

RIGOROUS QUANTIZATION OF THE HAMILTONIAN OF ECE2 SPECIAL  
RELATIVITY.

by

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ABSTRACT

It is shown that the hamiltonian of ECE2 special relativity can be quantized using at least four different classification schemes, each leading to different spectral results. The method used by Dirac uses an empirical choice of approximation which results in a zero classical hamiltonian and so is very restrictive if not unphysical. The schemes are illustrated with the rigorous quantization of the class one hamiltonian. If the spectral detail predicted by rigorous quantization is not observed, there is a major crisis in physics, because the philosophy of the Dirac equation would have been refuted, and along with it most of quantum field theory. In ECE theory the Dirac equation can be derived from Cartan geometry.

Keywords: ECE special relativity, quantization of the ECE2 hamiltonian.

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## 1. INTRODUCTION

In recent papers of this series {1 - 12} ECE2 theory has been developed in UFT314-320 and UFT322 - UFT332 from the Jacobi Cartan Evans (JCE) identity of UFT313. It has been shown that ECE2 leads to a type of special relativity in a space with non zero torsion and curvature. Therefore the well known equations of special relativity can be used in a new way, notably the lagrangian and hamiltonian. In immediately preceding papers it has been shown that orbital precession in a plane can be produced by simultaneous solution of the lagrangian and hamiltonian of ECE2 special relativity. The ECE2 hamiltonian is mathematically the same as the one used by Dirac to produce relativistic quantum mechanics using the SU(2) basis. For over ninety years it was thought that the procedure used by Dirac is rigorous and foundational, but it has been shown in UFT330 ff. that it depends on a subjective or empirical choice of approximation. When the hamiltonian is quantized rigorously, different spectral detail emerge from each choice of quantization. There are at least four classes of hamiltonian in the SU(2) basis. Therefore the Dirac equation is not foundational. This finding catalyzes a major crisis in physics because it shows that a new insight is needed in order to forge a rigorous relativistic quantum mechanics.

This paper is a short synopsis of detailed calculations given in the notes accompanying UFT333 on [www.aias.us](http://www.aias.us). Note 333(1) describes the classification scheme and proves that the classical hamiltonian  $H_0$  vanishes in the approximation used by Dirac. This approximation is therefore unphysical because it restricts the classical hamiltonian to zero, i.e. the hamiltonian can take only one value. This famous approximation happens to work but it is not known why it works. It is unphysical because the classical hamiltonian is obviously not restricted to zero in general. Note 333(2) works out the class four hamiltonian in terms of the Lande factor. Note 333(3) develops the class four hamiltonian without approximation.

Note 333(4) is a development of the classical energy levels of the H atom, which can be expressed in terms of the rest energy of special relativity multiplied by  $(d/n^2)$  where  $d$  is the fine structure constant and  $n$  the principal quantum number of the H atom. Notes 333(5) to 333(7) describe the rigorous development of the class one hamiltonian and form the basis of Section 2 of this paper. Section 3 is a computational and graphical analysis.

## 2. RIGOROUS QUANTIZATION OF THE CLASS ONE HAMILTONIAN

The classification scheme is based on the following four types of SU(2)

hamiltonian in ECE2 special relativity:

$$H_0 = \frac{1}{m} \underline{\sigma} \cdot \underline{p}_0 \frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot \underline{p}_0 + U \quad (1)$$

$$H_0 = \frac{\gamma}{m} \underline{\sigma} \cdot \underline{p}_0 \frac{\gamma}{1+\gamma} \underline{\sigma} \cdot \underline{p}_0 + U \quad (2)$$

$$H_0 = \frac{\gamma^2}{m} \underline{\sigma} \cdot \underline{p}_0 \frac{1}{1+\gamma} \underline{\sigma} \cdot \underline{p}_0 + U \quad (3)$$

$$H_0 = \frac{1}{m} \frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot \underline{p}_0 \underline{\sigma} \cdot \underline{p}_0 + U \quad (4)$$

Here  $m$  is the particle mass,  $\underline{p}_0$  the classical linear momentum,  $U$  the potential energy and  $\gamma$  is the Lorentz factor:

$$\gamma = \left( 1 - \frac{p_0^2}{m^2 c^2} \right)^{-1/2} \quad (5)$$

in which  $c$  is the vacuum speed of light. The classes one to four hamiltonians are defined by Eqs. (1) to (4) respectively. The classical hamiltonian is defined by:

$$H_0 := H - mc^2 \quad (6)$$

where  $H$  is the relativistic hamiltonian:

$$H = E + U \quad - (7)$$

where  $E$  is the relativistic total energy:

$$E = \gamma mc^2 = (c^2 p^2 + m^2 c^4)^{1/2} \quad - (8)$$

Here

$$\underline{p} = \gamma \underline{p}_0 \quad - (9)$$

is the relativistic momentum. It follows that:

$$H_0 = H - mc^2 = \frac{c^2 p^2}{E + mc^2} + U \quad - (10)$$

and the Dirac approximation is {1 - 12}:

$$H = mc^2 \quad - (11)$$

which means that the classical hamiltonian vanishes:

$$H_0 = 0. \quad - (12)$$

This is clearly unphysical, but the approximation leads to the famous description of spectral fine structure:

$$H_0 = \frac{c^2 p^2}{2mc^2 - U} + U \quad - (13)$$

through spin orbit interaction.

The rigorous development of the class one hamiltonian is described in Notes

333(5) to 333(7), of which this section is a synopsis. First note that:

$$p_0^2 = 2m(H_0 - \bar{U}) \quad (14)$$

so it follows that:

$$\frac{\gamma^2}{1+\gamma} = \left( \left( 1 - \frac{2(H_0 - \bar{U})}{mc^2} \right) + \left( 1 - \frac{2(H_0 - \bar{U})}{mc^2} \right)^{1/2} \right)^{-1} \quad (15)$$

In the H atom, using the non relativistic hydrogenic orbitals in a first approximation:

$$\langle \bar{U} \rangle = -2 \langle H_0 \rangle = -mc^2 \left( \frac{\alpha}{n} \right)^2 \quad (16)$$

as in Note 333(4). Quantization takes place by using:

$$-i\hbar \nabla \phi = \underline{p}_0 \phi \quad (17)$$

for the first  $\underline{p}_0$  in Eq. (1), and by using the function for the second  $\underline{p}_0$ . This is in fact an arbitrary procedure, but is the one which has been used for ninety years. This procedure gives:

$$H_0 \phi = -\frac{i\hbar}{m} \underline{\sigma} \cdot \nabla \left( \left( 1 - \frac{2(H_0 - \bar{U})}{mc^2} \right) + \left( 1 - \frac{2(H_0 - \bar{U})}{mc^2} \right)^{1/2} \right)^{-1} \underline{\sigma} \cdot \underline{p}_0 \phi + \bar{U} \phi \quad (18)$$

Using computer algebra it is found that:

$$\nabla \left( \frac{\gamma^2}{1+\gamma} \right) = - \frac{2 + \left( 1 - \frac{p_0^2}{m^2 c^2} \right)^{-1/2}}{\left( \frac{1 - p_0^2}{m^2 c^2} + \left( 1 - \frac{p_0^2}{m^2 c^2} \right)^{1/2} \right)^2} \frac{e^2}{4\pi \epsilon_0 m^2 c^2} \frac{1}{r^3} \quad (19)$$

for the Coulomb potential between the electron and proton of the H atom:

$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad - (20)$$

Defining:

$$A := \frac{2 + \left(1 - \frac{p_0^2}{m^2 c^2}\right)^{-1/2}}{\left(\left(\frac{1 - p_0^2}{m^2 c^2}\right)^{1/2} + 1 - \frac{p_0^2}{m^2 c^2}\right)^2} \quad - (21)$$

it is found that:

$$H_{so} \psi = -\frac{ie^2 \hbar A}{4\pi\epsilon_0 m^2 c^2 r^3} \underline{\sigma} \cdot \underline{r} \underline{\sigma} \cdot \underline{p}_0 \psi + U \psi \quad - (22)$$

By Pauli algebra:

$$\underline{\sigma} \cdot \underline{r} \underline{\sigma} \cdot \underline{p}_0 = \underline{r} \cdot \underline{p}_0 + i \underline{r} \times \underline{p}_0 \quad - (23)$$

in which the classical orbital angular momentum is:

$$\underline{L} = \underline{r} \times \underline{p}_0 \quad - (24)$$

So the rigorous class one spin orbit hamiltonian is:

$$E = \langle H_{so} \rangle = \frac{e^2 \hbar A}{4\pi\epsilon_0 m^2 c^2} \left\langle \frac{\underline{\sigma} \cdot \underline{L}}{r^3} \right\rangle \quad - (25)$$

This reduces to the Dirac result in the limit:

$$\gamma \rightarrow 1 \quad - (26)$$

If  $\underline{p}_0$  and A are regarded as functions, the Dirac fine structure is shifted as described in Section 3. For relativistic  $\underline{p}_0$  the shift becomes very large and should be experimentally observable. Such an experiment is a rigorous test of the Dirac equation, or more accurately,

the quantization of the hamiltonian of special relativity and the very foundations of relativistic quantum mechanics.

If the expectation value:

$$\left\langle \frac{p_0^2}{m^2 c^2} \right\rangle = \frac{1}{2} \left( \frac{d}{n} \right)^2 = \frac{2.662567 \times 10^{-5}}{n^2} \quad - (27)$$

is used an entirely different spectrum emerges from Eq. ( 1 ). The energy levels of this spectrum are:

$$E = \langle H_{so} \rangle = \frac{e^2 \hbar A}{4\pi \epsilon_0 m^2 c^3} \left\langle \frac{\sigma \cdot L}{r^3} \right\rangle \quad - (28)$$

$$= \frac{e^2 A}{16\pi \epsilon_0 m^2 c^3} \left( \frac{J(J+1) - L(L+1) - S(S+1)}{a_0^3 n^3 L(L+\frac{1}{2})(L+1)} \right)$$

in which the total angular momentum quantum number J is defined by:

$$J = L+S, L+S-1, \dots, |L-S| \quad - (29)$$

where L is the orbital angular momentum quantum number and S the spin quantum number.

In Eq. ( 28 ) A is defined by expectation values:

$$A = \frac{2 + \left( 1 - \left\langle \frac{p_0^2}{m^2 c^2} \right\rangle \right)^{-1/2}}{\left( \left( 1 - \left\langle \frac{p_0^2}{m^2 c^2} \right\rangle \right)^{1/2} + 1 - \left\langle \frac{p_0^2}{m^2 c^2} \right\rangle \right)^2} \quad - (30)$$

The selection rules for such a spectrum are:

$$\Delta J = 0, \pm 1, \quad - (31)$$

and

$$\Delta m_j = 0, \pm 1. \quad - (32)$$

Recall that Eq. ( 28 ) is the rigorous consequence of:

$$H_0 = \frac{1}{m} \underline{\sigma} \cdot \underline{p}_0 \frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot \underline{p}_0 + U \quad - (33)$$

which reduces to te classical hamiltonian:

$$H_0 \xrightarrow{\gamma \rightarrow 1} \frac{p_0^2}{2m} + U \quad - (34)$$

in the limit:

$$\gamma \rightarrow 1. \quad - (35)$$

In this development the classical hamiltonian can take any value as required, this eliminating the unphysical Dirac approximation QED.

The energy levels from Eq. ( 28 ) are plotted in Section 3. If these are not observed the ninety year old philosophy of the Dirac equation fails at a basic level.

### 3. COMPUTATIONAL AND GRAPHICAL ANALYSIS



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## REFERENCES

- {1} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, “The Principles of ECE Theory” (open source as UFT281-UFT288 and Spanish section, New Generation Publishing in press).
- {2} M .W. Evans, Ed., “Definitive Refutations of the Einsteinian General Relativity” (Cambridge International Science Publishing, CISP, 2012, and open source on [www.aias.us](http://www.aias.us)).
- {3} M .W. Evans, Ed., J. Found. Phys. Chem., (CISP 2011 and open source on [www.aias.us](http://www.aias.us)).
- {4} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, “Criticisms of the Einstein Field Equation” (CEFE, open source as UFT301, CISP 2010).
- {5} L. Felker, “The Evans Equations of Unified Field Theory” (Abramis 2007, UFT302).
- {6} H. Eckardt, “The ECE Engineering Model” (UFT303).
- {7} M. W. Evans, “Collected Scientometrics” (UFT307 and New Generation 2015).
- {8} M .W. Evans, H. Eckardt and D. W. Lindstrom, “Generally Covariant Unified Field Theory” (Abramis 2005 to 2011 in seven volumes, open source on [www.aias.us](http://www.aias.us))
- {9} M . W. Evans and L. B. Crowell, “Classical and Quantum Electrodynamics and the B(3) Field” (World Scientific, 2001, open source in the Omnia Opera Section of [www.aias.us](http://www.aias.us))
- {10} M .W. Evans and S. Kielich., Eds, “Modern Nonlinear Optics” (Wiley Interscience, New York, 1992, 1993, 1997, 2001) in two editions and six volumes.
- {11} M .W. Evans and J. P. Vigiier, “The Enigmatic Photon” (Kluwer 1994 to 2002 and open source on [www.aias.us](http://www.aias.us), Omnia Opera section, in five volumes hardback, five volumes softback).
- {12} M .W. Evans and A. A. Hasanein, “The Photomagnetron in Quantum Field Theory” (World Scientific 1994).