4FT 333(7): Energy Levels Using Expectation Values

These are:

$$E = \langle H_{SO} \rangle = \frac{-e^2 \hbar A}{4\pi \epsilon_0 m^2 c^2} \left\langle \frac{\underline{\sigma} \cdot \underline{L}}{r^3} \right\rangle \quad -(1)$$

$$= \frac{-e^2 A}{16\pi \epsilon_0 m^2 c^2} \frac{\left(J(J+1) - L(L+1) - S(S+1)\right)}{a_0^3 n^3 L\left(L+\frac{1}{2}\right)(L+1)}$$

where:

$$A = \frac{2 + \left(1 - \left(1 - \left\langle \frac{p_0^2}{m^2 c^2} \right\rangle\right)\right)^{-1/2}}{\left(\left(1 - \left\langle \frac{p_0^2}{m^2 c^2} \right\rangle\right)^{1/2} + \left(1 - \left\langle \frac{p_0^2}{m^2 c^2} \right\rangle\right)\right)^2} \quad -(2)$$

where $\left\langle \frac{p_0^2}{m^2 c^2} \right\rangle = \frac{2.662567 \times 10^{-5}}{n^2}$ $-(3)$

This is the result of the Hamiltonian:

$$H = \frac{1}{m} \underline{\sigma} \cdot \underline{p_0} \frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot \underline{p_0} + U \quad -(4)$$

labelled the class one Hamiltonian, where

$$\gamma = \left(1 - \frac{p_0^2}{m^2 c^2}\right)^{-1/2} \quad -(5)$$

In the non relativistic limit:

2)

$$\gamma \rightarrow 1 \quad -(6)$$

and eq. (4) becomes:

$$H = \frac{p_0^2}{2m} + U \quad -(7)$$

which is the classical hamiltonian. In the limit

$$p_0 << mc \quad -(8)$$

the factor A becomes:

$$A \rightarrow 1 \quad -(9)$$

and the usual spin orbit hamiltonian is obtained from eq. (1).