310(2): Rayleigh Jeans Density of States for Particles with Mass.

Consider the ECE wave equation:

$$(\square + R)\, q^a_\mu = 0 \quad -(1)$$

where q^a_μ is the Cartan tetrad and R is well defined from the tetrad postulate in the ECE papers. As in previous work it reduces in the limit of special relativity to:

$$R = \left(\frac{mc}{\hbar}\right)^2 \quad -(2)$$

where m is the particle mass, c the vacuum speed of light and $\hbar$ the reduced Planck constant.

The photon with mass is a boson defined by the Proca equation defined by the ECE hypothesis:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(3)$$

where A^a_μ is the generally covariant electromagnetic potential of ECE theory. So:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right) A^a_\mu = 0 \quad -(4)$$

For each state of polarization a:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right) A_\mu = 0 \quad -(5)$$

2) Rayleigh in 1900 assumed that:

$$m = 0 \quad -(6)$$

because the concept of photon mass was not well defined until Poincaré in 1905. So the Rayleigh Jeans density of states is based on:

$$\Box A_\mu = 0 \quad -(7)$$

which is the d'Alembert wave equation. Eq. (7) is

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) A_\mu = 0 \quad -(8)$$

Rayleigh further assumed that this equation can be used with each component of A_μ so it becomes:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) f = 0 \quad -(9)$$

This equation has the solution:

$$\omega^2 = c^2 \kappa^2 \quad -(10)$$

so it was assumed by Rayleigh that the wave velocity is c.

It was then assumed that:

$$\kappa^2 = \frac{\pi^2}{L^2}\left(n_1^2 + n_2^2 + n_3^2\right) \quad -(11)$$

where

$$V = L^3 \quad -(12)$$

is the volume of a cubic box of length L, and

3) where n_1, n_2 and n_3 are integers denoting modes of radiation. The total number of modes of frequency less than or equal to ω is given by:

$$n_1^2 + n_2^2 + n_3^2 \leq \frac{\omega^2 L^2}{c^2 \pi^2} \quad -(13)$$

This inequality defines the octant of a sphere of unit radius:

$$\boxed{n = \frac{\omega L}{c\pi}} \quad -(14)$$

It was pointed out by Jeans in 1905 that this is the octant of positive valued integers. This is why the law is called the Rayleigh Jeans density of states.

The unit volume of the octant is:

$$V_{oct} = \frac{1}{8}\left(\frac{4}{3}\pi n^3\right) \quad -(15)$$

From eqs. (14) and (15):

$$V_{oct} = \frac{1}{6}\frac{\omega^3}{c^3\pi^2} L^3 \quad -(16)$$

$$= \frac{1}{6}\frac{\omega^3}{c^3\pi^2} V$$

So the number of oscillators N in a volume V is:

$$\frac{N}{V} = \frac{V_{oct}}{V} = \frac{1}{6}\frac{\omega^3}{c^3\pi^2} \quad -(17)$$

4) Finally Rayleigh assumed that there were two polarizations:

$$a = (1) \text{ and } (2) \qquad -(18)$$

so he doubled the result in eq. (17) to give the Rayleigh Jeans density of state:

$$\boxed{\frac{N}{V} = \frac{1}{3}\frac{\omega^3}{c^3\pi^2}} \qquad -(19)$$

Note carefully that the existence of the $\underline{B}^{(3)}$ field modifies this result because:

$$a = (1), (2) \text{ and } (3) \qquad -(20)$$

Eq. (19) is true for monochromatic radiation only. For polychromatic radiation it becomes:

$$\frac{dN}{V} = \frac{1}{V}\left(N(\omega + d\omega) - N(\omega)\right) \qquad -(21)$$

$$= \frac{1}{3c^3\pi^2}\left((\omega + d\omega)^3 - \omega^3\right)$$

The consideration of photon or particle mass modifies this result by using the solution of the wave equation:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right) f = 0 \qquad -(22)$$

The solution of this equation may be found by

5) realizing that it is the quantized equivalent of the Einstein equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(23)$$

with:

$$E = \hbar\omega, \quad \underline{p} = \hbar \underline{k} \quad -(24)$$

So:

$$\omega^2 = c^2 k^2 + \left(\frac{mc^2}{\hbar}\right)^2 \quad -(25)$$

Denote the rest frequency by the de Broglie equation:

$$\hbar\omega_0 = mc^2 \quad -(26)$$

then

$$\boxed{c^2 k^2 = \omega^2 - \omega_0^2} \quad -(27)$$

So in the Rayleigh calculation ω^2 is replaced by $\omega^2 - \omega_0^2$.

The unit radius (14) becomes:

$$n = \left(\omega^2 - \omega_0^2\right)^{1/2} \frac{L}{c\pi} \quad -(28)$$

and the number of oscillators becomes: $\quad -(29)$

$$N = \frac{1}{8}\left(\frac{4}{3}\pi n^3\right) = \frac{1}{6}\left(\omega^2 - \omega_0^2\right)^{3/2} \frac{L^3}{c^3\pi^2}$$

so for two states of polarization the density

6) of states for a photon with mass is:

$$\boxed{\frac{N}{V} = \frac{1}{3c^2\pi^2}\left(\omega^2 - \omega_0^2\right)^{3/2}} \quad -(30)$$

for monochromatic radiation.

For polychromatic radiation:

$$\boxed{\frac{dN}{V} = \frac{1}{3c^2\pi^2}\left(\left((\omega + d\omega)^2 - \omega_0^2\right)^{3/2} - \left(\omega^2 - \omega_0^2\right)^{3/2}\right)} \quad -(31)$$

The same results are true for an ECE graviton and fermion with mass.