1) 339(4): Summary of Concepts

1) Free Particle

For the free particle, a hypothetical particle not in contact with the vacuum:

$$H = E = \left(c^2 p^2 + m^2 c^4\right)^{1/2} \quad -(1)$$

where H is the hamiltonian and E the total relativistic energy. For this hypothetical particle there is no potential energy U.

From eq. (1):

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(2)$$

which is the Einstein energy equation. It is well known that eq. (2) is equivalent to:

$$\underline{p} = \gamma m \underline{v_0} \quad -(3)$$

which is the relativistic momentum of a free particle of mass m. Here γ is the Lorentz factor:

$$\gamma = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} \quad -(4)$$

The mass m is the mass of the hypothetical free particle. It is hypothetical because a particle cannot exist without being in contact with the vacuum. In the standard special relativity it is assumed that the vacuum does not contain a potential energy U.

From eq. (1):

$$E^2 - m^2 c^4 = \left(E - mc^2\right)\left(E + mc^2\right) = c^2 p^2 \quad -(5)$$

so

$$E - mc^2 = \frac{c^2 p^2}{E + mc^2} = \frac{c^2 p^2}{(1+\gamma)mc^2} \quad -(6)$$

Therefore

$$H_0 := E - mc^2 = \frac{p^2}{(1+\gamma)m} \quad -(7)$$

This result can be expressed as:

$$H_0 = T = (\gamma - 1)mc^2 = \frac{p^2}{(1+\gamma)m} \quad -(8)$$

where T is the well known relativistic kinetic energy:

$$T = \left(\left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} - 1\right)mc^2$$

$$\xrightarrow[v_0 << c]{} \left(1 + \frac{v_0^2}{2c^2} + \dots - 1\right)mc^2 \quad -(9)$$

$$= \frac{1}{2}mv_0^2$$

which is the non-relativistic kinetic energy Q.E.D.

On the right hand side of eq. (8)

$$\gamma \xrightarrow[v_0 << c]{} 1 \quad -(10)$$

so in the non relativistic limit:

$$\frac{1}{2}mv_0^2 = \frac{p_0^2}{2m} \quad -(11)$$

where

$$\underline{p}_0 = m\underline{v}_0 \quad -(12)$$

Q.E.D.

In order to relate this theory to the g factor of the electron, the conventional theory uses the minimal prescription:

$$\underline{p} \rightarrow \underline{p} - e\underline{A} \quad -(13)$$

this conventional theory is followed for the sake of argument, and eq. (8) becomes:

$$H_0 = T = (\gamma - 1)mc^2 = \frac{(\underline{p} - e\underline{A})\cdot(\underline{p} - e\underline{A})}{m(1+\gamma)}$$

$$= \frac{1}{m(1+\gamma)}\left(p^2 - 2eA_p + e^2A^2\right) \quad -(14)$$

The Zeeman effect hamiltonian is:

$$H(\text{Zeeman}) = -\frac{2eA_p}{m(1+\gamma)} = -\frac{2e\underline{A}\cdot\underline{p}}{m(1+\gamma)} \quad -(15)$$

For an applied static magnetic field $\underline{B}$:

$$\underline{A} = \frac{1}{2}\underline{B}\times\underline{r} \quad -(16)$$

so

$$H(\text{Zeeman}) = -\frac{e}{m(1+\gamma)}\underline{B}\times\underline{r}\cdot\underline{p} \quad -(17)$$

$$= -\frac{e}{m(1+\gamma)}\underline{r}\times\underline{p}\cdot\underline{B}$$

$$= -\frac{e}{m(1+\gamma)}\underline{L}\cdot\underline{B}$$

where

$$\underline{L} = \underline{r}\times\underline{p} \quad -(18)$$

is the orbital angular momentum. Note carefully that $\underline{L}$ is the relativistic angular momentum because

$\underline{p}$ is the relativistic linear momentum.

In the non relativistic limit of eq. (17):

$$H(\text{Zeeman}) \rightarrow -\frac{e}{2m}\underline{L}_0 \cdot \underline{B} \quad -(19)$$

here

$$\underline{L_0} = \underline{r} \times \underline{p_0} \quad -(20)$$

is the non relativistic angular momentum. When electron spin is considered, eq. (19) becomes the hamiltonian of the <u>anomalous</u> Zeeman effect:

$$H_0(AZE) = -\frac{e}{2m}\left(\underline{L_0} + 2\underline{S}\right)\cdot\underline{B} \quad -(21)$$

and the correctly relativistic hamiltonian (17) becomes:

$$H(AZE) = -\frac{e}{m(1+\gamma)}\left(\underline{L} + 2\underline{S}\right)\cdot\underline{B} \quad -(22)$$

The g factor of the electron from eq. (21) is the factor highlighted in the following equation:

$$H_0(AZE) = -\frac{e}{2m}\left(\underline{L_0} + \boxed{2}\,\underline{S}\right)\cdot\underline{B} \quad -(23)$$

$$g = 2 \quad -(24)$$

in eq. (21). The non-relativistic <u>gyromagnetic</u> <u>ratio</u> is

$$\gamma_e = -\frac{e}{2m} \quad -(25)$$

eq. (23) becomes

$$H_0(AZE) = \gamma_e\left(\underline{L_0} + g\,\underline{S}\right)\cdot\underline{B} \quad -(26)$$

5) The relativistic gyromagnetic ratio is:

$$\boxed{\gamma_e = -\frac{e}{m(1+\gamma)}} \quad -(27)$$

and the relativistic g factor of the electron is:

$$\boxed{g = 1+\gamma} \quad -(28)$$

So eq. (22) can be written in the form of eq. (26)

The free particle Lorentz factor is defined by

$$\gamma = \frac{\hbar\omega}{mc^2} = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} \quad -(29)$$

where ω is the angular frequency of the free particle's matter wave. So the relativistic gyromagnetic ratio is:

$$\gamma_e = -\frac{e}{m\left(1+\frac{\hbar\omega}{mc^2}\right)} \quad -(30)$$

and the relativistic g factor of the electron is:

$$g = 1 + \frac{\hbar\omega}{mc^2} \quad -(31)$$

For a static particle:

$$\hbar\omega_0 = mc^2 \quad -(32)$$

where ω_0 is the rest angular frequency.

Therefore for a particle at rest:

6)

$$\gamma_e \rightarrow -\frac{e}{2m}, \quad g \rightarrow 2. \quad -(33)$$

For a particle moving at relativistic velocities, both the gyromagnetic ratio and the g factor depend on ω.

2) Particle in Contact with the Vacuum

The vacuum in ECE2 theory contains a potential energy:

$$U = e\phi_W. \quad -(34)$$

Therefore energy can be obtained from the ECE2 vacuum.

By definition:

$$W^\mu = \left(\frac{\phi_W}{c}, \underline{W}\right) = \frac{\hbar}{e}\Omega^\mu \quad -(35)$$

$$= \frac{\hbar}{e}\left(\Omega^0, \underline{\Omega}\right)$$

where W^μ is the potential four vector and Ω^μ the spin connection four vector. Therefore:

$$\phi_W = \frac{\hbar c}{e}\Omega^0 \quad -(36)$$

and

$$\boxed{U = \hbar c \Omega^0} \quad -(37)$$

By definition, the vacuum angular frequency is:

$$\boxed{\omega(\text{vac}) := c\Omega^0} \quad -(38)$$

and

$$\boxed{U = \hbar\omega(\text{vac})} \quad -(39)$$

7) The vacuum contains quanta of energy $\hbar\omega(vac)$.

The total energy of a particle in contact with the vacuum is therefore:

$$H = \hbar\omega + \hbar\omega(vac) \quad -(40)$$
$$= \hbar(\omega + \omega(vac))$$

and the total momentum of the particle in contact with the vacuum is

$$\underline{p} = \hbar(\underline{\kappa} + \underline{\kappa}(vacuum)) \quad -(41)$$

The vacuum is therefore populated with particles whose mass is defined by the de Broglie / Einstein equations:

$$\hbar\omega(vac) = \gamma(vac)m(vac)c^2 \quad -(42)$$

and

$$\hbar\underline{\kappa}(vac) = \gamma(vac)m(vac)\underline{v}_0(vac) \quad -(43)$$

The de Broglie Einstein equation of the free particle is therefore changed to:

$$E = \hbar\omega = \gamma mc^2 \quad -(44)$$

$$\rightarrow \hbar(\omega + \omega(vac)) = \gamma_1(m + m(vac))c^2$$

and the mass of the particle is increased to:

$$m \rightarrow m + m(vac) \quad -(45)$$

The g factor of the electron is changed to:

$$\boxed{g = 1 + \gamma_1} \quad -(46)$$

8) i.e.

$$g = 1 + \frac{\hbar(\omega + \omega(vac))}{(m + m(vac))c^2} \quad -(47)$$

$$= 1 + \frac{\hbar\omega}{(m + m(vac))c^2} + \frac{\hbar\omega(vac)}{(m + m(vac))c^2}$$

It is convenient to think of the interaction between m and m(vac) in terms of scattering theory and in terms of two Einstein energy equations:

$$E^2(total) = c^2 p^2(total) + m^2 c^4 \quad -(48)$$

for the hypothetical free particle of mass m, and one for the vacuum particle of mass m(vac):

$$E^2(total) = c^2 p^2(total) + m^2(vac)c^4 \quad -(49)$$

where

$$E(total) = \hbar(\omega + \omega(vac)) \quad -(50)$$

and

$$\underline{p}(total) = \underline{p}(\text{free particle}) + \underline{p}(\text{vacuum}) \quad -(51)$$

There can therefore be scattering of free particles from vacuum particles.

Eq. (49) leads to the g factor:

$$g = 1 + \frac{\hbar(\omega + \omega(vac))}{mc^2} \quad -(52)$$

and as in UFT338, to the anomalous g factor:

$$g = 1 + \frac{\hbar\omega}{mc^2} + \frac{\hbar\omega(vac)}{mc^2} \quad -(53)$$

$$\rightarrow \quad 2 + \frac{\hbar\omega(vac)}{mc^2} = 2.00231931 4$$

for a static electron.

This process is a fundamental shift in the de Broglie / Einstein equations of the free particle to give

$$E = \hbar(\omega + \omega(vac)) = \gamma mc^2 \quad -(54)$$

and

$$\underline{p} = \hbar(\underline{\kappa} + \underline{\kappa}(vac)) = \gamma m \underline{v}_0 \quad -(55)$$

where $\underline{v}_0$ is the classical velocity of the free particle. Having deduced $\omega(vac)$ from observation of g via eq. (53), the mass and velocity of the vacuum particle are obtained from:

$$E(vac) = \hbar\omega(vac) = \gamma(vac)\, m(vac)\, c^2 \quad -(56)$$

where $\gamma(vac)$ is given by:

$$g = 1 + \frac{\hbar(\omega + \omega(vac))}{mc^2} \quad -(57)$$

For a static particle:

$$g = 1 + \frac{\hbar\omega_0}{mc^2} + \frac{\hbar\omega(vac)}{mc^2} = 1 + \left(1 + \frac{\hbar\omega(vac)}{mc^2}\right) \quad -(58)$$

c) because:

$$\hbar\omega_0 = mc^2 \quad -(59)$$

All the motion is due to the vacuum particle, so:

$$1 + \frac{\hbar\omega(vac)}{mc^2} = \gamma(vac) = 1.00231931\text{4} \quad -(60)$$

$$= \left(1 - \frac{v_0^2(vac)}{c^2}\right)^{-1/2}$$

so

$$v_0(vac) = 0.068c \quad -(61)$$

Having determined $\omega(vac)$ and $\gamma(vac)$, it is possible to determine the mass of the vacuum particle from

$$\hbar\omega(vac) = \gamma(vac)\, m(vac)\, c^2 \quad -(62)$$

where

$$\omega(vac) = 0.002319314 \frac{mc^2}{\hbar} \quad -(63)$$

i.e.

$$m(vac) = \left(\frac{0.002319314}{1.002319314}\right) m \quad -(64)$$

where m can be taken to an excellent approximation to be the mass of the electron.