316(1): General Theory of Precession in Fluid Gravitation.

The general force equation is:

$$\underline{F} = m\underline{a} = -mMG\frac{\underline{r}}{r^3} \quad -(1)$$

where it has been assumed that:

$$M \gg m. \quad -(2)$$

Here, the mass m orbits a mass M. The vector distance between m and M is $\underline{r}$, and G is Newton's constant. The acceleration is defined by the convective derivative:

$$\underline{a} = \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\underline{v} \quad -(3)$$

In gravitational theory:

$$\underline{a} = \underline{g} \quad -(4)$$

so

$$\underline{F} = m\underline{g} = m\left(\frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\underline{v}\right) = -mMG\frac{\underline{r}}{r^3} \quad -(5)$$

The relevant gravitational field equation is:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m = \underline{\kappa} \cdot \underline{g} \quad -(6)$$

where ρ_m is the mass density and:

$$\underline{\kappa} = 2\left(\frac{1}{r^{(0)}}\underline{q} - \underline{\omega}\right) \quad -(7)$$

2) where $\underline{q}$ is the tetrad vector and $\underline{\omega}$ is the spin connection vector of ECE2 theory.

Therefore eqs. (1) and (6) can be solved simultaneously to give various types of precession.

In the absence of a gravitational monopole the complete set of field equations is:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad - (8)$$

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad - (9)$$

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m = \underline{\kappa} \cdot \underline{g} \quad - (10)$$

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2} \frac{\partial \underline{g}}{\partial t} = \frac{4\pi G}{c^2} \underline{J}_m = \underline{\kappa} \times \underline{\Omega} \quad - (11)$$

So in general Eqs (5) and (8) - (11) must be solved simultaneously. Here $\underline{\Omega}$ is the gravitomagnetic field. For "gravitostatics", akin to electrostatics, it is sufficient to solve eqns. (5) and (10) by computer.

In Cartesian coordinates, for a planar orbit:

$$\underline{g} = \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \underline{v} \quad - (12)$$

$$= [illegible]$$

where

$$\underline{v} = \frac{\partial \underline{R}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \underline{R} \quad - (13)$$

3) where

$$\underline{R} = \underline{R}\left(x(t), y(t), t\right) \quad -(14)$$

and

$$\underline{v} = \underline{v}\left(x(t), y(t), t\right) \quad -(15)$$

So:

$$\underline{v} = v_x \underline{i} + v_y \underline{j} \quad -(16)$$

$$= \dot{R}_x \underline{i} + \dot{R}_y \underline{j} + $$

$$+ \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y}\right)\left(R_x \underline{i} + R_y \underline{j}\right)$$

$$= \left(\dot{R}_x + v_x \frac{\partial R_x}{\partial x} + v_y \frac{\partial R_x}{\partial y}\right)\underline{i} \quad -(17)$$

$$+ \left(\dot{R}_y + v_x \frac{\partial R_y}{\partial x} + v_y \frac{\partial R_y}{\partial y}\right)\underline{j}$$

i.e.

$$v_x = \dot{R}_x + v_x \frac{\partial R_x}{\partial x} + v_y \frac{\partial R_x}{\partial y} \quad -(18)$$

$$v_y = \dot{R}_y + v_x \frac{\partial R_y}{\partial x} + v_y \frac{\partial R_y}{\partial y} \quad -(19)$$

These equations can be solved simultaneously for v_x and v_y.

In Cartesian coordinates the acceleration due to gravity is:

$$\underline{g} = \frac{\partial \underline{v}}{\partial t} + \left(\underline{v} \cdot \underline{\nabla}\right)\underline{v} \quad -(20)$$

$$= \dot{v}_x \underline{i} + \dot{v}_y \underline{j} + \left(v_x \frac{\partial}{\partial X} + v_y \frac{\partial}{\partial Y} \right) \left(v_x \underline{i} + v_y \underline{j} \right)$$

$$= \left(\dot{v}_x + v_x \frac{\partial v_x}{\partial X} + v_y \frac{\partial v_x}{\partial Y} \right) \underline{i} \quad -(21)$$

$$+ \left(\dot{v}_y + v_y \frac{\partial v_x}{\partial Y} + v_y \frac{\partial v_y}{\partial Y} \right) \underline{j}$$

So:

$$g_x = \dot{v}_x + v_x \frac{\partial v_x}{\partial X} + v_y \frac{\partial v_x}{\partial Y} \quad -(21)$$

$$g_y = \dot{v}_y + v_y \frac{\partial v_x}{\partial Y} + v_y \frac{\partial v_y}{\partial Y} \quad -(22)$$

where v_x and v_y are given by eqs. (18) and (19). The force equation (1) gives:

$$g_x = -mMG \frac{R_x}{R^3} \quad -(23)$$

$$g_y = -mMG \frac{R_y}{R^3} \quad -(24)$$

where

$$R^3 = \left(R_x^2 + R_y^2 \right)^{3/2} \quad -(25)$$

From Eq. (7):

$$\underline{\kappa} = \kappa_x \underline{i} + \kappa_y \underline{j} \quad -(26)$$

$$= 2\left(\left(\frac{v_x}{r^{(0)}} \underline{i} + \frac{v_y}{r^{(0)}} \underline{j} \right) - \omega_x \underline{i} + \omega_y \underline{j} \right)$$

5) Let:

$$a_x = 2\left(\frac{q_x}{r^{(0)}} - \omega_x\right) \quad -(27)$$

$$a_y = 2\left(\frac{q_y}{r^{(0)}} - \omega_y\right) \quad -(28)$$

then

$$\underline{\nabla} \cdot \underline{g} = a_x g_x + a_y g_y = 4\pi G \rho_m \quad -(29)$$

$$-(29)$$

Therefore eqs. (18), (19), (21), (22) and (29) can be solved simultaneously for R_x and R_y and $\dot{R}_x$ and $\dot{R}_y$. The input parameters are a_x and a_y.

Newtonian Limit

Eq. (1) gives:

$$\ddot{X} = -mMG\frac{X}{(X^2+Y^2)^{3/2}} \quad -(30)$$

$$\ddot{Y} = -mMG\frac{Y}{(X^2+Y^2)^{3/2}} \quad -(31)$$

Eq. (6) gives:

$$\frac{\partial \ddot{X}}{\partial X} + \frac{\partial \ddot{Y}}{\partial Y} = a_x \ddot{X} + a_y \ddot{Y} \quad -(32)$$

In the Newtonian limit:

6)

$$\underline{R}\,(X(t), Y(t), t) \longrightarrow \underline{r}(t) \quad -(33)$$

$$\underline{v}\,(X(t), Y(t), t) \longrightarrow \underline{v}(t) \quad -(34)$$

Eqs. (30), (31) and (32), when solved simultaneously, may give precession. Choice of a_x and a_y may give retrograde precession. The masses m and M may be chosen for the S2 star system.

It is known from previous work that simultaneous solution of Eqs. (30) and (31) produces an ellipse. The additional consideration of Eq. (32) may be enough to produce a precessing ellipse. Here:

$$\frac{\partial \ddot{X}}{\partial X} + \frac{\partial \ddot{Y}}{\partial Y} = a_x \ddot{X} + a_y \ddot{Y} = 4\pi G \rho_m \quad -(33)$$