Spinning top - represented by dumbbel

```
(%i1)  kill(all);
(%o0)  done
```

1 Coordinates

```
(%i1)  depends([r_1, r_2, r_3,theta,phi,psi,omega_1,omega_2,omega_3,T,U,L],
(%o1)  [r_1(t), r_2(t), r_3(t), θ(t), φ(t), Ψ(t), ω_1(t), ω_2(t), ω_3(t), T(t), U(t)
, L(t)]
```

2 Kinetic energy

```
(%i4)  v_1: diff(r_1,t) - omega_3*r_2+omega_2*r_3;
       v_2: diff(r_2,t) + omega_3*r_1-omega_1*r_3;
       v_3: diff(r_3,t) - omega_2*r_1+omega_1*r_2;
```

$$(\%o2)\quad \omega_2\, r_3-\omega_3\, r_2+\frac{\mathrm{d}}{\mathrm{d}\,t}\, r_1$$

$$(\%o3)\quad -\omega_1\, r_3+\frac{\mathrm{d}}{\mathrm{d}\,t}\, r_2+\omega_3\, r_1$$

$$(\%o4)\quad \frac{\mathrm{d}}{\mathrm{d}\,t}\, r_3+\omega_1\, r_2-\omega_2\, r_1$$

```
(%i5)  T: m/2*(v_1^2+v_2^2+v_3^2);
(%o5)
```

$$\frac{m\left(\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\, r_3+\omega_1\, r_2-\omega_2\, r_1\right)^2+\left(\omega_2\, r_3-\omega_3\, r_2+\frac{\mathrm{d}}{\mathrm{d}\,t}\, r_1\right)^2+\left(-\omega_1\, r_3+\frac{\mathrm{d}}{\mathrm{d}\,t}\, r_2+\omega_3\, r_1\right)^2\right)}{2}$$

3 Potential energy

```
(%i6)  U: -M*m*G/sqrt(r_1^2+r_2^2+r_3^2);
```

$$(\%o6)\quad -\frac{G\,M\,m}{\sqrt{{r_3}^2+{r_2}^2+{r_1}^2}}$$

4 Lagrange function

```
(%i7)  T;
(%o7)
```

$$\frac{m\left(\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\, r_3+\omega_1\, r_2-\omega_2\, r_1\right)^2+\left(\omega_2\, r_3-\omega_3\, r_2+\frac{\mathrm{d}}{\mathrm{d}\,t}\, r_1\right)^2+\left(-\omega_1\, r_3+\frac{\mathrm{d}}{\mathrm{d}\,t}\, r_2+\omega_3\, r_1\right)^2\right)}{2}$$

(%i8) U;

(%o8) $$-\frac{G\,M\,m}{\sqrt{r_3^{\,2}+r_2^{\,2}+r_1^{\,2}}}$$

(%i9) L: T - U;

(%o9)
$$\frac{m\left(\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\,r_3+\omega_1\,r_2-\omega_2\,r_1\right)^2+\left(\omega_2\,r_3-\omega_3\,r_2+\frac{\mathrm{d}}{\mathrm{d}\,t}\,r_1\right)^2+\left(-\omega_1\,r_3+\frac{\mathrm{d}}{\mathrm{d}\,t}\,r_2+\omega_3\,r_1\right)^2\right)}{2}+\frac{G\,M\,m}{\sqrt{r_3^{\,2}+r_2^{\,2}+r_1^{\,2}}}$$

5 Lagrange equations II

5.1 r_1 equation

(%i10) D5: diff(L, diff(r_1,t));

(%o10) $$m\left(\omega_2\,r_3-\omega_3\,r_2+\frac{\mathrm{d}}{\mathrm{d}\,t}\,r_1\right)$$

5.2 with time derivatives of omega_i

(%i11) `E5: expand(ratsimp(diff(D5,t) - diff(L,r_1) = 0));`

(%o11)
$$\frac{2\,m\,\omega_2\,r_3^2\left(\frac{d}{dt}\,r_3\right)}{r_3^2+r_2^2+r_1^2}+\frac{2\,m\,\omega_2\,r_2^2\left(\frac{d}{dt}\,r_3\right)}{r_3^2+r_2^2+r_1^2}+\frac{2\,m\,\omega_2\,r_1^2\left(\frac{d}{dt}\,r_3\right)}{r_3^2+r_2^2+r_1^2}+$$
$$\frac{m\,\omega_1\,\omega_3\,r_3^3}{r_3^2+r_2^2+r_1^2}+\frac{m\left(\frac{d}{dt}\,\omega_2\right)r_3^3}{r_3^2+r_2^2+r_1^2}-\frac{2\,m\,\omega_3\left(\frac{d}{dt}\,r_2\right)r_3^2}{r_3^2+r_2^2+r_1^2}-\frac{m\left(\frac{d}{dt}\,\omega_3\right)r_2\,r_3^2}{r_3^2+r_2^2+r_1^2}+$$
$$\frac{m\,\omega_1\,\omega_2\,r_2\,r_3^2}{r_3^2+r_2^2+r_1^2}+\frac{m\left(\frac{d^2}{dt^2}\,r_1\right)r_3^2}{r_3^2+r_2^2+r_1^2}-\frac{m\,\omega_3^2\,r_1\,r_3^2}{r_3^2+r_2^2+r_1^2}-\frac{m\,\omega_2^2\,r_1\,r_3^2}{r_3^2+r_2^2+r_1^2}+\frac{m\,\omega_1\,\omega_3\,r_2^2\,r_3}{r_3^2+r_2^2+r_1^2}+$$
$$\frac{m\left(\frac{d}{dt}\,\omega_2\right)r_2^2\,r_3}{r_3^2+r_2^2+r_1^2}+\frac{m\,\omega_1\,\omega_3\,r_1^2\,r_3}{r_3^2+r_2^2+r_1^2}+\frac{m\left(\frac{d}{dt}\,\omega_2\right)r_1^2\,r_3}{r_3^2+r_2^2+r_1^2}-\frac{2\,m\,\omega_3\,r_2^2\left(\frac{d}{dt}\,r_2\right)}{r_3^2+r_2^2+r_1^2}-$$
$$\frac{2\,m\,\omega_3\,r_1^2\left(\frac{d}{dt}\,r_2\right)}{r_3^2+r_2^2+r_1^2}-\frac{m\left(\frac{d}{dt}\,\omega_3\right)r_2^3}{r_3^2+r_2^2+r_1^2}+\frac{m\,\omega_1\,\omega_2\,r_2^3}{r_3^2+r_2^2+r_1^2}+\frac{m\left(\frac{d^2}{dt^2}\,r_1\right)r_2^2}{r_3^2+r_2^2+r_1^2}-$$
$$\frac{m\,\omega_3^2\,r_1\,r_2^2}{r_3^2+r_2^2+r_1^2}-\frac{m\,\omega_2^2\,r_1\,r_2^2}{r_3^2+r_2^2+r_1^2}-\frac{m\left(\frac{d}{dt}\,\omega_3\right)r_1^2\,r_2}{r_3^2+r_2^2+r_1^2}+\frac{m\,\omega_1\,\omega_2\,r_1^2\,r_2}{r_3^2+r_2^2+r_1^2}+$$
$$\frac{m\,r_1^2\left(\frac{d^2}{dt^2}\,r_1\right)}{r_3^2+r_2^2+r_1^2}-\frac{m\,\omega_3^2\,r_1^3}{r_3^2+r_2^2+r_1^2}-\frac{m\,\omega_2^2\,r_1^3}{r_3^2+r_2^2+r_1^2}+\frac{G\,M\,m\,r_1}{\left(r_3^2+r_2^2+r_1^2\right)^{3/2}}=0$$

(%i12) `E51: trigsimp(solve(E5, diff(r_1,t,2)));`

(%o12)
$$\left[\frac{d^2}{dt^2}\,r_1=-\left(\sqrt{r_3^2+r_2^2+r_1^2}\left(\left(2\,\omega_2\,r_3^2+2\,\omega_2\,r_2^2+2\,\omega_2\,r_1^2\right)\left(\frac{d}{dt}\,r_3\right)+\left(\omega_1\,\omega_3+\frac{d}{dt}\,\omega_2\right)r_3^3+\left(-2\,\omega_3\left(\frac{d}{dt}\,r_2\right)+\left(\omega_1\,\omega_2-\frac{d}{dt}\,\omega_3\right)r_2+\left(-\omega_3^2-\omega_2^2\right)r_1\right)r_3^2+\left(\left(\omega_1\,\omega_3+\frac{d}{dt}\,\omega_2\right)r_2^2+\left(\omega_1\,\omega_3+\frac{d}{dt}\,\omega_2\right)r_1^2\right)r_3+\left(-2\,\omega_3\,r_2^2-2\,\omega_3\,r_1^2\right)\left(\frac{d}{dt}\,r_2\right)+\left(\omega_1\,\omega_2-\frac{d}{dt}\,\omega_3\right)r_2^3+\left(-\omega_3^2-\omega_2^2\right)r_1\,r_2^2+\left(\omega_1\,\omega_2-\frac{d}{dt}\,\omega_3\right)r_1^2\,r_2+\left(-\omega_3^2-\omega_2^2\right)r_1^3\right)+G\,M\,r_1\right)\Big/\left(r_3^2+r_2^2+r_1^2\right)^{3/2}\right]$$

5.3 without time derivatives of omega_i

(%i13) `ev(%, [diff(omega_2,t)=0,diff(omega_3,t)=0]);`

(%o13)
$$\left[\frac{d^2}{dt^2}\,r_1=-\left(\sqrt{r_3^2+r_2^2+r_1^2}\left(\left(2\,\omega_2\,r_3^2+2\,\omega_2\,r_2^2+2\,\omega_2\,r_1^2\right)\left(\frac{d}{dt}\,r_3\right)+\omega_1\,\omega_3\,r_3^3+\left(-2\,\omega_3\left(\frac{d}{dt}\,r_2\right)+\omega_1\,\omega_2\,r_2+\left(-\omega_3^2-\omega_2^2\right)r_1\right)r_3^2+\left(\omega_1\,\omega_3\,r_2^2+\omega_1\,\omega_3\,r_1^2\right)r_3+\left(-2\,\omega_3\,r_2^2-2\,\omega_3\,r_1^2\right)\left(\frac{d}{dt}\,r_2\right)+\omega_1\,\omega_2\,r_2^3+\left(-\omega_3^2-\omega_2^2\right)r_1\,r_2^2+\omega_1\,\omega_2\,r_1^2\,r_2+\left(-\omega_3^2-\omega_2^2\right)r_1^3\right)+G\,M\,r_1\right)/\left(r_3^2+r_2^2+r_1^2\right)^{3/2}\right]$$

(%i14) `trigsimp(%);`

(%o14)
$$\left[\frac{d^2}{dt^2}\,r_1=-\left(\sqrt{r_3^2+r_2^2+r_1^2}\left(\left(2\,\omega_2\,r_3^2+2\,\omega_2\,r_2^2+2\,\omega_2\,r_1^2\right)\left(\frac{d}{dt}\,r_3\right)+\omega_1\,\omega_3\,r_3^3+\left(-2\,\omega_3\left(\frac{d}{dt}\,r_2\right)+\omega_1\,\omega_2\,r_2+\left(-\omega_3^2-\omega_2^2\right)r_1\right)r_3^2+\left(\omega_1\,\omega_3\,r_2^2+\omega_1\,\omega_3\,r_1^2\right)r_3+\left(-2\,\omega_3\,r_2^2-2\,\omega_3\,r_1^2\right)\left(\frac{d}{dt}\,r_2\right)+\omega_1\,\omega_2\,r_2^3+\left(-\omega_3^2-\omega_2^2\right)r_1\,r_2^2+\omega_1\,\omega_2\,r_1^2\,r_2+\left(-\omega_3^2-\omega_2^2\right)r_1^3\right)+G\,M\,r_1\right)/\left(r_3^2+r_2^2+r_1^2\right)^{3/2}\right]$$