310(5): Planck / Rayleigh Calculation of the Stefan Boltzmann Law with Photon Mass.

Using two polarizations as assumed by Rayleigh the conventional calculation starts with:

$$\frac{dN}{V} = \frac{1}{3c^3\pi^2}\left((\omega + d\omega)^3 - \omega^3\right)$$

$$= \frac{1}{c^3\pi^2}\left(\omega^2 d\omega + \omega(d\omega)^2 + \frac{(d\omega)^3}{3}\right) \quad -(1)$$

Rayleigh assumed incorrectly that:

$$\frac{dN}{V} = \frac{\omega^2}{c^3\pi^2}\, d\omega \quad -(2)$$

UFT 291 shows that it is not possible to omit the higher order infinitesimals and corrects the calculation by Lord Rayleigh in 1900.

For the sake of argument this note uses eq. (2) and corrects it for photon mass.

Rayleigh used the d'Alembert wave equation

$$\square f = 0 \quad -(3)$$

The presence of photon mass corrects eq. (3) to:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right) f = 0 \quad -(4)$$

2) as in previous notes for UFT 310. Eq. (4) is the quantized version of the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(5)$$

Now use:

$$E = \hbar \omega, \quad \underline{p} = \hbar \underline{\kappa} \quad -(6)$$

to find that:

$$c^2 \kappa^2 = \omega^2 - \omega_0^2 \quad -(7)$$

where ω_0 is the de Broglie rest frequency of the photon with mass:

$$\omega_0 = \frac{mc^2}{\hbar} \quad -(8)$$

In the original Rayleigh calculation:

$$c^2 \kappa^2 = \omega^2 . \quad -(9)$$

Eq. (7) can be written as:

$$c^2 \kappa^2 = \Omega^2 \quad -(10)$$

where

$$\Omega^2 = \omega^2 - \omega_0^2 \quad -(11)$$

Therefore Rayleigh's eq. (2) is corrected by photon mass to:

$$\frac{dN}{V} = \frac{\Omega^2}{c^3 \pi^2} \, d\Omega \quad -(12)$$

3) where:

$$\Omega = (\omega^2 - \omega_0^2)^{1/2} \quad -(13)$$

From eq. (13):

$$\frac{d\Omega}{d\omega} = \frac{\omega}{(\omega^2 - \omega_0^2)^{1/2}} \quad -(14)$$

so

$$\boxed{\frac{dN}{V} = \frac{\omega}{c^3 \pi^2} (\omega^2 - \omega_0^2)^{1/2} \, d\omega} \quad -(15)$$

It follows that:

$$\frac{dE}{V} = \langle \hbar\omega \rangle \frac{dN}{V} \quad -(16)$$

where

$$\langle \hbar\omega \rangle = \frac{\hbar\omega}{e^y - 1} \quad -(17)$$

and where

$$y = \frac{\hbar\omega}{kT} \quad -(18)$$

So

$$\frac{1}{V}\frac{dE}{d\omega} = \frac{\omega(\omega^2 - \omega_0^2)^{1/2}}{c^3 \pi^2} \langle \hbar\omega \rangle \quad -(19)$$

The total energy density in joules per cubic which is:

$$\frac{E}{V} = \frac{1}{c^3\pi^2}\int \omega\left(\omega^2-\omega_0^2\right)^{1/2}\langle \hbar\omega\rangle\, d\omega, \quad -(20)$$

$$\boxed{\frac{E}{V} = \frac{\hbar}{c^3\pi^2}\int \frac{\omega^2\left(\omega^2-\omega_0^2\right)^{1/2} d\omega}{e^{y}-1}} \quad -(21)$$

If this integral is analytical it can be worked out by Maxima.

The Stefan Boltzmann law is worked out by integrating over all frequencies:

$$\boxed{\frac{E}{V}(\text{Stefan Boltzmann}) = \frac{\hbar}{c^3\pi^2}\int_0^\infty \frac{\omega^2\left(\omega^2-\omega_0^2\right)^{1/2} d\omega}{e^{y}-1}} \quad -(22)$$

where

$$\omega_0 = \frac{mc^2}{\hbar}, \quad y = \frac{\hbar\omega}{kT}. \quad -(23)$$

Eq. (22) is the correction of the Stefan Boltzmann law

5)

for photon mass, QED.

In the usual standard model assumption:

$$m = ?\ 0 \quad -(24)$$

So:

$$\frac{E}{V} = \frac{\hbar}{c^3\pi^2}\int_0^\infty \frac{\omega^3\, d\omega}{e^y - 1} = \left(\frac{\pi^2 k^4}{15 c^3 \hbar^3}\right) T^4 \quad -(25)$$

Note carefully that eqns. (2) and (12) must be corrected for higher order infinitesimals as in UFT291, and for the consideration of the $B^{(3)}$ field. This will be the subject of the next note.

For the sake of argument, eq. (21) shows that the effect of photon mass is maximized for low frequency radiation, i.e. for:

$$\frac{E}{V} = \frac{\hbar}{c^3\pi^2}\int_0^{\omega_{max}} \frac{\omega^2\left(\omega^2 - \omega_0^2\right)^{1/2} d\omega}{e^y - 1} \quad -(26)$$

where

$$\boxed{\omega_{max} \gtrsim \omega_0} \quad -(27)$$

For monochromatic radiation such as a laser, the Rayleigh Jeans density of states is:

$$\frac{N}{V} = \frac{1}{3}\frac{\omega^3}{c^3\pi^2} \quad -(28)$$

and

$$\boxed{d\omega = 0.}, \quad -(29)$$

and as in note 310(3), eq. (8):

$$\frac{E}{V} = \langle \hbar\omega \rangle \frac{N}{V} \quad -(30)$$

$$= \frac{\hbar\omega^4}{3c^3\pi^2(e^y - 1)}$$

The presence of photon mass converts this to:

$$\frac{E}{V} = \frac{\hbar(\omega^2 - \omega_0^2)^2}{3c^3\pi^2(e^y - 1)} \quad -(31)$$

The flux density of the laser is:

$$\boxed{\Phi = \frac{cE}{V} = \frac{\hbar(\omega^2 - \omega_0^2)^2}{3c^2\pi^2(e^y - 1)}} \quad -(32)$$

7) in watts per square metre.

The equivalent result for a polychromatic beam is from eq. (21):

$$\Phi = \frac{cE}{V} = \frac{\hbar}{c^2\pi^2}\int \frac{\omega^2(\omega^2-\omega_0^2)^{1/2}}{e^y - 1}\, d\omega \quad -(33)$$

For a polychromatic beam of all frequencies:

$$\Phi = \frac{\hbar}{c^2\pi^2}\int_0^\infty \frac{\omega^2(\omega^2-\omega_0^2)^{1/2}}{e^y - 1}\, d\omega \quad -(34)$$

It would be very interesting to graph eqs. (32) to (34). Eq. (32) is the easiest to use. The photon mass can be estimated experimentally from eq. (32). It is given by:

$$(\omega^2 - \omega_0^2)^2 = \frac{3c^2\pi^2}{\hbar}(e^y - 1)\,\Phi(\text{exp})$$

$$= \frac{3c^2\pi^2}{\hbar}\left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)\Phi(\text{exp}) \quad -(35)$$

So:

8)

$$\omega_0^2 = \left(\frac{mc^2}{\hbar}\right)^2 = \omega^2 - \left(\frac{3c^2\pi^2}{\hbar}\left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)\Phi(\exp)\right)^{1/2} \qquad (36)$$

This gives an experimental method for finding the photon mass by measuring the frequency ω and the flux (density) Φ of monochromatic radiation. The flux density Φ is measured in watts per square metre.

So this is a very simple experiment.