319(2): The Newtonian Condition in ECE2.

In ECE2 the acceleration due to gravity is:

$$\underline{g} = -2\left(\underline{\nabla}\Phi + \frac{\partial \underline{Q}}{\partial t}\right) = 4\left(c\omega_0 \underline{Q} - \Phi\underline{\omega}\right) \quad -(1)$$

where the potential four vector is:

$$\Phi^\mu = \left(\frac{\Phi}{c}, \underline{Q}\right) \quad -(2)$$

Define the energy momentum four vector as:

$$U^\mu = \left(\frac{U}{c}, \underline{p}\right) \quad -(3)$$

where $\underline{p}$ is the linear momentum, then:

$$U^\mu = m\Phi^\mu \quad -(4)$$

So

$$U = m\Phi \quad -(5)$$

and

$$\underline{p} = m\underline{Q} \quad -(6)$$

Therefore:

$$\underline{g} = -\frac{2}{m}\left(\underline{\nabla}U + \frac{\partial \underline{p}}{\partial t}\right) \quad -(7)$$

$$= \frac{4}{m}\left(c\omega_0\underline{p} - U\underline{\omega}\right)$$

The Newtonian limit is reached when:

$$\underline{\nabla}U = \frac{\partial \underline{p}}{\partial t} \quad -(8)$$

So

$$\underline{\nabla}\,\underline{\Phi} = \frac{\partial \underline{Q}}{\partial t} \quad -(9)$$

So $$\underline{g} = -\underline{\nabla}\phi = -4\underline{\nabla}\,\underline{\Phi} \quad -(10)$$

the Newtonian ϕ is defined as:

$$\phi = 4\underline{\Phi} \quad -(11)$$

From eq. (1) the Newtonian condition is:

$$c\omega_0 \underline{Q} = -\underline{\Phi}\,\underline{\omega} \quad -(12)$$

i.e. $$c\omega_0 \underline{p} = -\underline{\omega}\,U \quad -(13)$$

A possible solution of eq. (1) is:

$$\underline{\nabla}\,\underline{\Phi} = c\omega_0 \underline{Q} \quad -(14)$$

and $$\frac{\partial \underline{Q}}{\partial t} = -\underline{\Phi}\,\underline{\omega} \quad -(15)$$

This means that:

$$\frac{1}{c}\frac{\partial}{\partial t}\underline{p} = 2\omega_0 \underline{p} \quad -(16)$$

and $$\underline{\nabla}\,U = -2\underline{\omega}\,U \quad -(17)$$

i.e. $$(2\omega_0, 2\underline{\omega}) = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right) \quad -(18)$$

or $$2\omega^\mu = \partial^\mu \quad -(19)$$

). The quantum condition is

$$p^\mu = i\hbar\, \partial^\mu \quad -(20)$$
$$= 2i\hbar\, \omega^\mu$$

i.e
$$\left(\frac{U}{c}, \underline{p}\right) = 2i\hbar\left(\omega_0, \underline{\omega}\right) \quad -(21)$$

or
$$U = 2i\hbar c\,\omega_0 \quad -(22)$$

and
$$\underline{p} = 2i\hbar\,\underline{\omega} \quad -(23)$$

Using the quantum condition:
$$\left(\frac{U}{c}, \underline{p}\right) = i\hbar\left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right) \quad -(24)$$

the Newtonian condition (8) becomes:

$$\underline{\nabla}\frac{\partial}{\partial t} + \frac{\partial}{\partial t}\underline{\nabla} = 0 \quad -(25)$$

i.e
$$\left(\underline{\nabla}\frac{\partial}{\partial t} + \frac{\partial}{\partial t}\underline{\nabla}\right)\phi = 0 \quad -(26)$$

or
$$\left[\underline{\nabla}, \frac{\partial}{\partial t}\right]\phi = 0 \quad -(27)$$

where $[\ ,\ ]$ denotes the anticommutator.

4) This is interesting because in quantum field theory the use of the anticommutator leads directly to the Pauli exclusion principle. Eq. (27) can be regarded as an equation of quantum gravity.

In ECE2 theory there is also the equation:

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(28)$$

where

$$\underline{\kappa} = 2 \left(\frac{\underline{\nabla}}{r^{(0)}} - \underline{\omega} \right) \quad -(29)$$

From eqs (1) and (28):

$$\nabla^2 \underline{\Phi} + \frac{\partial}{\partial t} \left(\underline{\nabla} \cdot \underline{Q} \right) = -2\pi G \rho_m \quad -(30)$$

i.e.

$$\boxed{\nabla^2 U + \frac{\partial}{\partial t} \left(\underline{\nabla} \cdot \underline{p} \right) = -2\pi G m \rho_m} \quad -(31)$$

In the Newtonian limit (8) eq. (31) reduces to the Poisson equation:

$$\nabla^2 U = -4\pi m \rho_m G \quad -(32)$$

From eq. (1) in the Newtonian limit:

$$c\omega_0 \underline{p} = -\underline{\omega}\,\Pi \quad -(33)$$

So:

$$\boxed{\underline{F} = m\underline{g} = 8c\omega_0 \underline{p} = -8\Pi\,\underline{\omega}} \quad -(34)$$

Eqs. (28) and (29) mean that:

$$\underline{\nabla}\cdot\underline{p} = \left(\frac{\underline{q}}{r^{(0)}} - \underline{\omega}\right)\cdot\underline{p} \quad -(35)$$

and

$$\underline{\nabla}\cdot\underline{\omega} = \left(\frac{\underline{q}}{r^{(0)}} - \underline{\omega}\right)\cdot\underline{\omega} \quad -(36)$$

So

$$\underline{\nabla} = \frac{\underline{q}}{r^{(0)}} - \underline{\omega} \quad -(37)$$

From eq. (18):

$$\boxed{\begin{aligned}\underline{\omega} &= -\frac{1}{2}\underline{\nabla} \quad -(38)\\ \underline{q} &= \frac{r^{(0)}}{2}\underline{\nabla} \quad -(39)\end{aligned}}$$

So

Eqs. (38) and (39) are the spin connection and tetrad vectors in the Newtonian limit.

Non Newtonian effects and counter gravitation are described by the conditions:

6)

$$\underline{\nabla} U \neq \frac{\partial \underline{p}}{\partial t} \quad -(40)$$

and

$$c\omega_0 \underline{p} \neq -\underline{\omega} U \quad -(41)$$

Zero gravitation is described by:

$$c\omega_0 \underline{p} = \underline{\omega} U \quad -(42)$$

and repulsive g is described by:

$$U \underline{\omega} > c\omega_0 \underline{p} \quad -(43)$$
