333(2): Development of the Class 4 Hamiltonian

The class 4 hamiltonian is:

$$H = \frac{1}{m}\frac{\gamma^2}{1+\gamma}\,\underline{\sigma}\cdot\underline{p}_0\,\underline{\sigma}\cdot\underline{p}_0 + U, \quad -(1)$$

and in the presence of a magnetic field:

$$H = \frac{1}{m}\left(\frac{\gamma^2}{1+\gamma}\right)\underline{\sigma}\cdot(\underline{p}_0 - e\underline{A})\,\underline{\sigma}\cdot(\underline{p}_0 - e\underline{A}) + U \quad -(2)$$

As in UFT 322:

$$H = \left(\frac{\gamma^2}{1+\gamma}\right)\left(\frac{p_0^2}{m} - \frac{e\hbar}{m} g_J m_J B_Z + \dots\right) \quad -(3)$$

$$+ \dots \quad -(4)$$

where the Landé factor is:

$$g_J = \frac{1}{2}\left(1 + \frac{J(J+1) + S(S+1) - L(L+1)}{J(J+1)}\right)$$

is the Landé factor. Note that as:

$$\gamma \to 1 \quad -(5)$$

Eq. (3) reduces to the usual hamiltonian:

$$H(\text{nonrelativistic}) = \frac{p_0^2}{2m} - \frac{e\hbar}{2m} g_J m_J B_Z + \dots \quad -(6)$$

As in previous work:

$$\frac{\gamma^2}{1+\gamma} \sim \frac{1}{2}\left(1 - \frac{U}{2mc^2} + \frac{1}{mc^2}\left(\frac{H_0}{2} + \frac{p_0^2}{2m}\right)\right) \quad -(7)$$

Checked by eq (7) of protocol

$$= \frac{1}{2}\left(1 + \frac{H_0}{2c^2} - \frac{H_0}{c^2} - \frac{H_0}{mc^2}\right) = \frac{1}{2}\left(1 - \frac{3}{2}\frac{H_0}{mc^2}\right)$$

) where
$$H_0 = \frac{p_0^2}{2m} + U \quad -(8)$$

Now use, in the H atom:

$$\langle H_0 \rangle = -\frac{mc^2}{2}\frac{\lambda_c}{a_0}\frac{\alpha}{n^2} = -\frac{me^4}{32\pi^2\hbar^2\epsilon_0^2 n^2} \quad -(9)$$

and
$$\left\langle \frac{p_0^2}{2m} \right\rangle = -\langle H_0 \rangle = \frac{me^4}{32\pi^2\hbar^2\epsilon_0^2 n^2} \quad -(10)$$

i.e.
$$\langle U \rangle = 2\langle H_0 \rangle = -\frac{me^4}{16\pi^2\hbar^2\epsilon_0^2 n^2} \quad -(11)$$

So
$$\left\langle \frac{\gamma^2}{1+\gamma} \right\rangle = \frac{1}{2}\left(1 - \frac{\langle H_0 \rangle}{mc^2} + \frac{\langle H_0 \rangle}{2mc^2} - \frac{\langle H_0 \rangle}{mc^2}\right) \quad -(12)$$
$$= \frac{1}{2}\left(1 - \frac{3}{2}\left\langle \frac{H_0}{mc^2} \right\rangle\right)$$

i.e.
$$\boxed{\left\langle \frac{\gamma^2}{1+\gamma} \right\rangle = \frac{1}{2}\left(1 + \frac{3}{4}\left(\frac{\lambda_c}{a_0}\right)\frac{\alpha}{n^2}\right)} \quad -(13)$$

From eqs. (3) and (13):

$$E = \langle H \rangle = \frac{1}{2}\left(1 + \frac{3}{4}\left(\frac{\lambda_c}{a_0}\right)\frac{\alpha}{n^2}\right)\left(\left\langle \frac{p_0^2}{m} \right\rangle - \frac{e\hbar}{m} g_J m_J B_z\right) + \dots$$

$$\frac{1}{2}\left(1 + \frac{3}{4}\left(\frac{\lambda_c}{a_0}\right)\frac{\alpha}{n^2}\right)\left(mc^2\frac{\lambda_c}{a_0}\frac{\alpha}{n^2} - \frac{e\hbar}{m} g_J m_J B_z\right) + \dots \quad -(14)$$

Eq. (14) results in a hyper fine structure splitting of the anomalous Zeeman effect spectrum.

The selection rules for eq. (14) are:

$$\Delta J = 0, \pm 1, \quad J = 0 \not\to J = 0 \quad -(15)$$

and

$$\Delta m_J = 0, \pm 1 \quad -(16)$$

The usual anomalous Zeeman effect is:

$$E = \frac{mc^2}{2} \frac{\lambda_c}{a_0} \frac{\alpha}{n^2} - \frac{e\hbar}{2m} g_J m_J B_Z \quad -(17)$$

In the H alpha transition of atomic H the line is split into six by eq. (17) and is further split by eq. (14). A program can be written to evaluate this type of splitting for all the transitions of atomic H.

These are called class four splittings.