333(4) : Triple Check on Energy Levels of H Atom

We have:

$$E = -\frac{mc^2}{2}\frac{\alpha}{a_0}\frac{\lambda_c}{n^2} \quad -(1)$$

where

$$\alpha = \frac{e^2}{4\pi\hbar c\epsilon_0} \quad -(2)$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2} \quad -(3)$$

$$\lambda_c = \frac{\hbar}{mc} \quad -(4)$$

So:

$$E = -\frac{mc^2}{2}\left(\frac{e^2}{4\pi\hbar c\epsilon_0}\right)\left(\frac{me^2}{4\pi\epsilon_0\hbar^2}\right)\left(\frac{\hbar}{mc}\right)\frac{1}{n^2}$$

$$= -\frac{me^4}{32\pi^2\hbar^2\epsilon_0^2 n^2} \quad -(5)$$

QED