333(5): Rigorous Evaluation of the Class One Hamiltonian

This is:

$$H_0 = \frac{1}{m}\,\underline{\sigma}\cdot\underline{p}_0\,\frac{\gamma^2}{1+\gamma}\,\underline{\sigma}\cdot\underline{p}_0 + U \quad -(1)$$

where

$$\gamma = \left(1 - \frac{p_0^2}{m^2c^2}\right)^{-1/2} \quad -(2)$$

and

$$p_0^2 = 2m(H_0 - U) \quad -(3)$$

So:

$$\frac{\gamma^2}{1+\gamma} = \left(\frac{1}{\gamma^2} + \frac{1}{\gamma}\right)^{-1} \quad -(4)$$

$$= \left(\left(1 - \frac{p_0^2}{m^2c^2}\right) + \left(1 - \frac{p_0^2}{m^2c^2}\right)^{1/2}\right)^{-1}$$

$$= \left(\left(1 - \frac{2(H_0 - U)}{mc^2}\right) + \left(1 - \frac{2(H_0 - U)}{mc^2}\right)^{1/2}\right)^{-1}$$

In the H atom:

$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(5)$$

and

$$H_0 = \langle H_0 \rangle = -\frac{mc^2}{2}\,\frac{\lambda_c}{a_0}\,\frac{\alpha}{n^2} \quad -(6)$$

in the notation of previous papers.

a) Therefore a quantization:

$$H_0\psi = -\frac{\hbar}{m}\,\underline{\sigma}\cdot\underline{\nabla}\left[\left(1-2\left(\frac{H_0-U}{mc^2}\right)+\left(1-2\left(\frac{H_0-U}{mc^2}\right)\right)^{1/2}\right)^{-1}\underline{\sigma}\cdot\underline{p}_0\psi\right] + U \quad -(7)$$

This expression can be worked out with computer algebra without any approximation. This method will show whether it gives spin orbit fine structure.

Eq. (7) is the rigorous quantization of the hamiltonian of special relativity, using the SU(2) basis.

It assumes that the first p_0 is an operator and that the second p_0 is a function. It also assumes that $\gamma^2/(1+\gamma)$ can be placed between $\underline{\sigma}\cdot\underline{p}_0$ and $\underline{\sigma}\cdot\underline{p}_0$.

It is possible to obtain spin orbit fine structure in the slow motion approximation or non-relativistic limit:

$$v_0 << c \quad -(8)$$

in which

$$H_0 - U << mc^2 \quad -(9)$$

In this case:

$$\left(1 - \frac{2(H_0 - U)}{mc^2}\right)^{1/2} \sim 1 - \frac{H_0 - U}{mc^2} \quad -(10)$$

So:

$$H_0\phi = -\frac{\hbar}{m}\,\underline{\sigma}\cdot\underline{\nabla}\left(\left(2 - 3\left(\frac{H_0 - U}{mc^2}\right)\right)^{-1}\underline{\sigma}\cdot\underline{p}_0\,\phi\right) + U\phi \quad -(11)$$

$$\sim -\frac{\hbar}{2m}\,\underline{\sigma}\cdot\underline{\nabla}\left(\left(1 + \frac{3}{2}\left(\frac{H_0 - U}{mc^2}\right)\right)\underline{\sigma}\cdot\underline{p}_0\,\phi\right) + U\phi$$

$$= -\frac{\hbar}{2m}\,\underline{\sigma}\cdot\underline{\nabla}\left(\left(1 - \frac{U}{2mc^2} + \frac{3}{2}\frac{H_0}{mc^2} - \frac{U}{mc^2}\right)\underline{\sigma}\cdot\underline{p}_0\,\phi\right) + U\phi$$

The experimentally observed fine structure is given by the term:

$$H_{01}\phi = -\frac{\hbar}{2m}\,\underline{\sigma}\cdot\underline{\nabla}\left(1 - \frac{U}{2mc^2}\right)\underline{\sigma}\cdot\underline{p}_0\,\phi + U\phi + \dots \quad -(12)$$

Added to this is the term:

$$H_{02}\phi = -\frac{\hbar}{2m}\,\underline{\sigma}\cdot\underline{\nabla}\left(\frac{3}{2}\frac{H_0}{mc^2} - \frac{U}{mc^2}\right)\underline{\sigma}\cdot\underline{p}_0\,\phi \quad -(13)$$

The experimentally observed fine structure is obtained if and only if it is assumed that

$$U = \langle U \rangle = 2\langle H_0 \rangle \quad -(14)$$

4) in which case:

$$\underline{\nabla}\left(\frac{3}{2}\frac{H_0}{mc^2} - \frac{U}{mc^2}\right) = \underline{0} \quad -(15)$$

In eq. (12) however it must be assumed that

$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(16)$$

in order to obtain the correct spin orbit fine structure. This is a completely arbitrary procedure, and the theory is in reality empirical.

The Dirac procedure is:

$$H_0 = \underline{\sigma}\cdot\underline{p}_0 \frac{c^2}{H-U+mc^2}\,\underline{\sigma}\cdot\underline{p}_0 + U \quad -(17)$$

$$= \underline{\sigma}\cdot\underline{p}_0 \frac{c^2}{2mc^2-U}\,\underline{\sigma}\cdot\underline{p}_0 + U$$

so Dirac assumed:

$$H = mc^2. \quad -(18)$$

However, this means:

$$H_0 = H - mc^2 = 0. \quad -(19)$$

So Dirac assumed:

$$\frac{\gamma^2}{1+\gamma} = \frac{1}{m\left(2 - \frac{U}{mc^2}\right)} \quad -(20)$$

>) So he assumed:

$$1 - \frac{2(H_0 - U)}{mc^2} + \left(1 - \frac{2(H_0 - U)}{mc^2}\right)^{1/2} = 2 - \frac{U}{mc^2} \quad -(21)$$

and

$$H_0 = 0 \quad -(22)$$

i.e

$$1 + \frac{2U}{mc^2} + \left(1 + \frac{2U}{mc^2}\right)^{1/2} = 2 - \frac{U}{mc^2} \quad -(22)$$

i.e

$$U = \frac{8}{9} mc^2 \quad -(23)$$

However, he also assumed:

$$U << mc^2 \quad -(24)$$

So the Dirac approximation is meaningless, QED.

This catalyzes a major crisis in physics.