323(4): Check on the Lorentz Transform of A^μ and J^μ

By definition: $A^\mu = \left(\frac{\phi}{c}, \underline{A}\right)$ — (1)

and $J^\mu = (c\rho, \underline{J})$ — (2)

Consider the Lorentz boost in the Z direction. then:

$$\begin{bmatrix} \phi'/c \\ A_x' \\ A_y' \\ A_z' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} \phi/c \\ A_x \\ A_y \\ A_z \end{bmatrix} \quad -(3)$$

so $$\phi' = \gamma(\phi - v_z A_z) \quad -(4)$$

$$A_z' = \gamma\left(A_z - \frac{\phi v_z}{c^2}\right) \quad -(5)$$

Similarly:

$$\begin{bmatrix} c\rho' \\ J_x' \\ J_y' \\ J_z' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} c\rho \\ J_x \\ J_y \\ J_z \end{bmatrix} \quad -(5a)$$

so $$\rho' = \gamma\left(\rho - \frac{v_z J_z}{c^2}\right) \quad -(6)$$

$$J_z' = \gamma(J_z - \rho v_z) \quad -(7)$$

For the general boost:

2)

$$\rho' = \gamma\left(\rho - \frac{1}{c^2}\underline{v}\cdot\underline{J}\right) \quad -(8)$$

$$\underline{J}' = \gamma\left(\underline{J} - \rho\underline{v}\right) + (\gamma-1)\left(\underline{J}\cdot\hat{\underline{v}}\right)\hat{\underline{v}} \quad -(9)$$

$$\phi' = \gamma\left(\phi - \underline{v}\cdot\underline{A}\right) \quad -(10)$$

$$\underline{A}' = \gamma\left(\underline{A} - \frac{\phi}{c^2}\underline{v}\right) + (\gamma-1)\left(\underline{A}\cdot\hat{\underline{v}}\right)\hat{\underline{v}} \quad -(11)$$

where

$$\hat{\underline{v}} = \frac{\underline{v}}{v} \quad -(12)$$

Errors in the Wikipedia Site "Classical e/m and s.r."

1) Eq. (9) is incorrectly written as:

$$\underline{J}' \overset{?}{=} \underline{J} - \gamma\rho\underline{v} + (\gamma-1)(\underline{J}\cdot\hat{\underline{v}})\hat{\underline{v}} \quad -(13)$$

2) Eq. (11) is incorrectly written as:

$$\underline{A}' \overset{?}{=} \underline{A} - \frac{\gamma\phi}{c^2}\underline{v} + (\gamma-1)(\underline{A}\cdot\hat{\underline{v}})\hat{\underline{v}} \quad -(14)$$

Covariance of the MH and ECE2 equations:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \rightarrow \partial_{\mu'} \tilde{F}^{\mu'\nu'} = 0 \quad -(15)$$

and

$$\partial_\mu F^{\mu\nu} = J^\nu \rightarrow \partial_{\mu'} F^{\mu'\nu'} = J^{\nu'} \quad -(16)$$