ECE2 fluid electrodynamics

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3 Numerical solutions from flow algorithms

The equations (79), (81) and (84) have been solved numerically by the finite element program FlexPDE. The 3D volume was chosen as for typical Navier-Stokes applications: a plenum box with a circular inlet at the bottom and an offset circular outlet at the top, see Fig. 1. The boundary conditions were set to $\mathbf{v} = \mathbf{0}$ at the borders of the box and a directional derivative perpendicular to the openings area was assumed. This allows for a free floating solution of the velocity field. As a test, a solution for the Navier-Stokes equation

$$(\mathbf{v} \cdot \boldsymbol{\nabla})\, \mathbf{v} + \boldsymbol{\nabla} p - \eta\, \nabla^2 \mathbf{v} = \mathbf{0}. \tag{85}$$

was computed, with η being a viscosity. The pressure term was added because the equation is otherwise homogeneous which means that there is no source term, leading to a solution which does not guarantee conservation of mass. The divergence of the pressure gradient is assumed to be in proportion to the divergence of the velocity field:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} p = P\, \boldsymbol{\nabla} \cdot \mathbf{v} \tag{86}$$

with a constant P for "penalty pressure". This represents an additional equation for determining the pressure. The result for the velocity is graphed in Fig. 2, showing a straight flow through the box which is perpendicular to the inlet and outlet surfaces as requested by boundary conditions.

Next the vorticity equation (79) was solved, again with the pressure term to guarantee solutions:

$$\nabla^2 \mathbf{w} + \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{w}) + \boldsymbol{\nabla} p = \mathbf{0}. \tag{87}$$

It is difficult to define meaningful boundary conditions because this is a pure flow equation for the vorticity $\mathbf{w}$. We used the same as for the Navier-Stokes equations. The result is graphed in Fig. 3. There is a flow-like structure with a divergence at the left, the flow is not symmetric. By definition, there should not

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be a divergence because of Eq. (78). We assume that the boundary conditions are not adequate for this type of equation.

The situation is more meaningful for Eq. (81) which we solved as

$$\boldsymbol{\nabla} \times \mathbf{w} + R\left(\left(\mathbf{v} \cdot \boldsymbol{\nabla}\right)\mathbf{v} - \mathbf{v} \times \mathbf{w}\right) + \boldsymbol{\nabla} p = \mathbf{0}. \tag{88}$$

The solution for $R = 1$ gives an inclined input and output flow (Fig. 4). In the middle hight of the box the flow is more over the sides, therefore the intensity of velocity is low in the middle plane shown. The divergence (not shown) is practically zero in this region. Fig. 5 shows a divergent and convergent flow in the XY plane, the flow goes over the full width of the box. Results for higher Reynolds numbers show no significant difference.

Finally we solved Eq. (84) which holds for a Beltrami flow:

$$\nabla^2 \mathbf{v} - R\left(\mathbf{v} \cdot \boldsymbol{\nabla}\right)\mathbf{v} - \boldsymbol{\nabla}\left(\boldsymbol{\nabla} \cdot \mathbf{v}\right) + \boldsymbol{\nabla} p = \mathbf{0}. \tag{89}$$

Here the flow is strongly enhanced in the middle region (Fig. 6). In the perpendicular plane a similar effect can be seen (Fig. 7). The field is not divergence-free there. For a Beltrami field we should have

$$\mathbf{w} \times \mathbf{v} = k\mathbf{v} \times \mathbf{v} = \mathbf{0}. \tag{90}$$

The vorticity $\mathbf{w}$ corresponding to Fig. 7 has been graphed in Fig. 8. There are indeed large regions where both $\mathbf{w}$ and $\mathbf{v}$ are parallel or antiparallel. The factor k seems to be location dependent, we did not constrain the Beltrami property by further means. Therefore the result is satisfactory. For larger R values the results remain similar again.

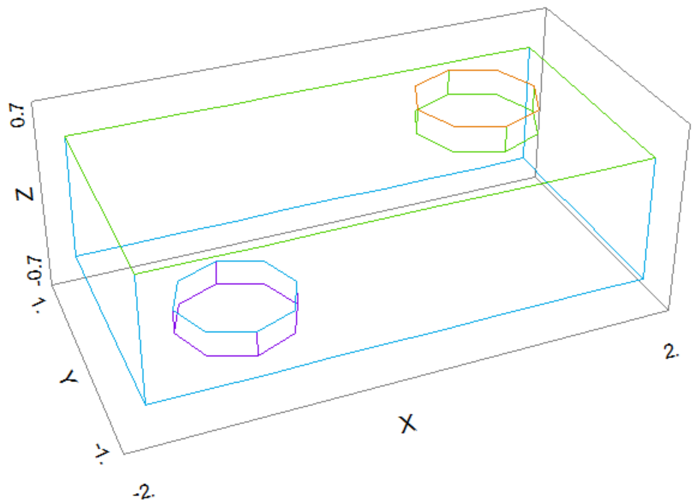

Figure 1: Geometry of FEM calculations.

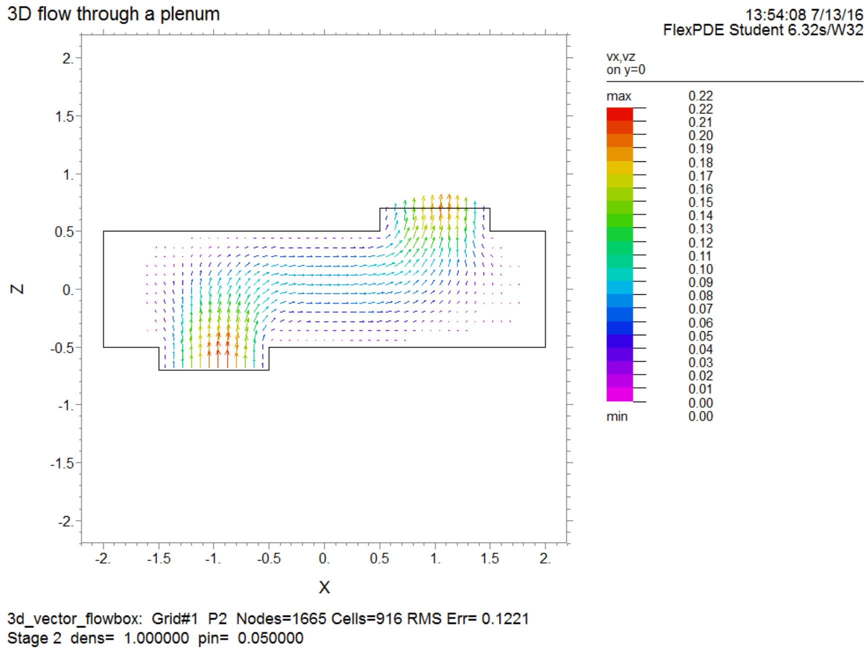

Figure 2: Velocity solution for Naview-Stokes Equation (85).

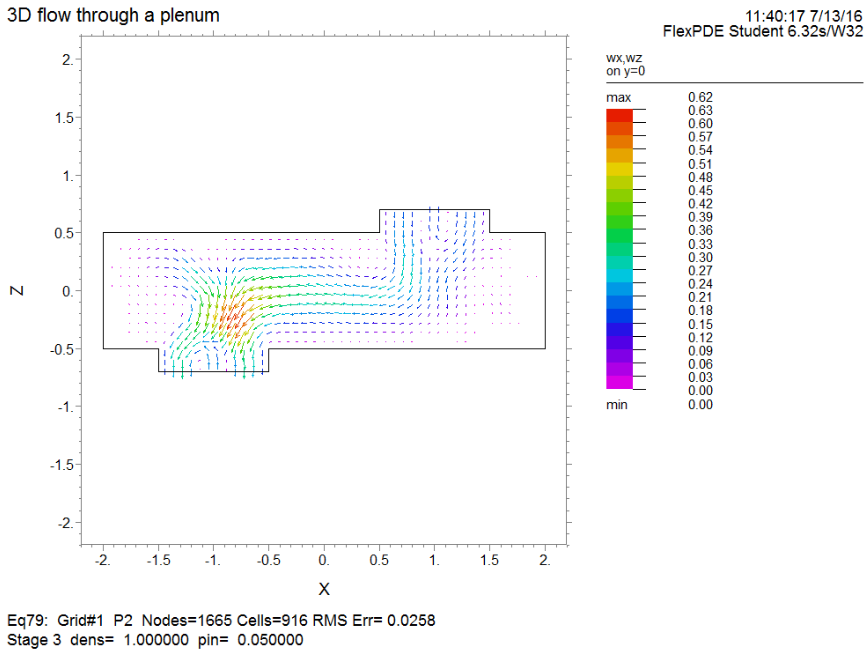

Figure 3: Vorticity solution for Equation (87).

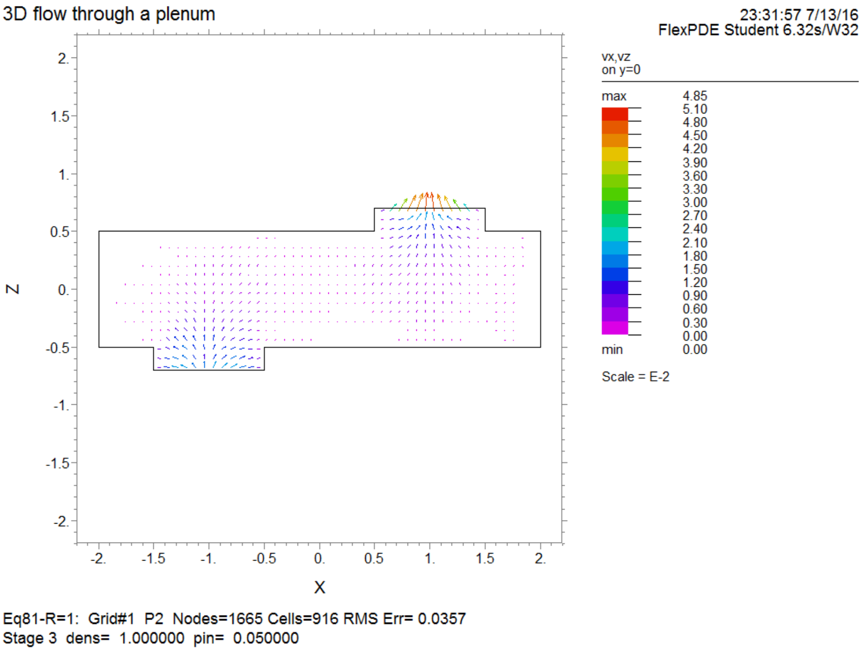

Figure 4: Velocity solution of Eq. (88) for $R = 1$, plane $Y = 0$.

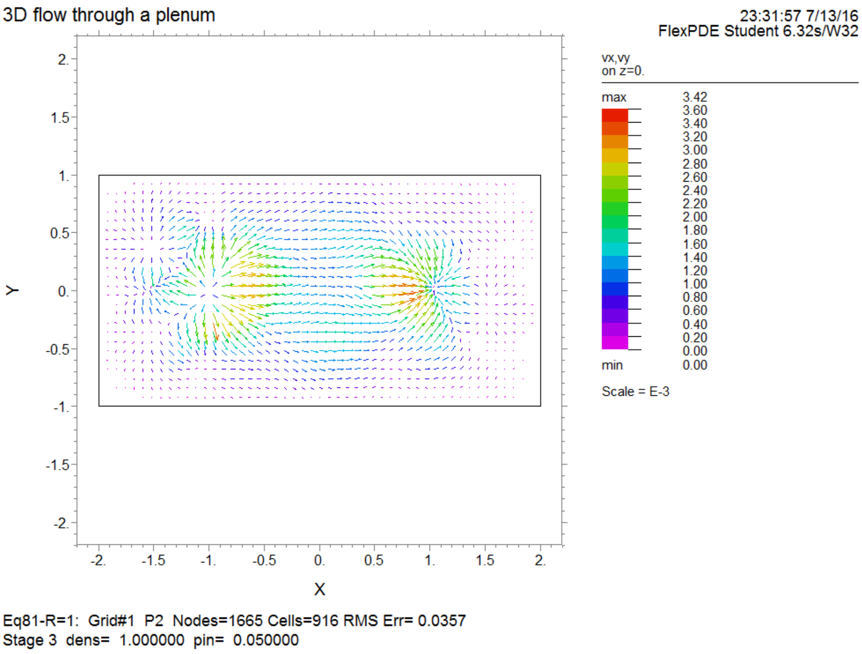

Figure 5: Velocity solution of Eq. (88) for $R = 1$, plane $Z = 0$.

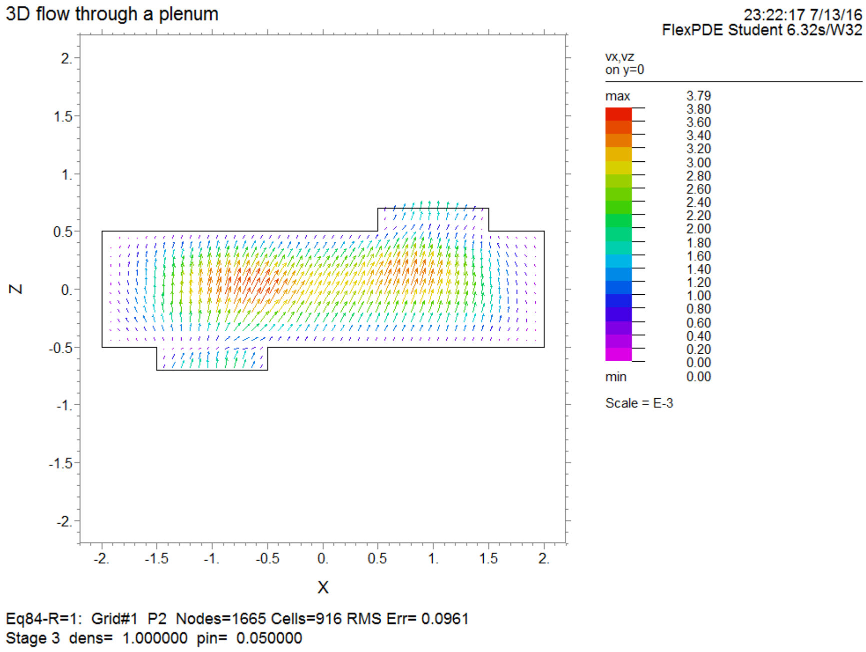

Figure 6: Beltrami solution of Eq. (89) for $R = 1$, plane $Y = 0$.

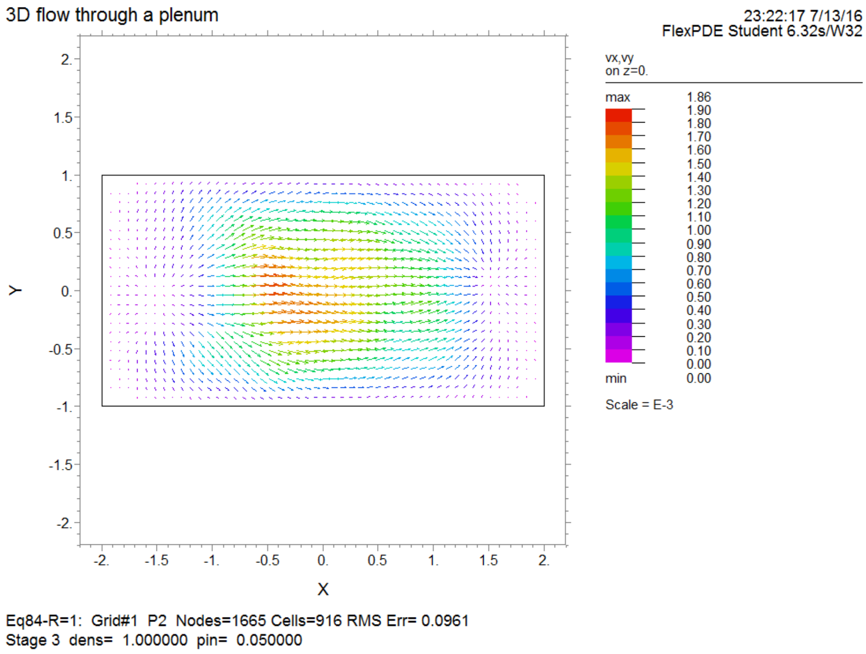

Figure 7: Beltrami solution of Eq. (89) for $R = 1$, plane $Z = 0$.

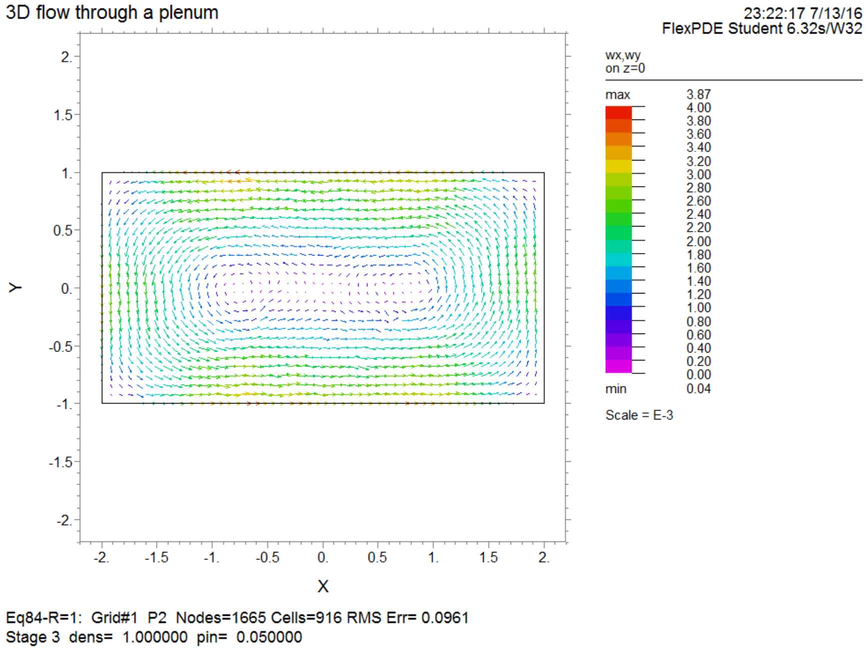

Figure 8: Vorticity of solution for Eq. (89) for $R = 1$, plane $Z = 0$.