.

$$\boxed{n' = \frac{c}{v} \quad \text{if } n'' = 0} \quad -(21)$$

Therefore the usual formula:

$$n' = c/v \quad -(22)$$

is true if and only if $n'' = 0$ or $\alpha = 0$.

4) Otherwise the velocity v is given by the quadratic:

$$x^2 - ax + b = 0 \quad -(23)$$

where:

$$a = \frac{2n'}{n'^2 + n''^2}, \quad b = \frac{1}{n'^2 + n''^2} \quad -(24)$$

i.e.

$$x = \frac{1}{2}\left(a \pm \left(a^2 - 4b\right)^{1/2}\right) \quad -(25)$$

$$= \frac{v}{c}$$

So:

$$\boxed{v = \frac{c}{2}\left(a \pm \left(a^2 - 4b\right)^{1/2}\right)} \quad -(26)$$

The power absorption coefficient is defined by:

$$\alpha = \frac{\omega \epsilon''}{n' c} = \left(\frac{N}{V}\right) \frac{|\mu_{gi}|^2}{6 \epsilon_0 V \hbar} \quad -(27)$$

So:

$$n' = \frac{\omega \epsilon''}{\alpha c} = \frac{\omega \epsilon''}{c} \frac{V}{d} \quad -(28)$$

where

$$d = \left(\frac{N}{V}\right) \frac{|\mu_{gi}|^2}{6 \epsilon_0 \hbar} \quad -(29)$$