336(4): The Aharonov Bohm Vacuum in ECE2

The Aharonov Bohm (AB) vacuum is defined by the presence of potentials and the absence of electromagnetic fields. The magnetic flux density in ECE2 is defined by

$$\underline{B} = \underline{\nabla} \times \underline{W} = \underline{\nabla} \times \underline{A} + 2\underline{\omega} \times \underline{A} \quad -(1)$$

where

$$\underline{W} = W^{(0)}\underline{\omega}, \quad \underline{A} = A^{(0)}\underline{q} \quad -(2)$$

Here $\underline{\omega}$ is the spin connection and $\underline{q}$ the tetrad. Therefore the geometry of the AB vacuum is defined by:

$$\underline{\nabla} \times \underline{\omega} = \underline{0} \quad -(3)$$

and

$$\underline{\nabla} \times \underline{q} = 2\underline{q} \times \underline{\omega}. \quad -(4)$$

Now use:

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{q} = 0. \quad -(5)$$

From eqs. (4) and (5):

$$\underline{\nabla} \cdot (\underline{q} \times \underline{\omega}) = 0 \quad -(6)$$

$$= \underline{\omega} \cdot \underline{\nabla} \times \underline{q} - \underline{q} \cdot \underline{\nabla} \times \underline{\omega}$$

From eqs. (3) and (6):

$$\underline{\omega} \cdot \underline{\nabla} \times \underline{q} = 0 \quad -(7)$$

i.e.

$$\boxed{\underline{W} \cdot \underline{\nabla} \times \underline{A} = 0} \quad -(8)$$

Under condition (8), the potentials $\underline{W}$ and $\underline{A}$

2) are non-zero, but the magnetic flux density $\underline{B}$ is zero.

In the well known Chambers experiment, an electron beam is affected in regions where there is no $\underline{B}$. This effect is therefore explained by:

$$\underline{p} \rightarrow \underline{p} - e\underline{A}_{vac} \quad -(9)$$

where $\underline{A}_{vac}$ is the AB vacuum potential. It is concluded that an ESR experiment could be carried out in regions where there is no magnetic field $\underline{B}$.

The electric field strength in ECE2 theory is defined by:

$$\underline{E} = -\underline{\nabla}\phi_W - \frac{\partial \underline{W}}{\partial t} \quad -(10)$$

so the electric AB effect occurs when:

$$\underline{\nabla}\phi_W = -\frac{\partial \underline{W}}{\partial t} \quad -(11)$$

In terms of curvature:

$$\underline{B} = W^{(0)}\underline{R}(\text{spin}) \quad -(12)$$

$$\underline{E} = cW^{(0)}\underline{R}(\text{orb}) \quad -(13)$$

where $\underline{R}(\text{spin})$ and $\underline{R}(\text{orb})$ are the spin and orbital curvature vectors.

Therefore the AB effects occur in regions where

3) the tetrad and spin connection are finite but where there is no spin or orbital curvature. Similarly the torsion tensor is zero in the AB effect, but the tetrad and spin connection are non-zero.

In minimal notation:

$$T = d \wedge q + \omega \wedge q = 0 \quad -(14)$$

$$R = d \wedge \omega + \omega \wedge \omega = 0 \quad -(15)$$

So

$$d \wedge q = -\omega \wedge q \quad -(16)$$

and

$$d \wedge \omega = -\omega \wedge \omega \quad -(17)$$

with

$$T = R = 0 \quad -(18)$$

Vacuum Defined by Absence of Charge Current Density.

Consider the ECE2 electromagnetic field equations:

$$\underline{\nabla} \cdot \underline{B} = \underline{\kappa} \cdot \underline{B} \quad -(19)$$

$$\underline{\nabla} \cdot \underline{E} = \underline{\kappa} \cdot \underline{E} \quad -(20)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = -\left(\kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E}\right) \quad -(21)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \frac{\kappa_0}{c}\underline{E} + \underline{\kappa} \times \underline{B} \quad -(22)$$

where

$$\kappa_0 = 2\left(\frac{q_0}{r^{(0)}} - \omega_0\right) \quad -(23)$$

$$\underline{\kappa} = 2\left(\frac{\underline{q}}{r^{(0)}} - \underline{\omega}\right) \quad -(24)$$

4) In the absence of charge current density:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad - (25)$$

$$\underline{\nabla} \cdot \underline{E} = 0 \quad - (26)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = \underline{0} \quad - (27)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \underline{0} \quad - (28)$$

and there is no magnetic or electric charge current density, but the electromagnetic field is not zero.
In the Aharonov Bohm vacuum both the charge densities and the fields are zero, but the potentials are not zero.
One possible solution for eqs. (25) to (28) is

$$\kappa_0 = 0 \quad - (29)$$

$$\underline{\kappa} = \underline{0} \quad - (30)$$

so

$$q_0 = r^{(0)} \omega_0 \quad - (31)$$

and

$$\underline{q} = r^{(0)} \underline{\omega} \quad - (32)$$

From eq. (32) it follows that:

$$\begin{aligned} \underline{\omega} \times \underline{A} &= A^{(0)} \underline{\omega} \times \underline{q} \\ &= A^{(0)} r^{(0)} \underline{\omega} \times \underline{\omega} \quad - (33) \\ &= \underline{0} \end{aligned}$$

So in the absence of charge current density, and in

→) the presence of electromagnetic fields:

$$\underline{B} = \underline{\nabla} \times \underline{W} = \underline{\nabla} \times \underline{A} \quad -(34)$$
$$\neq \underline{0}$$

It is seen that eq. (6) of the AB vacuum is satisfied by eq. (32), because the latter implies:

$$\underline{\nabla} \cdot (\underline{v} \times \underline{v}) = \underline{0}. \quad -(35)$$

Eq. (4) of the AB vacuum is also satisfied by eq. (32) provided that:

$$\underline{\nabla} \times \underline{v} = \underline{0} \quad -(36)$$

but eqs. (32) and (36) imply:

$$\underline{\nabla} \times \underline{\omega} = \underline{0} \quad -(37)$$

so it is concluded that eq. (32) implies:

$$\underline{B} = \underline{0} \quad -(38)$$

Therefore in order that charge current density vanish and fields remain finite solutions other than eq. (32) must be found, and κ_0 and $\underline{\kappa}$ must remain finite. The equations to be solved are:

$$\underline{\kappa} \cdot \underline{B} = 0 \quad -(39)$$
$$\underline{\kappa} \cdot \underline{E} = 0 \quad -(40)$$
$$\kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E} = \underline{0} \quad -(41)$$
$$\frac{\kappa_0}{c} \underline{E} + \underline{\kappa} \times \underline{B} = \underline{0} \quad -(42)$$

with:

$$\kappa_0 \neq 0, \quad \underline{\kappa} \neq \underline{0}. \quad -(43)$$

From eqs. (41) and (42):

$$\underline{B} = -\frac{1}{\kappa_0 c} \underline{\kappa} \times \underline{E} \quad -(44)$$

and

$$\underline{E} = -\frac{c}{\kappa_0} \underline{\kappa} \times \underline{B}. \quad -(45)$$

Therefore in eqs. (39) and (40):

$$\underline{\kappa} \cdot (\underline{\kappa} \times \underline{B}) = 0 \quad -(46)$$

and

$$\underline{\kappa} \cdot (\underline{\kappa} \times \underline{E}) = 0. \quad -(47)$$

Now use:

$$\underline{\kappa} \cdot (\underline{\kappa} \times \underline{B}) = \underline{B} \cdot (\underline{\kappa} \times \underline{\kappa}) = \underline{0} \quad -(48)$$

and

$$\underline{\kappa} \cdot (\underline{\kappa} \times \underline{E}) = \underline{E} \cdot (\underline{\kappa} \times \underline{\kappa}) = \underline{0} \quad -(49)$$

So the solutions are eqs. (44) and (45), QED. These must be solutions of:

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = \underline{0} \quad -(50)$$

and

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(51)$$

The plane wave solution of eqs. (50) and (51) is:

7) Fm eq. (45):

$$\underline{\kappa} \times \underline{E} = -\frac{c}{\kappa_0}\,\underline{\kappa} \times (\underline{\kappa} \times \underline{B}) \quad -(52)$$

so $$\kappa_0^2\,\underline{B} = \underline{\kappa}(\underline{\kappa} \cdot \underline{B}) - \kappa^2 \underline{B} \quad -(53)$$

where $$\kappa^2 = \underline{\kappa} \cdot \underline{\kappa} \quad -(54)$$

i.e $$(\kappa^2 + \kappa_0^2)\,\underline{B} = \underline{\kappa}(\underline{\kappa} \cdot \underline{B}) \quad -(55)$$

The plane wave solution of eqs. (50) and (51) are:

$$\underline{B} = \frac{B^{(0)}}{\sqrt{2}}(i\underline{i} + \underline{j})e^{i\phi} \quad -(56)$$

and $$\underline{E} = \frac{E^{(0)}}{\sqrt{2}}(\underline{i} - i\underline{j})e^{i\phi} \quad -(57).$$

Fm eqs. (55) and (56):

$$(\kappa^2 + \kappa_0^2)(i\underline{i} + \underline{j}) = \underline{\kappa}(\underline{\kappa} \cdot (i\underline{i} + \underline{j})) \quad -(58)$$

Comparing real parts:

$$(\kappa^2 + \kappa_0^2)\,\underline{j} = \kappa_y \underline{\kappa} \quad -(59)$$

Comparing imaginary parts:

$$(\kappa^2 + \kappa_0^2)\,\underline{i} = \kappa_x \underline{\kappa} \quad -(60)$$

Adding: $$(\kappa^2 + \kappa_0^2)(\underline{i} + \underline{j}) = (\kappa_x + \kappa_y)\,\underline{\kappa} \quad -(61)$$

so $$\boxed{\underline{\kappa} = \left(\frac{\kappa^2 + \kappa_0^2}{\kappa_x + \kappa_y}\right)(\underline{i} + \underline{j})} \quad -(62)$$