336(5): The Interaction of the electron with the Aharonov Bohm (AB) Vacuum.

Consider the Einstein energy equation of the free electron:

$$E^2 = p^2c^2 + m^2c^4 \quad -(1)$$

where

$$E = \gamma mc^2 \quad -(2)$$

and

$$\underline{p} = \gamma \underline{p_0} \quad -(3)$$

Here m is the mass of the electron, E is the total relativistic energy, p is the relativistic momentum, p_0 the classical or non relativistic momentum, c the speed of light in vacuo, and γ the Lorentz factor:

$$\gamma = \left(1 - \frac{p_0^2}{m^2c^2}\right)^{-1/2} \quad -(4)$$

Use the de Broglie / Einstein equations:

$$E = \gamma mc^2 = \hbar\omega \quad -(5)$$

$$\underline{p} = \gamma \underline{p_0} = \hbar \underline{\kappa} \quad -(6)$$

to show that:

$$\gamma = \frac{\hbar\omega}{mc^2} = \frac{\hbar\kappa}{mv_0} \quad -(7)$$

Here $\hbar$ is the reduced Planck constant, ω is the angular frequency of the electronic matter wave, κ the wavenumber of the electronic matter ~~w~~ wave and $\underline{v_0}$ the

2) classical velocity of the electron. From eqs. (1), (5) and (6) it follows ~~that~~:

$$\omega^2 = c^2\kappa^2 + \left(\frac{mc^2}{\hbar}\right)^2 \quad - (8)$$

for the electron matter wave.

The AB vacuum is defined by the four potential:

$$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right) \quad - (9)$$

The relativistic energy momentum is defined by:

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right) \quad - (10)$$

so eq (1) is

$$p^\mu p_\mu = m^2c^2 \quad - (11)$$

where

$$p_\mu = \left(\frac{E}{c}, -\underline{p}\right) \quad - (12)$$

The interaction of the electron with the AB vacuum is defined by:

$$p^\mu \rightarrow p^\mu - eA^\mu \quad - (13)$$

i.e <u>by</u> the <u>minimal</u> prescription. Note carefully that electric and magnetic fields have not been used. The AB vacuum contains potentials but no fields. Eqs (9) to (13) imply:

)

$$E \rightarrow E - e\phi \quad -(14)$$

nd

$$\underline{p} \rightarrow \underline{p} - e\underline{A}. \quad -(15)$$

So eq. (1) becomes:

$$\boxed{(E-e\phi)^2 = c^2(\underline{p}-e\underline{A})\cdot(\underline{p}-e\underline{A}) + m^2c^4} \quad -(16)$$

i.e.

$$(E-e\phi)^2 - m^2c^4 = c^2(\underline{p}-e\underline{A})\cdot(\underline{p}-e\underline{A})$$
$$= (E-e\phi-mc^2)(E-e\phi+mc^2), \quad -(17)$$

So

$$E - mc^2 = \frac{c^2(\underline{p}-e\underline{A})\cdot(\underline{p}-e\underline{A})}{E-e\phi+mc^2} + e\phi \quad -(18)$$

$$= \frac{c^2(\underline{p}-e\underline{A})\cdot(\underline{p}-e\underline{A})}{(1+\gamma)mc^2 - e\phi} + e\phi$$

using eq. (2).

Divide top and bottom by mc^2:

$$E - mc^2 = \frac{1}{m}\left(\frac{(\underline{p}-e\underline{A})\cdot(\underline{p}-e\underline{A})}{1+\gamma-\frac{e\phi}{mc^2}}\right) + e\phi$$

$$) = \frac{1}{m(1+\gamma)} \left(\frac{(\underline{p} - e\underline{A}) \cdot (\underline{p} - e\underline{A})}{1 - \frac{e\phi}{mc^2(1+\gamma)}} \right) + e\phi \quad -(19)$$

Now use the SU(2) basis and assume that:

$$E - mc^2 = \frac{1}{m(1+\gamma)} \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \left(1 - \frac{e\phi}{(1+\gamma)mc^2} \right)^{-1} \underline{\sigma} \cdot (\underline{p} - e\underline{A}) + e\phi \quad -(20)$$

To this point, the theory is rigorous. It reduces to the Dirac type theory when:

$$\gamma \rightarrow 1 \quad -(21)$$

i.e.

$$\hbar\omega \rightarrow mc^2 \quad -(22)$$

the de Broglie equation for the rest frequency of the electron is

$$\hbar\omega_0 = mc^2 \quad -(23)$$

In order to develop eq. (20) assume that:

$$e\phi \ll (1+\gamma)mc^2 \quad -(24)$$

So Eq. (20) becomes:

$$E - mc^2 \sim \frac{1}{(1+\gamma)m} \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \left(1 + \frac{e\phi}{(1+\gamma)mc^2} \right) \underline{\sigma} \cdot (\underline{p} - e\underline{A}) + e\phi \quad -(25)$$

$$= \frac{1}{(1+\gamma)m}\,\underline{\sigma}\cdot(\underline{p}-e\underline{A})\,\underline{\sigma}\cdot(\underline{p}-e\underline{A}) \qquad -(26)$$

$$+\frac{1}{(1+\gamma)m}\,\underline{\sigma}\cdot(\underline{p}-e\underline{A})\,\frac{e\phi}{(1+\gamma)mc^2}\,\underline{\sigma}\cdot(\underline{p}-e\underline{A})+e\phi$$

There are various ways of quantizing this equation using:

$$p^\mu\phi = i\hbar\,\partial^\mu\phi \qquad -(27)$$

where

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t},\,-\underline{\nabla}\right) \qquad -(28)$$

Therefore:

$$E\phi = i\hbar\frac{\partial\phi}{\partial t} \qquad -(29)$$

and

$$\underline{p}\,\phi = -i\hbar\,\underline{\nabla}\phi \qquad -(30)$$

Note carefully ~~that~~ the quantization procedure uses the relativistic p^μ. The application of eqs. (29) and (30) to eq. (26) can be carried out in a number of different ways. Each choice of quantization produces a different result. The correct choice, if there is such a thing, can be determined only by comparison with experimental data.

ESR Type Hamiltonian:

This is obtained with the first term on the right hand side of eq. (26), and the choice:

$$(E - mc^2)\phi = \frac{1}{m(1+\gamma)}\Big[\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla}\cdot\left(\underline{\sigma}\cdot(\underline{p}-e\underline{A})\right)\right)$$

$$-\underline{\sigma}\cdot e\underline{A}\ \underline{\sigma}\cdot(\underline{p}-e\underline{A})\phi\Big] + \ldots \quad -(31)$$

$$= \frac{ie\hbar}{m(1+\gamma)}\ \underline{\sigma}\cdot\underline{\nabla}\ \underline{\sigma}\cdot\underline{A}\ \phi + \ldots$$

Using Pauli algebra:

$$\underline{\sigma}\cdot\underline{\nabla}\ \underline{\sigma}\cdot\underline{A} = \underline{\nabla}\cdot\underline{A} + i\,\underline{\sigma}\cdot\underline{\nabla}\times\underline{A} \quad -(32)$$

so the real and physical part of eq. (31) is:

$$(E - mc^2)\phi = -\frac{e\hbar}{m(1+\gamma)}\ \underline{\sigma}\cdot\underline{\nabla}\times\underline{A}\ \phi \quad -(33)$$

The spin angular momentum of the electron is:

$$\underline{S} = \frac{\hbar}{2}\underline{\sigma} \quad -(34)$$

so

$$(E - mc^2)\phi = -\frac{2e}{m(1+\gamma)}\ \underline{S}\cdot\underline{\nabla}\times\underline{A}\ \phi \quad -(35)$$

In quantum theory:

$$S_Z\phi = m_s\hbar\phi \quad -(36)$$

where

$$m_s = -S, \ldots, S \quad -(37)$$

7) where S is the spin angular momentum quantum number of the electron, a fermion, so:

$$S = \frac{1}{2} \quad -(38)$$

and

$$m_S = -\frac{1}{2}, \frac{1}{2} \quad -(39)$$

Therefore:

$$(E - mc^2)\phi = \frac{-2e\hbar}{m(1+\gamma)} m_S (\underline{\nabla} \times \underline{A})_Z \quad -(40)$$

Here $(\underline{\nabla} \times \underline{A})_Z$ is the Z component of the curl of the AB vacuum vector potential $\underline{A}$.

Electron spin resonance is defined by:

$$\hbar\omega_{res} = -\frac{2e\hbar}{m(1+\gamma)} \left(-\frac{1}{2} - \left(\frac{1}{2}\right)\right) (\underline{\nabla} \times \underline{A})_Z \quad -(41)$$

so

$$\boxed{\omega_{res} = \frac{2e}{m(1+\gamma)} (\underline{\nabla} \times \underline{A})_Z} \quad -(42)$$

Therefore the effect of the vacuum of AB type is to cause ESR in the absence of a magnetic field.