Note 333(1): Classification Scheme for the New Hyperfine Structure.

The classification scheme is based on the following four types of SU(2) hamiltonian in special relativity.

Class 1

$$H_0(1) = \frac{1}{m} \underline{\sigma} \cdot \underline{p_0} \frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot \underline{p_0} + U \quad -(1)$$

Class 2

$$H_0(2) = \frac{\gamma}{m} \underline{\sigma} \cdot \underline{p_0} \frac{\gamma}{1+\gamma} \underline{\sigma} \cdot \underline{p_0} + U \quad -(2)$$

Class 3

$$H_0(3) = \frac{\gamma^2}{m} \underline{\sigma} \cdot \underline{p_0} \frac{1}{1+\gamma} \underline{\sigma} \cdot \underline{p_0} + U \quad -(3)$$

Class 4

$$H_0(4) = \frac{1}{m} \frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot \underline{p_0} \, \underline{\sigma} \cdot \underline{p_0} + U \quad -(4)$$

where

$$\gamma = \left(1 - \frac{p_0^2}{m^2 c^2}\right)^{-1/2} \quad -(5)$$

The relativistic momentum is:

$$\underline{p} = \gamma \underline{p_0} \quad -(6)$$

In the presence of a magnetic field:

$$\underline{p}_0 \longrightarrow \underline{p_0} - e\underline{A} \quad -(7)$$

In ECE and ECE2 these equations can be derived from Cartan geometry, and $\underline{A}$ related

2) to grounding as in UFT 317, and UFT 318.

The four classes of equations give four different types of hyperfine structure. In the presence of a magnetic field intricate new effects appear, notably in ESR and NMR. Eqs. (1) to (4) derive from the hamiltonian of special relativity

$$H = E + U \quad -(8)$$

where

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(9)$$

In ECE and ECE2 the Einstein energy equation (9) is derived from the tetrad postulate as in previous work. From eqs. (8) and (9):

$$(H - U)^2 - m^2 c^4 = c^2 p^2 \quad -(10)$$

so

$$H - U - mc^2 = \frac{c^2 p^2}{H - U + mc^2} \quad -(11)$$

i.e.

$$H_0 = H - mc^2 = \frac{c^2 p^2}{E + mc^2} + U$$

$$= \frac{p^2}{m(1+\gamma)} + U \quad -(12)$$

$$= \frac{\gamma^2 p_0^2}{m(1+\gamma)} + U$$

QED. Eqs. (1) to (4) are ways of expressing eq. (12) in the SU(2) basis.

3) The old approximation by Dirac and one of his contemporaries is:

$$H \sim mc^2 \quad (13)$$

but as shown in UFT322 eq. (13) leads to:

$$H_0 =? \; 0 \quad (14)$$

which is very restrictive if not unphysical. This fact was first discovered in UFT322.

There is no way of knowing a priori which scheme (1) to (4) should be used. They must be carefully investigated and systematically evaluated, then their new predictions compared with experimental data. This is a rigorous new test of relativistic quantum mechanics, especially in the areas of ESR and NMR.

Clearly the Dirac approximation (13) is very rough, because the correct hamiltonian is:

$$H = E + U = \gamma mc^2 + U \quad (15)$$

so Dirac assumed:

$$\gamma \rightarrow 1 \quad (16)$$

and

$$U << mc^2 \quad (17)$$

In the H atom:

$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad (18)$$

4)

Therefore the magnitude of U is

$$|U| = \frac{e^2}{4\pi\epsilon_0 r} \quad -(19)$$

So eq. (17) means:

$$\frac{e^2}{4\pi\epsilon_0 r} \ll mc^2 \quad -(20)$$

i.e.

$$r \gg \frac{e^2}{4\pi\epsilon_0 mc^2} \quad -(21)$$

Note that

$$\frac{e^2}{4\pi\epsilon_0 mc^2} = \frac{\hbar}{mc}\alpha \quad -(21)$$

where the fine structure constant is:

$$\alpha = \frac{e^2}{4\pi\hbar c\epsilon_0} = 0.00729351 \quad -(22)$$

and the Compton wavelength is

$$\lambda_c = \frac{\hbar}{mc} = 3.8616 \times 10^{-13}\,m \quad -(23)$$

So Dirac assumed:

$$r \gg 2.816 \times 10^{-15} \quad -(24)$$

and

$$v_0 \ll c \quad -(25)$$

The Bohr radius is

$$a_0 = 5.29177 \times 10^{-11}\,m \quad -(26)$$

) the restrictive nature of eq. (15) only becomes clear when it is realized that:

$$\gamma m c^2 + U \sim m c^2 \quad -(27)$$

So

$$\gamma \sim 1 - \frac{U}{mc^2} \quad -(28)$$

The correct γ is given by eq. (5), and in the same approximation as used by Dirac,

$$p_0^2 << m^2 c^2 \quad -(29)$$

$$\gamma \sim 1 + \frac{1}{2} \frac{p_0^2}{m^2 c^2} \quad -(30)$$

From eqs. (28) and (30):

$$\frac{p_0^2}{2m} = -U \quad -(31)$$

So

$$H_0 = \frac{p_0^2}{2m} + U =? \; 0 \quad -(32)$$

This makes it clear that the Dirac approximation is at best very restrictive, at worst unphysical.

So it is replaced by eqs. (1) to (4)