310(8): Self Consistent Calculation of Photon Mass

Starting from eq. (28) of the previous note:

$$\frac{M}{V} = m\left(\frac{2\zeta(3)}{\pi^2}\left(\frac{k}{c\hbar}\right)^3 T^3 - \frac{kT m_0^2 c}{\pi^2 \hbar^3}\right) \quad -(1)$$

where m is the photon mass, M is the missing mass of the universe and V is the volume of the universe. This equation can be written as:

$$Bm^3 - mA + \rho = 0 \quad -(2)$$

where

$$\rho = \frac{M}{V} \quad -(3)$$

is the density of the missing mass. Here:

$$B = \frac{kTc}{\pi^2 \hbar^3} \quad -(4)$$

and

$$A = \frac{2\zeta(3)}{\pi^2}\left(\frac{k}{c\hbar}\right)^3 T^3 \quad -(5)$$

with

$$\zeta(3) = 1.20206, \quad -(6)$$

$$k = 1.38066 \times 10^{-23}\ JK^{-1}$$

$$c = 2.997925 \times 10^{8}\ ms^{-1}$$

$$\hbar = 1.05459 \times 10^{-34}\ Js$$

The temperature is $T = 2.7K. \quad -(7)$

2) The density must be positive:

$$\rho \geqslant 0 \quad -(8)$$

so from eq. (2):

$$mA \geqslant Bm^3 \quad -(9)$$

i.e

$$\boxed{m^2 \leqslant \frac{A}{B}} \quad -(10)$$

where

$$\frac{A}{B} = 2\zeta(3)\left(\frac{k^2 T^2}{c^4}\right) \quad -(11)$$

$$= 2\zeta(3)\left(\frac{kT}{c^2}\right)^2$$

therefore

$$\boxed{2\zeta(3)\left(\frac{kT}{mc^2}\right) \geqslant 1} \quad -(12)$$

for

$$\rho \geqslant 0 \quad -(13)$$

At

$$T = 2.7K \quad -(14)$$

it follows that

$$\boxed{m \leqslant 6.43 \times 10^{-40} \text{ kilograms}}$$

$-(15)$

3) If the universe is infinite then

$$m = 6.43 \times 10^{-40} \text{ kg} \quad -(16)$$

and this is an upper bound a photon mass.

The basic assumption is that:

$$M = Nm \quad -(17)$$

where M is the missing mass and where N is the number of photons in the universe. From the de Broglie equation:

$$\hbar\omega = \gamma mc^2 \quad -(18)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(19)$$

so

$$m = \left(1 - \frac{v^2}{c^2}\right)^{1/2} \frac{\hbar\omega}{c^2} \quad -(20)$$

in general. The rest mass is:

$$m_0 = \frac{\hbar\omega_0}{c^2} \quad -(21)$$

By definition the number density of photons from eq. (3) of note 310(7) is:

4) $$\frac{N}{V} = \frac{1}{\pi^2 c^3}\left(\int \frac{\omega^2 d\omega}{e^y - 1} - \omega_0^2 \int \frac{d\omega}{e^y - 1}\right) \quad -(22)$$

here ω_0 is the rest frequency.

In the approximation:

$$\hbar\omega \ll kT \quad -(23)$$

the propagation of the photons is determined by:

$$\frac{\omega}{\omega_0} = \exp\left(-\frac{\alpha l}{2}\right) \quad -(24)$$

where α is the power absorption coefficient of inter-stellar matter and l the distance over which the photon has propagated from the source.

However in the present calculation:

$$T = 2.7K \quad -(25)$$

so the complete expression must be used in eq. (24):

$$\frac{\Phi}{\Phi_0} = \exp(-\alpha l) \quad -(26)$$

The photon rest mass must be very small because frequencies of radiation are observed down to the sub hertzian range.

5) For example if ω_0 is one radian per second:

$$m_0 = \frac{\hbar}{c^2} = 1.54 \times 10^{-50} \text{ kgm.} \quad -(27)$$

Therefore the rest mass is determined by the lowest observable frequency and so:

$$m_0 < 1.54 \times 10^{-50} \text{ kg} \quad -(28)$$

when

$$v = 0. \quad -(29)$$

<u>This is the mass of a photon at rest</u>.

In the contemporary interpretation of special relativity the mass of a particle is a fundamental property and does not vary. Therefore the de Broglie equation (18) is interpreted as:

$$\hbar\omega = \gamma m_0 c^2 \quad -(30)$$

and

$$\omega = \frac{\gamma m_0 c^2}{\hbar} \quad -(31)$$

It follows that the mass m appearing in eq. (1) is:

$$\boxed{m = \gamma m_0} \quad -(32)$$

In the approximation (24) and using eqs. (21)

) and (30):

$$\frac{\omega}{\omega_0} = \gamma = \exp\left(-\frac{\alpha \ell}{2}\right) \quad -(33)$$

so the photon velocity is given by:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \exp\left(-\frac{\alpha \ell}{2}\right) \quad -(34)$$

i.e

$$1 - \frac{v^2}{c^2} = \exp(\alpha \ell) \quad -(35)$$

and

$$\boxed{\frac{v^2}{c^2} = 1 - \exp(\alpha \ell)} \quad -(36)$$

The power absorption coefficient of interstellar space is:

$$\boxed{\alpha = \frac{1}{\ell} \log_e\left(1 - \frac{v^2}{c^2}\right)} \quad -(37)$$

With this interpretation the mass of a photon moving with velocity v is:

$$\boxed{m = \gamma m_0 = m_0 \exp\left(-\frac{\alpha \ell}{2}\right)} \quad -(38)$$

and the photon mass follows a Beer Lambert law.

7) The power absorption coefficient of the interstellar space is related to Hubble's constant, so m / m_0 can be calculated.

Finally the second term in eq. (1) can be neglected and we obtain:

$$m = \gamma m_0 = 5.29 \times 10^{-40} \text{ kg} \quad -(39)$$

where

$$\gamma = \exp\left(-\frac{\alpha \ell}{2}\right) \quad -(40)$$

Therefore the rest mass is:

$$m_0 = 5.29 \times 10^{-40} \exp\left(\frac{\alpha \ell}{2}\right) \quad -(41)$$