510(1): Comparison of the Planck Oscillator Theory with Bose Einstein and Fermi Dirac Statistics

The Planck oscillator theory uses the Maxwell Boltzmann distribution for the probability that a system is in state of energy E_n defined by:

$$E_n = n\hbar\omega \quad -(1)$$

where

$$n = 0, 1, 2, \ldots \infty. \quad -(2)$$

The energy of one photon is:

$$E = \hbar\omega. \quad -(3)$$

The Maxwell Boltzmann probability is:

$$P_n = \frac{\exp(-E_n/(kT))}{\sum_n \exp(-E_n/(kT))} \quad -(4)$$

The partition function of the Bose Einstein statistics is well known to be:

$$Z = \sum_{n=0}^{\infty} \exp\left(\frac{n(\mu - E)}{kT}\right) \quad -(5)$$

The denominator of eq. (4) is eq. (5) with:

$$\mu = 0. \quad -(6)$$

Here μ is the chemical potential. The Bose Einstein statistics were introduced by Bose in

a) 1924 for photons and extended by Einstein to atoms. They apply to a quantum system of non interacting photons and bosons. The quantum system exchanges energy with a reservoir at a temperature T and chemical potential μ.

Eq. (5) can be written as:

$$Z = \sum_{n=0}^{\infty} \left(\exp\left(\frac{\mu - E}{kT} \right) \right)^{n} \quad -(6)$$

$$= \frac{1}{1 - \exp\left(\frac{(\mu - E)}{kT} \right)}$$

The average number of particles is given from thermodynamics by:

$$\langle n \rangle = kT \frac{1}{Z} \left(\frac{\partial Z}{\partial \mu} \right) \quad -(7)$$

$$= \frac{1}{\exp\left(\frac{E - \mu}{kT} \right) - 1}$$

In the Planck theory the same quantity is given by:

$$\langle n \rangle = \frac{1}{\exp\left(\frac{\hbar \omega}{kT} \right) - 1} \quad -(8)$$

) so the Planck and Bose / Einstein theories are the same when

$$\boxed{E - \mu = \hbar\omega} \quad -(9)$$

In the Planck theory the average energy of an oscillator is:

$$\langle \hbar\omega \rangle = \hbar\omega \langle n \rangle \quad -(10)$$

$$= \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1}$$

Therefore there is a similar expression for Bose Einstein statistics:

$$\langle E - \mu \rangle = \frac{E - \mu}{\exp\left(\frac{E-\mu}{kT}\right) - 1} \quad -(11)$$

Fermi Dirac statistics apply to an ensemble of fermions, notably electrons. Photons are bosons. The Pauli exclusion principle applies in this case and its derivation from ECE theory is described in essay 24. In Fermi Dirac statistics there are only two possible states of the partition function (5):

4)

a) No particle ($E = 0$);

b) One particle (energy E).

So:

$$Z = \exp\left(\frac{0(\mu - 0)}{kT}\right) + \exp\left(\frac{1(\mu - E)}{kT}\right) \quad -(12)$$

So:

$$\langle n \rangle = \frac{kT}{Z}\frac{\partial Z}{\partial \mu} = \frac{1}{\exp\left(\frac{E-\mu}{kT}\right) + 1} \quad -(13)$$

and

$$\langle E - \mu \rangle = \frac{E - \mu}{\exp\left(\frac{E-\mu}{kT}\right) + 1} \quad -(14).$$

Comparing eqs. (11) and (14) it is seen that there is a sign change in the denominator.

It follows that an ensemble of photons has $n+1$ terms in the partition function Z while an ensemble of fermions has only two terms, $n = 0$ and $n = 1$. An electron gas can be treated as a Planck oscillator theory. In other words there exists an electron oscillator theory in which the electron has energy:

$$E = \hbar\omega = \gamma m c^2 \quad -(15)$$

5) If it is assumed that for an electron ensemble:

$$E - \mu = \hbar\omega = \gamma m c^2 \quad -(16)$$

then

$$\langle \hbar\omega \rangle = \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) + 1} \quad -(17)$$

is the mean energy:

$$\langle \hbar\omega \rangle = \hbar\omega \langle n \rangle \quad -(18)$$

and the absorption of an electron beam can be developed in direct analogy with the absorption of a photon beam.

There will be Evans / Morris effects in the absorption of an electron beam, or of any boson or fermion beam.

The next note will evaluate the density of states for an electron gas. This will be similar to the density of states for a gas of photons with mass or a gas of gravitons with mass