Analytical calculation of precession from the ECE2 relativity

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3 Computer algebra and graphical results

We present some methods of showing the relativistic shifts of the orbital ellipse. First we want to get an impression on the Newtonian orbit $r(\phi)$ and the orbital derivatives $dr/d\phi$ and $(dr/d\phi)^2$, see Fig. 1. $r(\phi)$ oscillates between perihelion and aphelion, both derivative functions have zeros at these positions. A precession means shift of theses zero crossings. For the subsequent calculations we have to express the major axis a, the angular momentum L and the non-relativistic Hamiltonian H in terms of orbital parameters:

$$a = \frac{\alpha}{1-\epsilon^2}, \tag{45}$$

$$L = m\sqrt{\alpha M G}, \tag{46}$$

$$H = -\frac{mMG}{2a}. \tag{47}$$

In note 373(5) The function $(dr/d\phi)^2$ was separated from the Hamiltonian of Newtonian and relativistic theory. Computer algebra gives for this function from Newtonian theory:

$$\left(\frac{dr}{d\phi}\right)_N^2 = \frac{\alpha^2\,\epsilon^2 \sin(\phi)^2}{(\epsilon\cos(\phi)+1)^4}, \tag{48}$$

from the relativistic theory (with precessing orbit):

$$\left(\frac{dr}{d\phi}\right)_{rel}^2 = \frac{\alpha^2\,\epsilon^2 \sin(\phi)^2}{(\epsilon\cos(\phi)+1)^4} - \frac{3}{mc^2}\left(\frac{GM\,(\epsilon\cos(\phi)+1)}{\alpha} - \frac{GM\,(1-\epsilon^2)}{2\alpha}\right)^2. \tag{49}$$

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There is an additional term subtracted from $(dr/d\phi)^2$ of the Newtonian orbit to obtain the function for the precessing orbit. Both functions and their difference are graphed in Fig. 2. Both functions should be positive because they are squares but the relativistic function shows up negative values. There is a region of imaginary values for $dr/d\phi$ near to $\phi = 0$ and $\phi = 2\pi$. This seems not to be very satisfactory but the function for precession crosses zero at other values than for the Newtonian orbit. In so far the effect of precession is visible.

Instead of computing $(dr/d\phi)^2$ in dependence of ϕ, Eqs. (48-49) can be re-arranged to obtain the dependence of radius r:

$$\left(\frac{dr}{d\phi}\right)_N^2 = \frac{\left(\epsilon^2 - 1\right) r^4 + 2\alpha r^3}{\alpha^2} - r^2, \tag{50}$$

$$\left(\frac{dr}{d\phi}\right)_{rel}^2 = \frac{c^2}{GM\alpha} \left(1 - \frac{1}{\left(\frac{1}{mc^2}\left(\frac{GMm}{r} - \frac{GM\,(1-\epsilon^2)m}{2\alpha}\right) + 1\right)^2} \right) r^4 - r^2. \tag{51}$$

Then the results of Fig. 3 are obtained. It can be seen that both the perihelion and aphelion (represented by zero crossings) are shifted by relativistic effects. The strength of these effects is modeled in Fig. 4 by using different values of c. It can clearly be seen that the deviation form the Newtonian orbit is increased for smaller c, i.e. stronger relativistic effects.

In note 373(6) the differencence between the Hamiltonians (1) and (3), $H_0 - H$, has been investigated. From the result an expression for $(dr/d\phi)^2$ can be computed. For this, the equation has to be resolved for v^2 first. This gives a highly complicated equation with four solutions. Two solutions are complex, one is $v = c$ and the fourth is real-valued. We used the fourth solution and inserted Eq. (5). Then a highly complicated expression for $(dr/d\phi)^2$ follows. Unfortunately it is complex. The real and imaginary part are plotted in Fig. 5. The result depends on the choice of constant H_0. With $H = -3.75$ and $H_0 = -3.70$ the real part in Fig. 5 starts to become positive at the minimum radius of the ellipse. In so far this end behaves correctly, there is no zero crossing of $(dr/d\phi)^2$ at the other end ($r \approx 2$). The impact of H_0 on the real part of the solution is graphed in Fig. 6 for three different values of H_0 with $H = -3.75$ each. Increase of H_0 leads to a shift of zero crossing to lower radii.

Alternatively we did the following: We resolved both the Newtonian and relativistic Hamiltonian (1) and (3) separately according to v^2. From H follows the non-relativistic form, Eq. (5). Equating both solutions for v^2 gives an equation for $(dr/d\phi)^2$, containing H_0 as a parameter. The calculation has the benefit of not leading to complex-valued results (although the formula is complicated). The result is plotted in Fig. 7. The positive part now is on the left hand side of the zero crossing, i.e. the relativistic effect of the aphelion is modeled. The radial range is shifted to higher radii by the relativistic effects ($H_0 > H$) but the result is less sensitive than in Fig. 6.

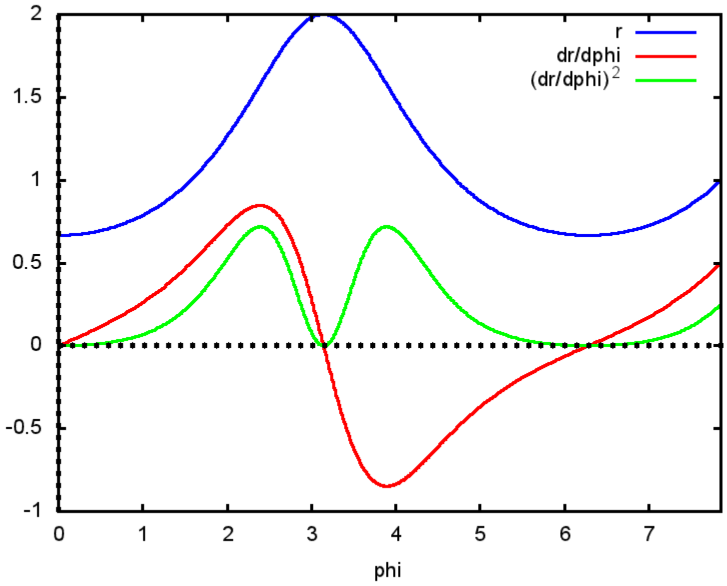

Figure 1: $r(\phi)$, $dr/d\phi$ and $(dr/d\phi)^2$ for a Newtonian elliptic orbit.

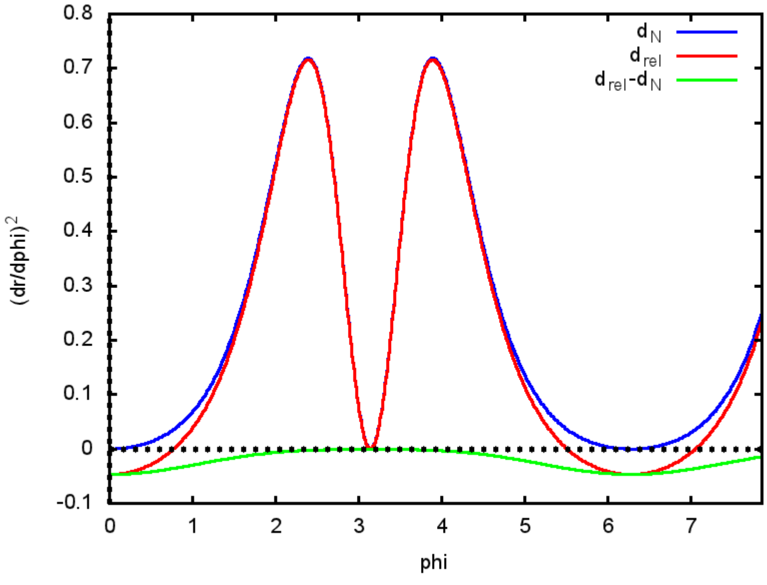

Figure 2: Angular dependence of $(dr/d\phi)^2$ from note 373(5).

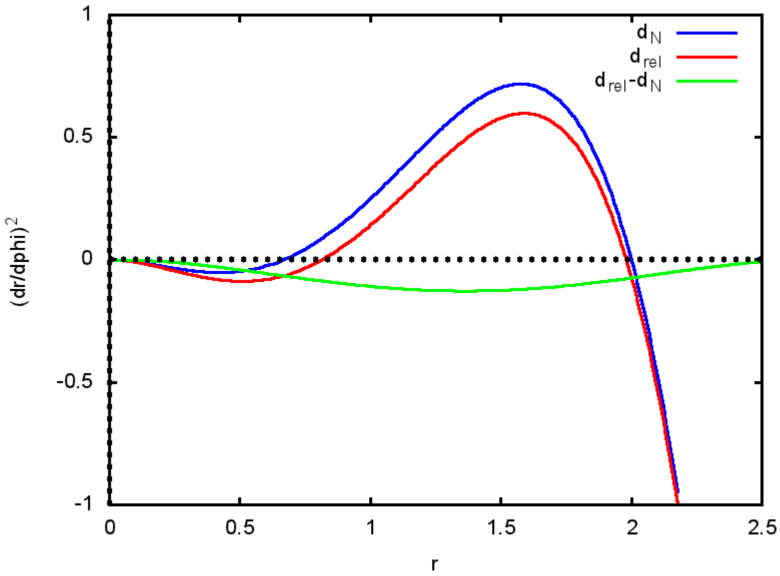

Figure 3: Radial dependence of $(dr/d\phi)^2$ from note 373(5).

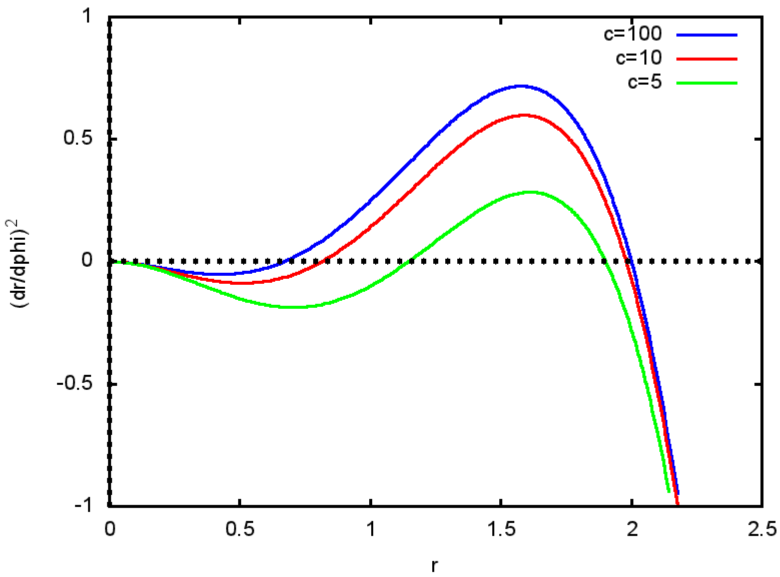

Figure 4: Strength of relativistic effects in Fig. 3, described by varying c.

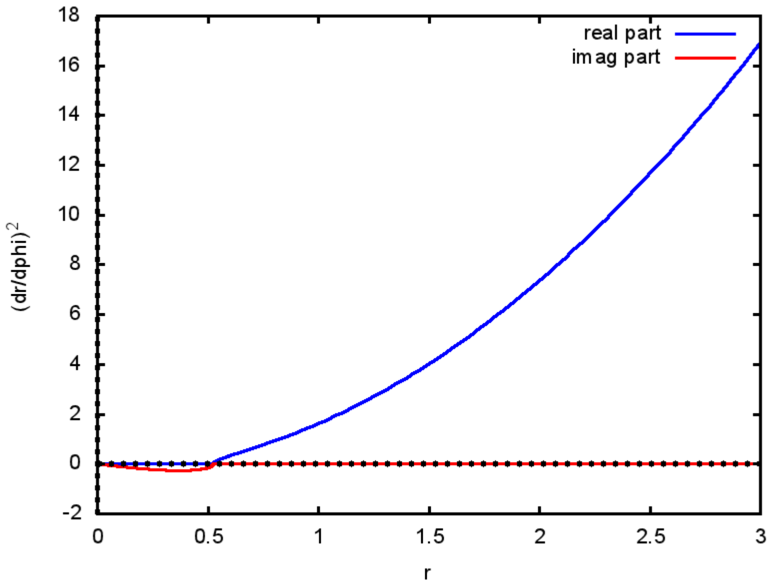

Figure 5: Real and imaginary part of $(dr/d\phi)^2$ from note 373(6).

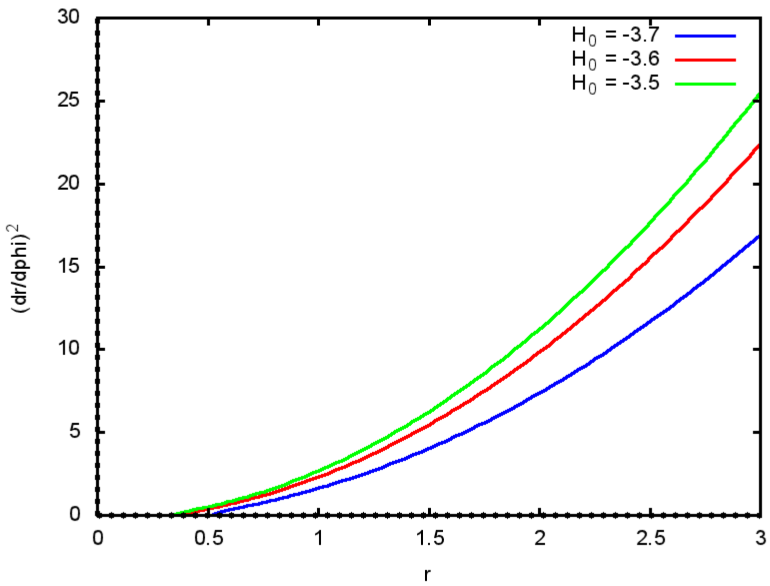

Figure 6: Strength of relativistic effects in Fig. 5, described by varying H_0.

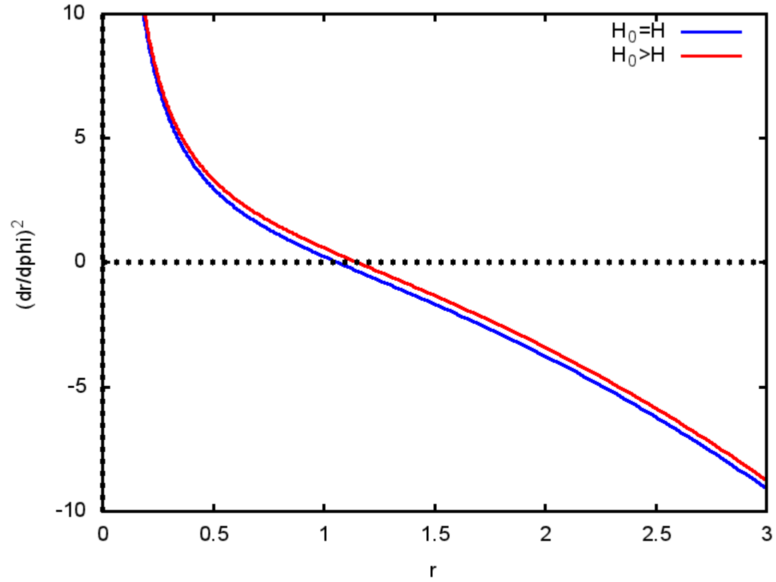

Figure 7: $(dr/d\phi)^2$ from equating the Newtonian and relativistic velocity terms.