373(7): Calculation of the Experimental $dr/d\phi$ at the Perihelion or Distance of Closest Approach

Experimentally, it is known that after 2π radians, the perihelion has advanced by:

$$\phi = 2\pi \left(1 + \frac{3mG}{\alpha c^2} \right) \quad - (1)$$

From the static ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos\phi} \quad - (2)$$

it follows that:

$$\frac{dr}{d\phi} = \frac{\epsilon r^2}{\alpha} \sin\phi \quad - (3)$$

The perihelion, or distance of closest approach of m to M is defined by

$$\phi = 2\pi \quad - (4)$$

$$r = \frac{\alpha}{1 + \epsilon} \quad - (5)$$

i.e.

and

$$\frac{dr}{d\phi} = 0. \quad - (6)$$

However, if

$$\phi = 2\pi \left(1 + \frac{3mG}{\alpha c^2} \right) \quad - (7)$$

then

$$\frac{dr}{d\phi} = \frac{\epsilon r^2}{\alpha} \sin\left(2\pi \left(1 + \frac{3mG}{\alpha c^2} \right) \right) \quad - (8)$$

$$= \frac{\epsilon \alpha}{(1+\epsilon)^2} \sin\left(2\pi \left(1 + \frac{3mG}{\alpha c^2} \right) \right) \quad - (9)$$

using eq. (5).

Now use:

2) $\sin(A+B) = \sin A \cos B + \cos A \sin B$ — (10)

So:

$$\sin\left(2\pi\left(1+\frac{3MG}{\alpha c^2}\right)\right) = \sin 2\pi \cos\left(\frac{6\pi MG}{\alpha c^2}\right) + \cos 2\pi \sin\left(\frac{6\pi MG}{\alpha c^2}\right)$$

$$= \sin\left(\frac{6\pi MG}{\alpha c^2}\right) \sim \frac{6\pi MG}{\alpha c^2} \quad -(11)$$

using the small angle formula:

$$\sin x \sim x \quad -(12)$$

Therefore at the perihelion, the precession produces the change:

$$\Delta\left(\frac{dr}{d\phi}\right) = \frac{6\pi MG}{c^2}\frac{\epsilon}{(1+\epsilon)^2} \quad -(13)$$

in the differential function $dr/d\phi$. Using:

$$r_0 = \frac{2MG}{c^2} = 2{,}950 \text{ m} \quad -(14)$$

and the Earth's eccentricity of:

$$\epsilon = 0.0167 \quad -(15)$$

it is found that the experimental change in $dr/d\phi$ is:

$$\boxed{\Delta\left(\frac{dr}{d\phi}\right) = 299.46 \text{ metres}} \quad (16)$$

This change is produced by ECE2 relativity.