376(2): Numerical Solution of the ECE2 Field Equations of Gravitation.

In the notation of UFT318 the field equations are:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad -(1)$$

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad -(2)$$

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(3)$$

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2}\frac{\partial \underline{g}}{\partial t} = \underline{\kappa} \times \underline{\Omega} = \frac{4\pi G}{c^2} \underline{J}_m \quad -(4)$$

From eqs. (3) and (4) the continuity equation follows:

$$\frac{\partial \rho_m}{\partial t} + \underline{\nabla} \cdot \underline{J}_m = 0 \quad -(5)$$

i.e.

$$\frac{1}{c^2}\frac{\partial}{\partial t}(\underline{\kappa} \cdot \underline{g}) + \underline{\nabla} \cdot (\underline{\kappa} \times \underline{\Omega}) = 0 \quad -(6)$$

In general:

$$\underline{g} = g_X \underline{i} + g_Y \underline{j} + g_Z \underline{k} \quad -(7)$$

$$\underline{\Omega} = \Omega_X \underline{i} + \Omega_Y \underline{j} + \Omega_Z \underline{k} \quad -(8)$$

$$\underline{\kappa} = \kappa_X \underline{i} + \kappa_Y \underline{j} + \kappa_Z \underline{k} \quad -(9)$$

so there are nine unknowns, the components of $\underline{g}$, $\underline{\Omega}$ and $\underline{\kappa}$.

There are also nine equations as follows:

$$\frac{\partial \Omega_X}{\partial X} + \frac{\partial \Omega_Y}{\partial Y} + \frac{\partial \Omega_Z}{\partial Z} = 0 \quad -(10)$$

$$\frac{\partial g_Z}{\partial Y} - \frac{\partial g_Y}{\partial Z} = \frac{\partial \Omega_X}{\partial t} \quad -(11)$$

2)

$$\frac{\partial g_X}{\partial Z} - \frac{\partial g_Z}{\partial X} = \frac{\partial \Omega_Y}{\partial t} \quad -(12)$$

$$\frac{\partial g_Y}{\partial X} - \frac{\partial g_X}{\partial Y} = \frac{\partial \Omega_Z}{\partial t} \quad -(13)$$

$$\frac{\partial g_X}{\partial X} + \frac{\partial g_Y}{\partial Y} + \frac{\partial g_Z}{\partial Z} = \kappa_X g_X + \kappa_Y g_Y + \kappa_Z g_Z$$

$$= 4\pi G \rho_m \quad -(14)$$

$$\frac{\partial \Omega_Z}{\partial Y} - \frac{\partial \Omega_Y}{\partial Z} - \frac{1}{c^2}\frac{\partial g_X}{\partial t} = \kappa_Y \Omega_Z - \kappa_Z \Omega_Y = \frac{4\pi G}{c^2} J_{mX}$$

$$-(15)$$

$$\frac{\partial \Omega_X}{\partial Z} - \frac{\partial \Omega_Z}{\partial X} - \frac{1}{c^2}\frac{\partial g_Y}{\partial t} = \kappa_Z \Omega_X - \kappa_X \Omega_Z = \frac{4\pi G}{c^2} J_{mY}$$

$$-(16)$$

$$\frac{\partial \Omega_Y}{\partial X} - \frac{\partial \Omega_X}{\partial Y} - \frac{1}{c^2}\frac{\partial g_X}{\partial t} = \kappa_X \Omega_Y - \kappa_Y \Omega_X = \frac{4\pi G}{c^2} J_{mZ}$$

$$-(17)$$

and the continuity equation (6) gives:

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left(\kappa_X g_X + \kappa_Y g_Y + \kappa_Z g_Z\right)$$

$$= \frac{\partial}{\partial X}\left(\kappa_Y \Omega_Z - \kappa_Z \Omega_Y\right) + \frac{\partial}{\partial Y}\left(\kappa_Z \Omega_X - \kappa_X \Omega_Z\right)$$

$$+ \frac{\partial}{\partial Z}\left(\kappa_X \Omega_Y - \kappa_Y \Omega_X\right) \quad -(18)$$

So there are nine equations in nine unknowns and the system is exactly determined.

3) Gravito Statics

The equations of gravitostatics are:

$$\underline{\nabla} \times \underline{g} = \underline{0} \quad -(19)$$

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(20)$$

$$\frac{\partial \underline{g}}{\partial t} = \underline{0} \quad -(21)$$

and the continuity equation:

$$\frac{\partial}{\partial t} (\underline{\kappa} \cdot \underline{g}) = 0 \quad -(22)$$

So there are six equations in six unknowns, the components of $\underline{g}$ and $\underline{\kappa}$.

These equations are ECE2 invariant and relativistic. In the non relativistic Newtonian limit:

$$\underline{g} = - MG \frac{\underline{r}}{r^3} \quad -(23)$$

In fluid gravitation:

$$\underline{g} = \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \underline{v} \quad -(24)$$

$$\underline{v} = \frac{\partial \underline{R}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \underline{R} \quad -(25)$$

where

$$\underline{v} = \underline{v}(R(x), R(y), t) \quad -(26)$$

$$= v_x \underline{i} + v_y \underline{j} + v_z \underline{k}$$

$$\underline{R} = \underline{R}(x(t), y(t), t) \quad -(27)$$

$$= R_x \underline{i} + R_y \underline{j} + R_z \underline{k}$$

4) The relativistic hamiltonian is:

$$H = \gamma m c^2 + U \quad -(28)$$

and the relativistic lagrangian is:

$$\mathcal{L} = -\frac{mc^2}{\gamma} + U \quad -(29)$$

where

$$\gamma = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} \quad -(30)$$

In classical one particle dynamics:

$$\underline{g} = \frac{d\underline{v}_0}{dt} = \frac{d^2\underline{r}}{dt^2}. \quad -(31)$$

in the non relativistic limit. The ECE2 covariant relativistic velocity is:

$$\underline{v} = \gamma \underline{v}_0 \quad -(32)$$

$$= \gamma \frac{d\underline{r}}{dt} = \frac{d\underline{r}}{d\tau}$$

and the relativistic $\underline{g}$ is:

$$\underline{g} = \frac{d\underline{v}}{d\tau} = \gamma^4 \frac{d^2\underline{r}}{dt^2} \quad -(33)$$

The Minkowski force equation is:

$$\underline{F} = m\underline{g} = \gamma^4 \frac{d^2\underline{r}}{dt^2} \quad -(34)$$

Therefore it is possible to write the relativistic orbit as the solution of:

5)

$$\underline{F} = m\gamma^4 \frac{d^2 \underline{r}}{dt^2} = -mMG\frac{\underline{r}}{r^3} \quad -(35)$$

In Cartesian Coordinates:

$$\left(1 - \frac{v_0^2}{c^2}\right)^{-2} \left(\ddot{X}\underline{i} + \ddot{Y}\underline{j}\right) = -mG\frac{(X\underline{i} + Y\underline{j})}{(X^2+Y^2)^{3/2}} \quad -(36)$$

So:

$$\ddot{X} = -mG\left(1 - \frac{\dot{X}^2 + \dot{Y}^2}{c^2}\right)^2 \frac{X}{(X^2+Y^2)^{3/2}} \quad -(37)$$

$$\ddot{Y} = -mG\left(1 - \frac{\dot{X}^2 + \dot{Y}^2}{c^2}\right)^2 \frac{Y}{(X^2+Y^2)^{3/2}} \quad -(38)$$

Eqs. (37) and (38) should produce precession.

Magneto Gravito Statics

The equations are:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad -(39)$$

$$\frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad -(40)$$

$$\underline{\nabla} \times \underline{\Omega} = \underline{\kappa} \times \underline{\Omega} = \frac{4\pi G}{c^2} \underline{J}_m \quad -(41)$$

and from the continuity equation:

$$\underline{\nabla} \cdot (\underline{\kappa} \times \underline{\Omega}) = 0 \quad -(42)$$

So there are six equations in six unknowns, the components of $\underline{\kappa}$ and $\underline{\Omega}$.