

## CHAPTER FOUR

### ORBITAL THEORY

In this chapter ECE2 covariance is developed to produce the Lorentz force equation, and to develop quantization and orbital theory. In the opening section the gravitomagnetic Biot Savart and Ampere laws are developed and these laws are applied to planar orbits and the current of mass density of the planar orbit. The method is generally valid and can be used on all scales. The gravitomagnetic field responsible for the centrifugal force of planar orbits can be calculated. Following sections of this chapter develop ECE2 quantization and precessional theory, together with other aspects of orbital theory.

From chapter three, the ECE2 equations of gravitomagnetism are

and

where the field tensors are defined as:

and

In these equations  $g$  denotes the gravitational field and  $g_{\text{mag}}$  the gravitomagnetic field. It is assumed that the gravitomagnetic charge / current density vanishes. The contravariant index notation means that:

Lorentz transformation {1 - 10} of the field tensors gives the result:

where  $\gamma$  is the Lorentz factor:

in which  $v$  is the non relativistic velocity. In the rest frame:

Eqs. ( ) and ( ) exactly parallel electrodynamics {1 - 10}:

and

where  $E$  is the electric field strength in volts per metre and  $B$  is the magnetic flux density. In the non relativistic limit:

the gravitomagnetic Lorentz force is

In plane polar coordinates the orbital velocity of a mass  $m$  attracted to a mass  $M$  is, in general:

where the unit vectors of the cylindrical polar system are cyclically related as follows

For a planar orbit, the acceleration in general is:

where  $r$  is the radial vector defined by:

and where the angular velocity vector is:

The planar orbital force is therefore:

where  $G$  is Newton's constant. Eq. ( ) is the 1689 Leibnitz equation of orbits.

The orbital force equation can be written as:

where

is the velocity due to a rotating frame first inferred by Coriolis in 1835. The orbital force equation is the Lorentz force equation if:

and

Therefore is the gravitomagnetic field responsible for the centrifugal force of any planar orbit. The velocity due to the rotating frame of the plane polar coordinates is:

In the non-relativistic limit the electromagnetic Lorentz transforms are:

and the gravitomagnetic Lorentz transforms are:

The primes indicate the field in the observer frame in which the velocity of a charge or mass is non zero. The Biot Savart law of magnetism is obtained from Eq. ( ) with:

which means that there is no magnetic field in the rest frame, the frame in which the electric charge does not move. The electromagnetic Biot Savart law is therefore:

in S. I. Units. The prime in Eq. ( ) means that the law is written in the observer frame, the frame in which the velocity  $v$  of the electric charge is non-zero. In the usual electrostatics textbooks the prime is omitted by convention and the law becomes:

The Biot Savart law can be written {1 - 12} as:

which is the Ampere law of magnetostatics, describing the magnetic flux density generated by a current loop of any shape. It follows in ECE2 theory that:

so the current density of electrodynamics is:

Here:

The electromagnetic charge current density is:

In exact analogy, the ECE2 gravitomagnetic mass / current density is:

Therefore:

and the current of mass density is:

Now use:

Eqs. ( ) and ( ) are general to any orbit.

For the Hooke / Newton inverse square law:

the orbit in plane polar coordinates is the conic section {1 - 12}:

and the orbital linear velocity is:

where the semi major axis of an ellipse, for example, is:

Here  $\theta$  is the half right latitude and  $e$  is the eccentricity. Some examples of the gravitomagnetic field are developed and graphed later on in this chapter.

The ECE2 gravitomagnetic field can be calculated for dynamics in general and for a three dimensional orbit. For the planar part of this orbit the gravitomagnetic Ampere law can be used to calculate the light deflection due to gravitation and the precession of the perihelion from the ECE2 field equations.

In planar orbital theory it is well known {1 - 12} that the angular velocity is defined from a Lagrangian analysis in terms of the angular momentum of the system comprised of a mass  $m$  orbiting a mass  $M$ :

In this case the gravitomagnetic field is:

and the current of mass density is:

where the unit vectors are defined as:

in terms of the Cartesian unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ . From a hamiltonian analysis for a force law:

it follows that:

in order that the orbit be the conic section ( ). For an ellipse:

and for a hyperbola:

where  $a$  is the semi major axis for the ellipse.

The ECE2 gravitomagnetic Ampere law ( ) was first developed in UFT117 and UFT119, and orbital theory can be described by this law.

In cylindrical polar coordinates the position vector is:

the velocity vector is:



and the acceleration vector is:

In dynamics in general the gravitomagnetic field is:

If the gravitational potential energy is defined as:

the lagrangian is:

where the kinetic energy is:

There are three Euler Lagrange equations:

Eq. ( ) gives the Leibnitz equation:

Eq. ( ) gives:

and Eq. ( ) gives the conserved angular momentum

In cylindrical polar coordinates:

so for a three dimensional orbit  $L$  is not perpendicular to the orbital plane. It follows from Eq. ( ) that the  $Z$  component of angular momentum is:

and is a conserved constant of motion:

if the angular velocity is defined as in Eq. ( ). The total angular momentum is defined by:

and is not conserved, i.e. :

The Binet equation {1 - 12} of orbits is defined by Eqs. ( ) and ( ) and is:

It gives the force law for any orbit, not only the conic section orbits.

For planar orbits it can be shown {1 - 12} that:

so the velocity is:

and the acceleration is:

In three dimensions the position vector is:

and the angular momentum vector is:

so a planar orbit of any kind is embedded in the three dimensions defined by  
and Z. In the usual planar orbit theory it is assumed that

in Eq. ( ).

Eq. ( ) can be defined as:

so:

so a conic section orbit in cylindrical coordinates is defined in general by:

In general the lagrangian is:

and the potential energy and force depend on  $Z$  as well as on  $r$ . Most generally, the velocity in the observer frame is:

and the acceleration in the observer frame is:

The vector definitions ( ) and ( ) are equivalent to:

and

The gravitomagnetic field is proportional to the vector product of  $v$  from Eq. ( )

and  $a$  from Eq. ( ).

Using these concepts, the phenomena of light deflection due to gravitation and perihelion precession can be described straightforwardly by ECE2 theory as follows.

A precessing orbit can be modelled by:

and advances by:

IN the solar system,  $x$  is very close to unity. Later on in this chapter an exact ECE2 theory of the perihelion precession is developed by simultaneous solution of the ECE2 lagrangian and hamiltonian. However Eq. ( ) is accurate in the solar system to a very good approximation. Since  $x$  is very close to unity:

to an excellent approximation. From Eqs. ( ) and ( ) the force necessary for the precessing orbital model ( ) is:

Note carefully that this is not the force law of the incorrect Einstein theory, whose claims to accuracy are nullified because its underlying geometry is incorrect {1 - 12}. For light grazing the sun, the orbit is a hyperbola with a very large eccentricity, so the path of the light grazing the sun is almost a straight line. At the distance of closest approach (  $R$  ):

The angle of deflection of the light is defined by:

and the gravitomagnetic field is:

To an excellent approximation:

and the gravitomagnetic field at closest approach is:

Now use:

to find that:

The angle of deflection is therefore:

For light deflection by the sun:

so the gravitomagnetic field for light deflection by the sun is:

The precession of the perihelion of a planet such as Mercury is defined by the  $Z$  component of the gravitomagnetic field as follows:

where

Here  $b$  is the perihelion. Therefore at the perihelion:

and

The observed precession of the perihelion of Mercury is

and at the perihelion:

so

and to an excellent approximation:

thus justifying Eq. ( ). The required experimental data are:

so the gravitomagnetic field responsible for the precession of the perihelion of Mercury is:

As in UFT323 the concept of Lorentz transform can be extended to the Lorentz transform of frames, so in ECE2 the transform becomes one of a generally covariant unified field theory. The primed frame is the Newtonian or inertial frame whose axes are at rest. The notes accompanying UFT323 give clarifying examples. In contrast to the usual concept of the Lorentz transform in special relativity a particle may move in the primed Newtonian frame. In the original theory by Lorentz, the particle is at rest in its own frame of reference, known as the “rest frame”. The unprimed frame in this theory can move in any way with respect to the Newtonian or primed frame, so the 1835 theory by Coriolis is developed into a generally covariant unified field theory.

This theory produces the following force equation for orbits:

and is therefore the generally covariant Leibnitz force equation. It is shown as follows that this equation can describe precessional effects in orbits. The 1835 Coriolis theory is



recovered in the limit:

from which:

In conventional notation the well known Coriolis theory is:

in which:

for all planar orbits  $\{1 - 12\}$ . So for planar orbits:

The 1689 Leibnitz equation is:

which is recovered from the general theory using:

Therefore in the Leibnitz equation one frame moves with respect to another with the circular part of the orbital velocity

This is the angular part of the total orbital velocity:

The Leibnitz orbital equation produces the conic section:

whereas the observed orbit is accurately modelled by:

so the precession is due to the generalization of Eq. ( ) to Eq. ( ). In the Coriolis limit the gravitomagnetic field is given by Eq. ( ), so Eq. ( ) becomes

Using:

and:

Eq. ( ) reduces to:

The relativistic correction is due to an effective potential  $V$  defined by:

used with the lagrangian:

and the Euler Lagrange equation:

From Eqs. ( ) and ( ) the orbit due to Eq. ( ) is:

in which the Lorentz factor is defined by:

The force given by the Binet Eq. ( ) must be the same as the force given by Eq. ( )

so:

At the perihelion:

so:

so  $x$  can be found in terms of . The velocity of the Lorentz transform is defined by:

so:

The precession can be worked out in terms of  $\dot{\omega}$  for the orbit of the earth about the sun, and this is done later on in this chapter.

The Lorentz force equation of ECE2 theory can be solved by using the relativistic Binet equation for force, and its integral form for the hamiltonian. The relativistic Binet equation is derived from the Sommerfeld hamiltonian and the relativistic orbital velocity can be calculated straightforwardly and used to derive the observed velocity curve of a whirlpool galaxy and the precisely observed deflection of light due to gravity. These are major advances in understanding that overthrow the obsolete Einstein theory.

The property of ECE2 covariance means that the well known equations and ideas of special relativity can be used in orbital theory. The Lorentz transform is sufficient to produce the velocity curve of a whirlpool galaxy and the famous result of light deflection due to gravitation. Therefore these phenomena are explained by ECE2 theory straightforwardly. The relativistic Binet force equation is equivalent to the ECE2 Lorentz force equation derived earlier in this chapter. The former can be derived from the well known lagrangian of special relativity. The integral form of the Binet equation allows the evaluation of the hamiltonian for any orbit and the Binet force equation allows the evaluation of the central force and gravitational potential for any orbit. The methods can be exemplified with use of the plane polar coordinates and a precessing planar orbit. However it can be applied to three dimensional orbits.

It is shown as follows that the solution of the ECE2 Lorentz force equation for a planar orbit is:

The relativistic Binet equation for a planar orbit is:

in which the Lorentz factor is:

and in which the velocity used in the Lorentz factor is:

In the Lorentz force equation  $\mathbf{v}$  is the velocity of one frame with respect to another.

The relativistic Binet equation is derived from the lagrangian of special relativity:

where  $U$  is a central potential. The hamiltonian of special relativity can be derived from the lagrangian  $\{1 - 12\}$  and is:

where the total relativistic energy is:

The hamiltonian ( ) can be written as the Sommerfeld hamiltonian:

where:

is the relativistic kinetic energy. In the non relativistic limit:

.

The Euler Lagrange equations of the system are:

For a central potential that depends on  $r$  but on  $\theta$  they produce the results .

and:

Eq. ( ) defines the relativistic angular momentum:

which is a constant of motion:

Eq. ( ) defines the relativistic force equation of the orbit:

in which:

Here:

so:

where

In general this is a complicated expression that must be developed with computer algebra.

The Binet equation is defined by making a change of variable:

where:

From Eq. ( ) it follows that:

and

The orbital velocity is therefore:

and the integral form of the relativistic Binet equation is found directly from the Sommerfeld hamiltonian:

in which:

So the relativistic orbital velocity is:

Note carefully that

is a constant of motion, so the relativistic Binet force equation is:

which is the required solution of the ECE2 Lorentz force equation ( ), Q. E. D.

In the non relativistic limit the integral form of the Binet equation is:



and the Binet force equation in the non relativistic limit is the well known {1 - 12}:

For the precessing conic section ( ) for example, the central force is:

and the gravitational potential is:

The hamiltonian is:

In the Newtonian limit:

and the following well known results are recovered:

It follows that the Einstein theory is not needed to describe a precessing elliptical orbit. It can be derived classically as above. The Einstein theory gives an incorrect force law {1 - 12} that is the sum of terms that are inverse squared in r and inverse fourth power in r.

The correct expression is given in Eq. ( ).

The relativistic orbital velocity ( ) gives the correct experimental result for the velocity curve of a whirlpool galaxy using the hyperbolic spiral orbit of a star moving outwards from the galactic centre:

From Eqs. ( ) and ( ) the velocity curve of the spiral galaxy is:

and goes to the observed constant plateau:

These results amount to a strong indication that ECE2 is preferred by Ockham's Razor, and by observation, to the Einstein theory, because the latter fails completely to produce the velocity curve of a whirlpool galaxy {1 - 12}. The non relativistic Newtonian orbital velocity is:

so:

and the Newton theory fails completely in a whirlpool galaxy. The Einsteinian orbit is claimed to be able to reproduce the precessing ellipse ( ), so:

and the Einsteinian general relativity also fails completely in a whirlpool galaxy.

The relativistic orbital velocity from Eq. ( ) is:

where  $v$  is the non relativistic orbital velocity:

in plane polar coordinates. In light deflection by gravitation:

It follows from Eqs. ( ) and ( ) that:

and that there is an upper bound to the non relativistic velocity. This simple inference of ECE2 theory exactly explains light deflection by gravitation as follows.

The non relativistic orbital velocity is:

where the semi major axis is:

The distance of closest approach is:

It follows that:

Light grazing the sun follows a hyperbolic trajectory with a very large eccentricity:

so the orbit is almost a straight line. From Eqs. ( ) and ( ):

and the angle of deflection is:

This is often known as the Newtonian result. However, the non relativistic velocity from Eq. ( ) is:

so the angle of deflection is:

which is exactly the precisely measured experimental result, QED. The Einstein theory is not needed to produce this result.

The lagrangian and hamiltonian of ECE2 (those of special relativity) can be solved simultaneously using numerical scatter plot methods as in UFT325 on [www.aias.us](http://www.aias.us). The result is the precise orbit, the precisely defined precessing ellipse without any further assumption or theory. Note carefully that this is not the Einsteinian result, which is based on

an incorrect geometry without torsion. As shown in several UFT papers, the Einstein result produces a mirage of precision when the precession is tiny as in the solar system, over the full range of angle it gives a wildly incorrect orbit { 1 - 12 } and is known to be incorrect in many other ways. ECE2 is a great improvement because it gives a precessing ellipse directly from the simultaneous solution of the ECE2 lagrangian and hamiltonian - a precise, correct and general result. ECE2 gives the precessing orbit without any empiricism. The true precessing orbit is not that of Einstein, and is not the model ( ). The non relativistic Newton theory gives no precession at all.

The hamiltonian and lagrangian of ECE2 are given by Eqs. ( ) and ( ) respectively. It is assumed that the gravitational potential is:

The orbital velocity is defined by the infinitesimal line element of special relativity { 1 - 12 }:

where  $d\tau$  is the infinitesimal of proper time, the time in the frame moving with the object m orbiting an object M. It follows that the non relativistic velocity is:

The Euler Lagrange equations for this system produce Eq. ( ) as shown already.

As shown in Note 325(9) and by computer algebra, the Einstein theory gives an exceedingly complicated orbit and diverges, so the Einstein theory when correctly tested over its complete range gives an unphysical result, in fact it gives complete nonsense. UFT325 on [www.aias.us](http://www.aias.us) was the first paper to point htis out in irrefutable detail using computer algebra to eliminate human error. The Einstein theory is therefore obsolete. The basic incorrectness

of the Einstein theory can be demonstrated easily as follows. The Einsteinian hamiltonian and lagrangian are well known to be:

and

where:

and where

is known in the obsolete physics as the Schwarzschild radius. The conserved angular momentum of the Einstein theory is:

and it follows that:

and

Therefore the Einsteinian orbital velocity can be worked out from:

using Eqs. ( ) and ( ), giving the result:

As:

and at the distance of closest approach:

the Newtonian angle of deflection is changed to:

and this is not the experimental result, Q. E. D. As shown in detail in papers such as UFT150 and UFT155 on [www.aias.us](http://www.aias.us), by now classic papers, Einstein obtained the twice Newton result by a series of invalid approximations.

The Euler Lagrange equation ( ) gives the relativistic Leibnitz orbital equation of ECE2:

In the limit:

Eq. ( ) becomes the 1689 Leibnitz equation:

which gives a non precessing orbit. The Newtonian or non relativistic orbital velocity is:

From Eqs. ( ) and ( ) it can be shown that:

This result is graphed later in this chapter, in which a synopsis of UFT325 Section 3 is also given, a Section in which computer algebra is used to show that simultaneous solution of the lagrangian and hamiltonian of ECE2 theory gives the true precessing elliptical orbit for the first time in scientific history. The solution is a stable orbit and not an unstable, unphysical orbit as in the Einstein theory.

The property of ECE2 covariance means that quantization of ECE2 theory can be developed straightforwardly, as in UFT326, which is reviewed briefly as follows. These quantization schemes accompany a new axiom introduced logically by ECE2 theory, that the maximum value of the non relativistic velocity  $v$  is:

As shown already, this axiom immediately results in the precisely observed light deflection due to gravity, now known with claimed high precision. This axiom allows a particle with mass to travel at  $c$ . This fact is observed experimentally when electrons are accelerated to very close to  $c$ . The usual dogma of the obsolete physics claimed that only “massless particles” such as the photon can travel at  $c$ . ECE2 allows the photon with mass to travel at  $c$ .

The fundamental equations for the quantization schemes are the Einstein / de



Broglie equations:

and

The hamiltonian and lagrangian are defined as in Eqs. ( ) and ( ). The relativistic total energy is given by the well known Einstein equation:

This can be factorized in two ways:

and

each of which may be quantized using:

for the relativistic energy  $E$  and relativistic momentum  $p$  used in Eqs. ( ) and ( ).

The relativistic Schroedinger equation is obtained from Eqs. ( ) and ( )

and can be developed using various types of quantization as described in the notes of

UFT326 on [www.aias.us](http://www.aias.us). In the non relativistic limit:

the following result is obtained:

The relativistic Schroedinger equation of ECE2 may also be expressed as:

as explained in Note 326(8). The equation is therefore developed from the familiar Schroedinger equation using:

the non relativistic Schroedinger equation being defined by:

The relativistic Schroedinger equation may be developed as:

where:

and where the Coulomb potential is:

So the energy levels of the H atom are shifted by:

and this allows  $E_n$  to be found from the spectrum of the H atom. In the usual Dirac quantization scheme:

the rough approximation:

is used, implying:

Using these approximations in Eq. ( ) leads to:

where:

The energy levels of the H atom are shifted in the Dirac approximation to:

and this can be evaluated in the approximation of hydrogenic wave functions.

In the usual interpretation of special relativity the non relativistic velocity

of the Lorentz transform is allowed to reach  $c$ , the universal constant known as the vacuum speed of light. The experimentally untestable assumption:

results however in an unphysical infinity:

obscurely known in the obsolete physics as the hyper relativistic limit. The obsolete physics dealt with this unphysical infinity by inventing the massless particle. The relativistic momentum became indeterminate, zero multiplied by infinity, for this massless particle. A photon without mass became a dogmatic feature of the obsolete physics but at the same time introduced many severe difficulties {1 - 12} and obscurities which were acknowledged by the dogmatists themselves. The ECE2 axiom ( ) removes all these difficulties straightforwardly. Under condition ( ) the relativistic velocity  $v$  reaches  $c$  and the Lorentz factor remains finite:

As shown already, the ECE2 axiom ( ) immediately gives the observed light deflection due to gravitation. It also gives the correct  $O(3)$  little group of the Poincare group for a particle with mass, allows canonical quantization without problems, produces the Proca equation, and is also compatible with the  $B(3)$  field {1 - 12}. The ECE2 axiom ( ) introduces photon mass theory which refutes the Higgs boson and the entire structure of the obsolete physics. It removes the Gupta Bleuler condition, which was very obscure, and allows canonical quantization to take place self consistently. Some methods of measuring have been suggested already in order to test the axiom ( ) experimentally.

As described in notes such as 326(6) on [www.aiaa.us](http://www.aiaa.us) the relativistic Schroedinger equation may also be written as:

whose solution is:

where:

This leads to relativistic quantum theory and also an expression for  $\psi$  :

which may be used with Eq. ( ) to measure  $\psi$  and  $\psi^*$  experimentally.

Further details and numerical development are given in UFT326 and in numerical methods in UFT326 Section 3 summarized later in this chapter.

