374(1): Orbital Precession in an Incompressible Fluid.

In the notation of UFT363 the relevant lagrangian is

$$\mathcal{L} = \frac{1}{2} m \left(\left(1 + \frac{\partial R_r}{\partial r} \right)^2 \dot{r}^2 + \dot{\theta}^2 r^2 \right) + \frac{mMG}{r} \quad -(1)$$

where

$$\Omega^1{}_{01} = \frac{\partial R_r}{\partial r} . \quad -(2)$$

The Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(3)$$

and

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(4)$$

The hamiltonian is:

$$H = \text{constant} = \frac{1}{2} m \left(\left(1 + \frac{\partial R_r}{\partial r} \right)^2 \dot{r}^2 + \dot{\theta}^2 r^2 \right) - \frac{mMG}{r} \quad -(5)$$

The functions r, θ, $\dot{r}$ and $\dot{\theta}$ can be found from eqs. (3) and (4), and can be used in eq. (5) to find $\partial R_r / \partial r$ in terms of the constant H. The differential orbital function is

$$\frac{dr}{d\theta} = \frac{\dot{r}}{\dot{\theta}} \quad -(6)$$

2) and the orbit is:

$$r = \int \frac{dr}{d\theta} \, d\theta \quad -(7)$$

In recent numerical work by co author Horst Eckardt it has been found that the orbit is a precessing orbit.

If the spacetime is assumed to be an incompressible fluid:

$$\underline{\nabla} \cdot \underline{v} = 0 \quad -(8)$$

and

$$\frac{\partial \underline{\Omega}}{\partial t} + \underline{\nabla} \times (\underline{\Omega} \times \underline{v}) = \underline{0} \quad -(9)$$

where the vorticity is:

$$\underline{\Omega} = \underline{\nabla} \times \underline{v} \quad -(10)$$

In plane polar coordinates:

$$\underline{v} = v_r \, \underline{e}_r + v_\theta \, \underline{e}_\theta \quad -(11)$$

and

$$\underline{\nabla} \cdot \underline{v} = \frac{1}{r} \left(\frac{\partial (r v_r)}{\partial r} + \frac{\partial v_\theta}{\partial \theta} \right)$$

$$= \frac{1}{r} \left(v_r + \dot{\theta} \frac{\partial r}{\partial \theta} \right) + \frac{\partial v_r}{\partial r} + \frac{\partial \dot{\theta}}{\partial \theta}$$

$$= \frac{1}{r} \left(\left(1 + \frac{\partial R_r}{\partial r} \right) \dot{r} + \dot{\theta} \frac{\partial r}{\partial \theta} \right) + \frac{\partial}{\partial r} \left(\left(1 + \frac{\partial R_r}{\partial r} \right) \dot{r} \right) + \frac{\partial \dot{\theta}}{\partial \theta}$$

$$= 0$$

3) in an incompressible fluid....

So eq. (11) gives another equation which can be solved simultaneously with eqs (3) and (4).

Finally:

$$\underline{\Omega} = \underline{\nabla} \times \underline{v} = \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r v_\theta\right) - \frac{\partial v_r}{\partial r}\right)\underline{k} \quad (12)$$

and eqs. (9) and (10) give another equation, so the complete solution is the simultaneous solution of Eqs. (3), (4), (8) and (9) in an incompressible fluid spacetime.

This method can be greatly developed for other types of spacetime.