```
(%i1) kill(all);
(%o0) done
```

1 Eq.(10)

```
(%i1) L1: matrix([gamma,0,0,-theta*gamma],
      [0,1,0,0],
      [0,0,1,0],
      [-theta*gamma,0,0,gamma]);
```

(%o1)
$$\begin{bmatrix} \Gamma & 0 & 0 & -\theta\,\Gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\theta\,\Gamma & 0 & 0 & \Gamma \end{bmatrix}$$

```
(%i2) L2: matrix([gamma,0,0,theta*gamma],
      [0,1,0,0],
      [0,0,1,0],
      [theta*gamma,0,0,gamma]);
```

(%o2)
$$\begin{bmatrix} \Gamma & 0 & 0 & \theta\,\Gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \theta\,\Gamma & 0 & 0 & \Gamma \end{bmatrix}$$

```
(%i3) theta: v/c;
```

(%o3) $\frac{v}{c}$

```
(%i4) gamma: 1/sqrt(1-theta^2);
```

(%o4) $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

```
(%i5) (ev(L1.L2));
```

(%o5)
$$\begin{bmatrix} \frac{1}{1-\frac{v^2}{c^2}}-\frac{v^2}{c^2\left(1-\frac{v^2}{c^2}\right)} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{1-\frac{v^2}{c^2}}-\frac{v^2}{c^2\left(1-\frac{v^2}{c^2}\right)} \end{bmatrix}$$

```
(%i6) ratsimp(%);
```

$$(\%o6)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 Eqs.(27-30)

```
(%i7) P: [E/c, Px, Py, Pz];
```

$$(\%o7)\ \left[\frac{E}{c}, Px, Py, Pz\right]$$

```
(%i8) LP: factor(L1.P);
```

$$(\%o8)\begin{bmatrix} \frac{(E - c\ Pz\ \theta)\ \Gamma}{c} \\ Px \\ Py \\ -\frac{(\theta\ E - c\ Pz)\ \Gamma}{c} \end{bmatrix}$$

```
(%i9) kill(gamma);
(%o9) done
```

```
(%i10) ev(LP);
```

$$(\%o10)\begin{bmatrix} \frac{(E - Pz\ v)\ gamma}{c} \\ Px \\ Py \\ -\frac{\left(\frac{v\ E}{c} - c\ Pz\right)\ gamma}{c} \end{bmatrix}$$

3 Eqs.(31,34)

```
(%i11) J: [c*rho, Jx, Jy, Jz];
(%o11) [c ρ, Jx, Jy, Jz]
```

```
(%i12) LJ: factor(L1.J);
```

$$(\%o12)\begin{bmatrix} -(Jz\ \theta - c\ \rho)\ gamma \\ Jx \\ Jy \\ -(c\ \rho\ \theta - Jz)\ gamma \end{bmatrix}$$

(%i13) ev(%);

$$(\%o13)\quad \begin{bmatrix} -\left(\frac{Jz\ v}{c} - c\ \rho\right) gamma \\ Jx \\ Jy \\ -(\rho\ v - Jz)\ gamma \end{bmatrix}$$

4 Eqs.(53,54)

(%i14) kill(all);

(%o0) done

(%i1) theta[x]: v[x]/c; theta[y]: v[y]/c; theta[z]: v[z]/c; theta[0]: v/c

$$(\%o1)\quad \frac{v_x}{c}$$

$$(\%o2)\quad \frac{v_y}{c}$$

$$(\%o3)\quad \frac{v_z}{c}$$

$$(\%o4)\quad \frac{v}{c}$$

(%i5)
```
Lambda: matrix([gamma, -gamma*theta[x], -gamma*theta[y], -gamma*th
[-gamma*theta[x], 1+(gamma-1)*(theta[x]/theta[0])^2, (gamma-1)*the
(gamma-1)*theta[x]*theta[z]/theta[0]^2],
[-gamma*theta[y], (gamma-1)*theta[y]*theta[x]/theta[0]^2, 1+(gamma
(gamma-1)*theta[y]*theta[z]/theta[0]^2],
[-gamma*theta[z], (gamma-1)*theta[x]*theta[z]/theta[0]^2, (gamma-1
1+(gamma-1)*(theta[z]/theta[0])^2]);
```

$$(\%o5)\quad \begin{bmatrix} gamma & -\frac{v_x\ gamma}{c} & -\frac{v_y\ gamma}{c} & -\frac{v_z\ gamma}{c} \\ -\frac{v_x\ gamma}{c} & \frac{v_x^2\ (gamma-1)}{v^2}+1 & \frac{v_x\ v_y\ (gamma-1)}{v^2} & \frac{v_x\ v_z\ (gamma-1)}{v^2} \\ -\frac{v_y\ gamma}{c} & \frac{v_x\ v_y\ (gamma-1)}{v^2} & \frac{v_y^2\ (gamma-1)}{v^2}+1 & \frac{v_y\ v_z\ (gamma-1)}{v^2} \\ -\frac{v_z\ gamma}{c} & \frac{v_x\ v_z\ (gamma-1)}{v^2} & \frac{v_y\ v_z\ (gamma-1)}{v^2} & \frac{v_z^2\ (gamma-1)}{v^2}+1 \end{bmatrix}$$

(%i6) J4: [c*rho, J[x], J[y], J[z]];

(%o6) $[c\ \rho, J_x, J_y, J_z]$

(%i7) Lambda.J4;

(%o7)
$$\begin{bmatrix} -\frac{v_z\,J_z\,gamma}{c}-\frac{v_y\,J_y\,gamma}{c}-\frac{v_x\,J_x\,gamma}{c}+c\,\rho\,gamma \\ -\rho\,v_x\,gamma+\frac{v_x\,v_z\,J_z\,(gamma-1)}{v^2}+\frac{v_x\,v_y\,J_y\,(gamma-1)}{v^2}+J_x\left(\frac{v_x^2\,(gamma-1)}{v^2}+1\right) \\ -\rho\,v_y\,gamma+\frac{v_y\,v_z\,J_z\,(gamma-1)}{v^2}+\frac{v_x\,J_x\,v_y\,(gamma-1)}{v^2}+J_y\left(\frac{v_y^2\,(gamma-1)}{v^2}+1\right) \\ -\rho\,v_z\,gamma+\frac{v_y\,J_y\,v_z\,(gamma-1)}{v^2}+\frac{v_x\,J_x\,v_z\,(gamma-1)}{v^2}+J_z\left(\frac{v_z^2\,(gamma-1)}{v^2}+1\right) \end{bmatrix}$$

(%i8) ratsimp(%);

(%o8)
$$\begin{bmatrix} -\frac{(v_z\,J_z+v_y\,J_y+v_x\,J_x-c^2\,\rho)\,gamma}{c} \\ \frac{(v_x\,v_z\,J_z+v_x\,v_y\,J_y+v_x^2\,J_x-\rho\,v^2\,v_x)\,gamma-v_x\,v_z\,J_z-v_x\,v_y\,J_y+(v^2-v_x^2)\,J_x}{v^2} \\ \frac{(v_y\,v_z\,J_z+v_y^2\,J_y+(v_x\,J_x-\rho\,v^2)\,v_y)\,gamma-v_y\,v_z\,J_z+(v^2-v_y^2)\,J_y-v_x\,J_x\,v_y}{v^2} \\ \frac{(v_z^2\,J_z+(v_y\,J_y+v_x\,J_x-\rho\,v^2)\,v_z)\,gamma+(v^2-v_z^2)\,J_z+(-v_y\,J_y-v_x\,J_x)\,v_z}{v^2} \end{bmatrix}$$

(%i9) A4: [phi/c, A[x], A[y], A[z]];

(%o9) $\left[\frac{\phi}{c}, A_x, A_y, A_z\right]$

(%i10) Lambda.A4;

(%o10)
$$\begin{bmatrix} -\frac{v_z\,A_z\,gamma}{c}-\frac{v_y\,A_y\,gamma}{c}-\frac{v_x\,A_x\,gamma}{c}+\frac{\phi\,gamma}{c} \\ -\frac{\phi\,v_x\,gamma}{c^2}+\frac{v_x\,v_z\,A_z\,(gamma-1)}{v^2}+\frac{v_x\,v_y\,A_y\,(gamma-1)}{v^2}+A_x\left(\frac{v_x^2\,(gamma-1)}{v^2}+1\right) \\ -\frac{\phi\,v_y\,gamma}{c^2}+\frac{v_y\,v_z\,A_z\,(gamma-1)}{v^2}+\frac{v_x\,A_x\,v_y\,(gamma-1)}{v^2}+A_y\left(\frac{v_y^2\,(gamma-1)}{v^2}+1\right) \\ -\frac{\phi\,v_z\,gamma}{c^2}+\frac{v_y\,A_y\,v_z\,(gamma-1)}{v^2}+\frac{v_x\,A_x\,v_z\,(gamma-1)}{v^2}+A_z\left(\frac{v_z^2\,(gamma-1)}{v^2}+1\right) \end{bmatrix}$$

(%i11) ratsimp(%);

(%o11)

$$\begin{bmatrix} -\dfrac{(v_z A_z + v_y A_y + v_x A_x - \phi)\, gamma}{c} \\ \dfrac{(c^2 v_x v_z A_z + c^2 v_x v_y A_y + c^2 v_x^2 A_x - \phi v^2 v_x)\, gamma - c^2 v_x v_z A_z - c^2 v_x v_y A_y + (c^2 v^2 - c^2 v_x^2) A_x}{c^2 v^2} \\ \dfrac{(c^2 v_y v_z A_z + c^2 v_y^2 A_y + (c^2 v_x A_x - \phi v^2) v_y)\, gamma - c^2 v_y v_z A_z + (c^2 v^2 - c^2 v_y^2) A_y - c^2 v_x A_x v_y}{c^2 v^2} \\ \dfrac{(c^2 v_z^2 A_z + (c^2 v_y A_y + c^2 v_x A_x - \phi v^2) v_z)\, gamma + (c^2 v^2 - c^2 v_z^2) A_z + (-c^2 v_y A_y - c^2 v_x A_x) v_z}{c^2 v^2} \end{bmatrix}$$

5 Proving that (gamma-1)/v^2 = gamma^2/((gamma+1)*

(%i12) assume(c>0, v>0);

(%o12) [c>0, v>0]

(%i13) gamma: 1/sqrt(1-v^2/c^2);

(%o13) $$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

(%i14) f1: (gamma-1)/v^2;

(%o14) $$\frac{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1}{v^2}$$

(%i15) f2: gamma^2/((gamma+1)*c^2);

(%o15) $$\frac{1}{c^2\left(1-\frac{v^2}{c^2}\right)\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}+1\right)}$$

(%i16) ratsimp(f1-f2);

(%o16) 0