316(2): Vector Formulation of the JCE Identity

Consider initially the Cartan identity:

$$D_\mu T^a{}_{\nu\rho} + D_\rho T^a{}_{\mu\nu} + D_\nu T^a{}_{\rho\mu} := R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} \quad -(1)$$

In UFT 254 it was shown that the vector formulation of this identity is:

$$\underline{\nabla} \cdot \underset{\sim}{\underline{T}}^a(\text{spin}) + \underline{\omega}^a{}_b \cdot \underline{T}^b(\text{spin}) = \underline{q}^b \cdot \underline{R}^a{}_b(\text{spin}) \quad -(2)$$

The JCE identity of UFT 313 is:

$$D_\mu R^a{}_{\lambda\nu\rho} + D_\rho R^a{}_{\lambda\mu\nu} + D_\nu R^a{}_{\lambda\rho\mu} := R^a{}_{\lambda\mu\alpha} T^\alpha{}_{\nu\rho} + R^a{}_{\lambda\rho\alpha} T^\alpha{}_{\mu\nu} + R^a{}_{\lambda\nu\alpha} T^\alpha{}_{\rho\mu} \quad -(3)$$

which can be written as:

$$D_\mu R^a{}_{b\nu\rho} + D_\rho R^a{}_{b\mu\nu} + D_\nu R^a{}_{b\rho\mu} := R^a{}_{b\mu\alpha} T^\alpha{}_{\nu\rho} + R^a{}_{b\rho\alpha} T^\alpha{}_{\mu\nu} + R^a{}_{b\nu\alpha} T^\alpha{}_{\rho\mu} \quad -(4)$$

By direct analogy with eq. (2) the vector formulation of eq. (4) may be written as follows, with summation over repeated c and α indices:

2)

$$\underline{\nabla} \cdot \underline{R}^a{}_b(\text{spin}) + \underline{\omega}^a{}_c \cdot \underline{R}^c{}_b(\text{spin})$$
$$= \underline{R}^a{}_{b\alpha}(\text{spin}) \cdot \underset{\sim}{T}^\alpha(\text{spin}) \quad -(5)$$
$$= \underline{R}^a{}_{bc}(\text{spin}) \cdot \underset{\sim}{T}^c(\text{spin})$$

This reduces to:

$$\underline{\nabla} \cdot \underline{R}(\text{spin}) + \underline{\omega}_c \cdot \underline{R}^c(\text{spin}) = \underline{R}_c(\text{spin}) \cdot \underline{T}^c(\text{spin}) \quad -(6)$$

using:

$$\underline{R}(\text{spin}) = e_a e^b \underline{R}^a{}_b(\text{spin}) \quad -(7)$$

Multiply eq. (6) by:

$$e^c e_c = -2 \quad -(8)$$

to obtain:

$$-2\underline{\nabla} \cdot \underline{R}(\text{spin}) + \underline{\omega} \cdot \underline{R}(\text{spin}) = \underline{R}(\text{spin}) \cdot \underline{T}(\text{spin}) \quad -(9)$$

So:

$$\boxed{\underline{\nabla} \cdot \underline{R}(\text{spin}) = \frac{1}{2}\underline{R}(\text{spin}) \cdot \left(\underline{\omega} - \underset{\sim}{T}(\text{spin})\right)} \quad -(10)$$

Finally note:

3)
$$\underline{B} = W^{(0)} \underline{R}(\text{spin}) \quad -(11)$$

to obtain:

$$\boxed{\underline{\nabla} \cdot \underline{B} = \frac{1}{2} \underline{B} \cdot \left(\underline{\omega} - \underline{T}(\text{spin}) \right)} \quad -(12)$$

The magnetic monopole is zero if and only if:

$$\underline{\omega} = \underline{T}(\text{Spin}) \quad -(13)$$

In these equations:

$$\underline{R}^a{}_b(\text{spin}) = \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \quad -(14)$$

and:

$$\underline{R}^a{}_b(\text{orb}) = -\underline{\nabla}\, \omega^a{}_{0b} - \frac{1}{c} \frac{\partial \underline{\omega}^a{}_b}{\partial t} - \omega^a{}_{0c} \underline{\omega}^c{}_b + \underline{\omega}^a{}_c \omega^c{}_{0b} \quad -(15)$$

with:

$$\underline{B}^a{}_b = W^{(0)} \underline{R}^a{}_b(\text{spin}) \quad -(16)$$

and

$$\underline{E}^a{}_b = c W^{(0)} \underline{R}^a{}_b(\text{orb}) \quad -(17)$$

Finally the tangent indices are removed with:

$$\underline{B} = e_a e^b \underline{B}^a{}_b \quad -(18)$$

and

$$\underline{E} = e_a e^b \underline{E}^a{}_b \quad -(19)$$

It follows that:

4) $$\underline{B} = W^{(0)}\left(\underline{\nabla}\times\underline{\omega} - \underline{\omega}_c \times \underline{\omega}^c\right) \quad -(20)$$

$$= W^{(0)}\,\underline{\nabla}\times\underline{\omega}$$

Define the magnetic flux potential by:

$$\underline{W} = W^{(0)}\,\underline{\omega} \quad -(21)$$

and

$$\boxed{\underline{B} = \underline{\nabla}\times\underline{W}} \quad -(22)$$

Similarly:

$$\underline{R}\,(\text{anti}) = -\underline{\nabla}\,\omega_0 - \frac{1}{c}\frac{\partial\underline{\omega}}{\partial t} - \omega_{0c}\,\underline{\omega}^c + \underline{\omega}_c\,\omega_0^c \quad -(23)$$

and

$$\underline{E} = c\,W^{(0)}\left(-\underline{\nabla}\,\omega_0 - \frac{1}{c}\frac{\partial\underline{\omega}}{\partial t} - \omega_{0c}\,\underline{\omega}^c + \underline{\omega}_c\,\omega_0^c\right) \quad -(24)$$
