373(1): Comparison of Lagrangian and Spin Connection Theories of Orbital Precession.

In the spin connection theory (UFT 363):

$$\ddot{r} = \frac{r}{(1+\Omega^1{}_{01})^2}\dot{\phi}^2 - \frac{mG}{(1+\Omega^1{}_{01})^2 r^2} \quad -(1)$$

In the Lagrangian theory (UFT 372):

$$\ddot{r} = r\dot{\phi}^2 - \frac{mG}{r^2}\left(\frac{1}{\gamma}\left(1-\frac{\dot{r}^2}{c^2}\right)\right) \quad -(2)$$

If it is assumed that:

$$\Omega^1{}_{01} \ll 1 \quad -(3)$$

then eq. (1) can be written as:

$$\ddot{r} \sim r\dot{\phi}^2 - \frac{mG}{(1+\Omega^1{}_{01})^2 r^2} \quad -(4)$$

so

$$\frac{1}{(1+\Omega^1{}_{01})^2} \sim \frac{1}{\gamma}\left(1-\frac{\dot{r}^2}{c^2}\right) \quad -(5)$$

and

$$\boxed{\Omega^1{}_{01} \sim \left(\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{\dot{r}^2}{c^2}\right)\right)^{-1/2} - 1} \quad -(6)$$

For plane polar coordinates:

2)

$$v^2 = \dot{r}^2 + r^2 \dot{\phi}^2 \quad -(7)$$

Under condition (6), the fluid gravitation theory and lagrangian theory of precession are the same. Both theories are part of the ECE 2 unified field theory.

Comparison with Experimental Data.

The astronomical data can be summarized for small precessions by the empirical formula:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\phi)} \quad -(8)$$

where

$$x = 1 + \frac{3MG}{\alpha c^2} \quad -(9)$$

Here α is the half right latitude and ϵ the eccentricity. For a given orbit, both are observable. From eq. (8):

$$\frac{dr}{d\phi} = \frac{x \epsilon r^2}{\alpha} \sin(x\phi) \quad -(10)$$

in which:

$$\sin^2(x\phi) + \cos^2(x\phi) = 1 \quad -(11)$$

It is known that the eq. (8) is an approximate model of the true orbits given by integrating eqs (1) and (2). However, this method allows comparison with the well known astronomical data.

Using numerical methods, find:

$$\frac{dr}{d\phi} = \frac{dr}{dt}\frac{dt}{d\phi} = \frac{\dot{r}}{\dot{\phi}} \quad -(12)$$

from eqs. (1) and (2) using numerical methods. Eq. (12) can be compared directly with eq. (10), Q.E.D.

1) From the fluid gravitation theory, eq. (1), the experimental result is explained by the spin connection $\Omega^1{}_{01}$.

2) From the lagrangian theory, eq. (2), the experimental result is explained directly by expressing $\dot{r}$ in terms of $\dot{\phi}$:

$$r\dot{\phi}^2 = A - B\dot{r}^2 \quad -(13)$$

where

$$A = \ddot{r} + \frac{mG}{r^2\gamma} \quad -(14)$$

$$B = \frac{mG}{\gamma r^2 c^2} \quad -(15)$$

Therefore:

$$\left(\frac{\dot{\phi}}{\dot{r}}\right)^2 = \frac{1}{r}\left(\frac{A}{\dot{r}^2} - B\right) \quad -(16)$$

$$= \left(\frac{\alpha}{x\epsilon r^2 \sin(x\phi)}\right)^2$$

Q.E.D

From eq. (16) the experimentally observed x can be expressed in terms of the lagrangian theory:

$$\frac{x\epsilon r^2}{\alpha}\sin(x\phi) = \left(\frac{r}{\frac{A}{\dot{r}^2} - B}\right)^{1/2} \quad -(17)$$

so

$$\sin(x\phi) = \frac{\alpha}{x\epsilon}\left(r^3\left(\frac{A}{\dot{r}^2} - B\right)\right)^{-1/2} \quad -(18)$$

$$= \left(1 - \cos^2(x\phi)\right)^{1/2}$$

Therefore:

$$1 - \cos^2(x\phi) = \left(\frac{\alpha}{x\epsilon}\right)^2\left(r^3\left(\frac{A}{\dot{r}^2} - B\right)\right)^{-1} \quad -(19)$$

From eq. (8):

$$\cos(x\phi) = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(20)$$

so

$$\left(\frac{\alpha}{x\epsilon}\right)^2 = r^3\left(\frac{A}{\dot{r}^2} - B\right)\left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right) \quad -(21)$$

Under this condition the astronomical result is given exactly.

) A more direct comparison is obtained by using the
kinetic energies. In UFT363:

$$T = \frac{1}{2} m \left(\left(1+\Omega^1{}_{01}\right)^2 \dot{r}^2 + \dot{\phi}^2 r^2 \right) \quad -(22)$$

In UFT372:

$$\mathcal{L} = -mc^2 \left(1 - \frac{1}{c^2}\left(\dot{r}^2 + \dot{\phi}^2 r^2\right)\right)^{1/2} - U \quad -(23)$$

and

$$H = \gamma mc^2 + U \quad -(24)$$

The hamiltonian of UFT363 is

$$H_1 = \frac{1}{2} mv^2 + U \quad -(25)$$

where

$$v^2 = \left(1+\Omega^1{}_{01}\right)^2 \dot{r}^2 + \dot{\phi}^2 r^2 \quad -(26)$$

In Eq. (25),

$$T = \frac{1}{2} mv^2 \quad -(27)$$

is the non relativistic kinetic energy corrected with the
spin connection. In eq. (24):

$$E = \gamma mc^2 = T_{rel} + E_0 \quad -(28)$$
$$= (\gamma - 1) mc^2 + mc^2$$

where the relativistic kinetic energy is:

$$T = (\gamma - 1) mc^2 \quad -(29)$$

and where the rest energy is:

$$E_0 = mc^2 \quad -(30)$$

The hamiltonian (24) is therefore:

$$H = (\gamma - 1)mc^2 + mc^2 + U \quad -(31)$$

which is the relativistic hamiltonian, the eq. (25)

a non relativistic hamiltonian modified by Ω'_{01}.

In order to make a meaningful comparison, define:

$$H_0 = H - mc^2 = (\gamma - 1)mc^2 + U \quad -(32)$$

and compare with:

$$H_1 = \frac{1}{2}mv^2 + U \quad -(33)$$

where v^2 is given by Eq. (26). Take the non-relativistic limit of T_{rel}, i.e.:

$$T_{rel} \xrightarrow[v^2 \ll c^2]{} mc^2\left(\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1\right) \quad -(34)$$

$$= mc^2\left(1 + \frac{v^2}{2c^2} + \frac{3}{8}\left(\frac{v}{c}\right)^4 + \frac{5}{16}\left(\frac{v}{c}\right)^6\right) - mc^2$$

$$= \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \frac{5}{16}m\frac{v^6}{c^4} + \cdots$$

So

$$H_0 \rightarrow \frac{1}{2}mv^2 + m\left(\frac{3v^4}{8c^2} + \frac{5v^6}{16c^4} + \cdots\right) + U \quad -(35)$$

It is now possible to make a meaningful comparison of eqs. (33) and (35) to give:

$$\frac{1}{2}m\left(\left(1+\Omega'_{01}\right)^2\dot{r}^2+\dot{\phi}^2r^2\right)$$
$$=\frac{1}{2}m\left(\dot{r}^2+\dot{\phi}^2r^2\right)+\frac{3}{8}\frac{m}{c^2}\left(\dot{r}^2+\dot{\phi}^2r^2\right)^2$$
$$+\ \ldots \quad -(36)$$

so:
$$\left(1+\Omega'_{01}\right)^2\dot{r}^2+\dot{\phi}^2r^2=\dot{r}^2+\dot{\phi}^2r^2+\frac{3}{4c^2}\left(\dot{r}^2+\dot{\phi}^2r^2\right)^2 \quad -(37)$$

i.e.
$$\left(1+\Omega'_{01}\right)^2\dot{r}^2=\dot{r}^2+\frac{3}{4}\left(\frac{\dot{r}^2+\dot{\phi}^2r^2}{c}\right)^2 \quad -(38)$$

It follows that:
$$\left(1+\Omega'_{01}\right)^2=1+\frac{3}{4}\left(\frac{\dot{r}^2+\dot{\phi}^2r^2}{c\dot{r}}\right)^2 \quad -(39)$$

The quantities on the right hand side of Eq. (39) refer to the Newtonian limit, in which:
$$v^2=MG\left(\frac{2}{r}-\frac{1}{a}\right) \quad -(40)$$
$$=\dot{r}^2+\dot{\phi}^2r^2$$

where a is the semi major axis of the ellipse:
$$r=\frac{\alpha}{1+\epsilon\cos\phi} \quad -(41)$$

Here:
$$\dot{r}=\frac{dr}{dt}=\frac{dr}{d\phi}\frac{d\phi}{dt} \quad -(42)$$

) where:
$$\frac{dr}{d\phi} = \frac{\epsilon r^2}{\alpha}\sin\phi \quad -(43)$$
and
$$\frac{d\phi}{dt} = \frac{L}{mr^2} \quad -(44)$$
where the angular momentum L is defined by:
$$L^2 = m^2 MG\alpha \quad -(45)$$
Therefore:
$$\dot{r} = \frac{\epsilon r^2}{\alpha}\frac{L}{mr^2}\sin\phi = \frac{\epsilon L}{m\alpha}\sin\phi \quad -(46)$$
Here:
$$\sin\phi = \left(1 - \cos^2\phi\right)^{1/2} \quad -(47)$$
where
$$\cos\phi = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(48)$$

Therefore under these conditions the spin connection can be evaluated in terms of observable orbital parameters:
$$\left(1+\Omega^1{}_{01}\right)^2 = 1 + \frac{3}{4}\left(\frac{MG}{c\dot{r}}\left(\frac{2}{r} - \frac{1}{a}\right)\right)^2 \quad -(49)$$
where
$$\dot{r} = \frac{\epsilon L}{m\alpha}\sin\phi, \quad -(50)$$
$$L = m\left(MG\alpha\right)^{1/2} \quad -(51)$$
$$\sin\phi = \left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)^{1/2} \quad -(52)$$