336(3): Relation between the Anomalous g Factor of the Electron and the Vacuum Vector Potential.

Consider eqn. (54) of Note 336(2): — (1)

$$(E-mc^2)\psi = -\frac{2e}{m(1+\gamma)}\,\underline{S}\cdot(\underline{B}+\underline{\nabla}\times\underline{A}_{vac})\psi$$

In the non-relativistic limit:

$$\gamma \to 1 \quad -(2) \qquad -(3)$$

So $$(E-mc^2)\psi = -\frac{2e}{2m}\,\underline{S}\cdot(\underline{B}+\underline{\nabla}\times\underline{A}_{vac})\psi$$

The factor 2 in the numerator is the g factor of the electron from the semiclassical theory. In the Dirac approximation, which is the Dirac value of g:

$$g = 2. \quad -(4)$$

Experimentally, it is found that the g factor of the electron is

$$g = 2.00231914, \quad -(5)$$

so:

$$2\underline{S} \to 2.00231914\,\underline{S} \quad -(6)$$

So:

$$2\underline{S}\cdot(\underline{B}+\underline{\nabla}\times\underline{A}_{vac}) \quad -(7)$$

$$= 2.00231914\,\underline{S}\cdot\underline{B}$$

The effective magnetic field acting on the electron beam is:

$$2\underline{B}_{eff} = 2\left(\underline{B} + \underline{\nabla} \times \underline{A}_{vac}\right) = 2.00231931 4\,\underline{B} \quad -(8)$$

So

$$\underline{B}_{eff} = 1.00116\,\underline{B} \quad -(9)$$

So the anomalous g factor of the electron can be thought of in terms of the vacuum potential $\underline{A}_{vac}$.

In ECE2 theory the vacuum potential can be thought of either as:

$$\underline{B} = \underline{\nabla} \times \underline{A}_{vac} - \underline{\omega} \times \underline{A}_{vac} \quad -(10)$$

$$= \underline{0}$$

or

$$\underline{B} = \underline{\nabla} \times \underline{W} \quad -(11)$$

where

$$\underline{W} = W^{(0)}\underline{\omega} \quad -(12)$$

The vacuum is thought of containing potentials but no fields. Experimentally, the Aharonov Bohm effect is due to regions where there are potentials but no fields.

Therefore in ECE2 vacuum:

$$\underline{\nabla} \times \underline{A}_{vac} = \underline{\omega} \times \underline{A}_{vac} \quad -(13)$$

and

$$\underline{\nabla} \times \underline{W}_{vac} = \underline{0} \quad -(14)$$

This means:

$$\underline{\nabla} \times \underline{q} = \underline{\omega} \times \underline{q} \quad -(15)$$

and

$$\underline{\nabla} \times \underline{\omega} = 0. \quad -(16)$$

because

$$\underline{A} = A^{(0)} \underline{q} \quad -(17)$$

and

$$\underline{W} = W^{(0)} \underline{\omega} \quad -(18)$$

Eqs. (15) and (16) can be solved simultaneously to give the ~~related~~ vector q and spin connection vector $\underline{\omega}$ of the ECE2 vacuum.