```
(%i1)  kill(all);
(%o0)  done
```

1 Definitions

```
(%i1)  cross(a,b) :=  [a[2]*b[3] - a[3]*b[2],
                       a[3]*b[1] - a[1]*b[3],
                       a[1]*b[2] - a[2]*b[1]];
```

$$(\%o1)\quad \mathrm{cross}(a,b):=[a_2\,b_3-a_3\,b_2,\ a_3\,b_1-a_1\,b_3,\ a_1\,b_2-a_2\,b_1]$$

```
(%i2)  div(a) := diff(a[1],x) + diff(a[2],y) + diff(a[3],z);
```

$$(\%o2)\quad \mathrm{div}(a):=\frac{\mathrm{d}}{\mathrm{d}\,x}\,a_1+\frac{\mathrm{d}}{\mathrm{d}\,y}\,a_2+\frac{\mathrm{d}}{\mathrm{d}\,z}\,a_3$$

```
(%i3)  curl(a) := [diff(a[3],y) - diff(a[2],z),
                   diff(a[1],z) - diff(a[3],x),
                   diff(a[2],x) - diff(a[1],y)];
```

$$(\%o3)\quad \mathrm{curl}(a):=[\frac{\mathrm{d}}{\mathrm{d}\,y}\,a_3-\frac{\mathrm{d}}{\mathrm{d}\,z}\,a_2,\ \frac{\mathrm{d}}{\mathrm{d}\,z}\,a_1-\frac{\mathrm{d}}{\mathrm{d}\,x}\,a_3,\ \frac{\mathrm{d}}{\mathrm{d}\,x}\,a_2-\frac{\mathrm{d}}{\mathrm{d}\,y}\,a_1]$$

```
(%i4)  curl_s(a) := [1/(r*sin(theta))*(diff(sin(theta)*a[3],theta) - diff(a
                    1/(r*sin(theta))*diff(a[1],phi) - 1/r*diff(r*a[3],r),
                    1/r*(diff(r*a[2],r) - diff(a[1],theta))];
```

$$(\%o4)\quad curl_s(a):=[\frac{1}{r\sin(\theta)}\left(\frac{\mathrm{d}}{\mathrm{d}\,\theta}\left(\sin(\theta)\,a_3\right)-\frac{\mathrm{d}}{\mathrm{d}\,\varphi}\,a_2\right),\ \frac{1}{r\sin(\theta)}\left(\frac{\mathrm{d}}{\mathrm{d}\,\varphi}\,a_1\right)-\frac{1}{r}\left(\frac{\mathrm{d}}{\mathrm{d}\,r}\left(r\,a_3\right)\right),\ \frac{1}{r}\left(\frac{\mathrm{d}}{\mathrm{d}\,r}\left(r\,a_2\right)-\frac{\mathrm{d}}{\mathrm{d}\,\theta}\,a_1\right)]$$

```
(%i5)  /* transform vector from spherical to cartesian coordinates */
       Transform_s_c(V) := block([S_s_cart,r,theta,phi,VT],
           S_s_cart: matrix([sin(theta)*cos(phi), cos(theta)*cos(phi), -sin
       [sin(theta)*sin(phi), cos(theta)*sin(phi), cos(phi)],
       [cos(theta), -sin(theta), 0]),
           r: sqrt(x^2+y^2+z^2),
           phi: atan2(y,x),
           theta: acos(z/sqrt(x^2+y^2+z^2)),
           VT: (factor(ev(S_s_cart.V))),
           [VT[1,1], VT[2,1], VT[3,1]]
       )$
```

2 Antisymm- Eqs

```
(%i6)  depends([A_cart], [x,y,z]);
(%o6)  [A_cart(x,y,z)]
```

```
(%i9)  E5: diff(A_cart[3],y)-omega_y*A_cart[3] = -(diff(A_cart[2],z)-omega_
       E6: diff(A_cart[1],z)-omega_z*A_cart[1] = -(diff(A_cart[3],x)-omega_
       E7: diff(A_cart[2],x)-omega_x*A_cart[2] = -(diff(A_cart[1],y)-omega_
```

$$\text{(\%o7)}\quad \frac{\mathrm{d}}{\mathrm{d}\,y}\,A_{cart_3}-A_{cart_3}\,\omega_y=A_{cart_2}\,\omega_z-\frac{\mathrm{d}}{\mathrm{d}\,z}\,A_{cart_2}$$

$$\text{(\%o8)}\quad \frac{\mathrm{d}}{\mathrm{d}\,z}\,A_{cart_1}-A_{cart_1}\,\omega_z=A_{cart_3}\,\omega_x-\frac{\mathrm{d}}{\mathrm{d}\,x}\,A_{cart_3}$$

$$\text{(\%o9)}\quad \frac{\mathrm{d}}{\mathrm{d}\,x}\,A_{cart_2}-A_{cart_2}\,\omega_x=A_{cart_1}\,\omega_y-\frac{\mathrm{d}}{\mathrm{d}\,y}\,A_{cart_1}$$

2.1 Step 1: start with A, transform to cartes. coord.

```
(%i10) assume(r>0);
```

$$\text{(\%o10)}\quad [r>0]$$

```
(%i11) A1: [0,0,(I*a^2*mu_0*r*sin(theta)*((15*a^2*r^2*sin(theta)^2)/(8*(r^2
```

$$\text{(\%o11)}\quad \left[0,0,\frac{I\,a^2\,\mu_0\,r\,\sin(\theta)\left(\frac{15\,a^2\,r^2\,\sin(\theta)^2}{8\left(r^2+a^2\right)^2}+1\right)}{4\left(r^2+a^2\right)^{3/2}}\right]$$

```
(%i12) A2: [0,0,(I*a^2*mu_0*r*sin(theta)*((15*a^2*r^2*sin(theta)^2)/(8*(r^2
```

$$\text{(\%o12)}\quad \left[0,0,\frac{I\,a^2\,\mu_0\,\sin(\theta)\left(\frac{15\,a^2\,\sin(\theta)^2}{8\,r^2}+1\right)}{4\,r^2}\right]$$

```
(%i13) A3: [0,0,(I*a^2*mu_0*r*sin(theta)*(1))/(4*(r^2)^(3/2))];
```

$$\text{(\%o13)}\quad \left[0,0,\frac{I\,a^2\,\mu_0\,\sin(\theta)}{4\,r^2}\right]$$

```
(%i14) A: A3;
```

$$\text{(\%o14)}\quad \left[0,0,\frac{I\,a^2\,\mu_0\,\sin(\theta)}{4\,r^2}\right]$$

```
(%i15) A_cart: Transform_s_c(A);
```

$$\text{(\%o15)}\quad \left[-\frac{I\,a^2\,\mu_0\,y}{4\left(z^2+y^2+x^2\right)^{3/2}},\frac{I\,a^2\,\mu_0\,x}{4\left(z^2+y^2+x^2\right)^{3/2}},0\right]$$

```
(%i18)  factor(diff(A_cart,x));
        factor(diff(A_cart,y));
        factor(diff(A_cart,z));
```

$$(\%o16)\quad \left[\frac{3\,I\,a^2\,\mu_0\,x\,y}{4\left(z^2+y^2+x^2\right)^{5/2}},\frac{I\,a^2\,\mu_0\left(z^2+y^2-2\,x^2\right)}{4\left(z^2+y^2+x^2\right)^{5/2}},0\right]$$

$$(\%o17)\quad \left[-\frac{I\,a^2\,\mu_0\left(z^2-2\,y^2+x^2\right)}{4\left(z^2+y^2+x^2\right)^{5/2}},-\frac{3\,I\,a^2\,\mu_0\,x\,y}{4\left(z^2+y^2+x^2\right)^{5/2}},0\right]$$

$$(\%o18)\quad \left[\frac{3\,I\,a^2\,\mu_0\,y\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},-\frac{3\,I\,a^2\,\mu_0\,x\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},0\right]$$

2.2 Step 2: Compute omega from antisymm. eqs.

```
(%i21)  E5a: ev(E5),diff;
        E6a: ev(E6),diff;
        E7a: ev(E7),diff;
```

$$(\%o19)\quad 0=\frac{I\,a^2\,\mu_0\,\omega_z\,x}{4\left(z^2+y^2+x^2\right)^{3/2}}+\frac{3\,I\,a^2\,\mu_0\,x\,z}{4\left(z^2+y^2+x^2\right)^{5/2}}$$

$$(\%o20)\quad \frac{I\,a^2\,\mu_0\,\omega_z\,y}{4\left(z^2+y^2+x^2\right)^{3/2}}+\frac{3\,I\,a^2\,\mu_0\,y\,z}{4\left(z^2+y^2+x^2\right)^{5/2}}=0$$

$$(\%o21)\quad -\frac{I\,a^2\,\mu_0\,\omega_x\,x}{4\left(z^2+y^2+x^2\right)^{3/2}}+\frac{I\,a^2\,\mu_0}{4\left(z^2+y^2+x^2\right)^{3/2}}-\frac{3\,I\,a^2\,\mu_0\,x^2}{4\left(z^2+y^2+x^2\right)^{5/2}}=-\frac{I\,a^2\,\mu_0\,\omega_y\,y}{4\left(z^2+y^2+x^2\right)^{3/2}}+\frac{I\,a^2\,\mu_0}{4\left(z^2+y^2+x^2\right)^{3/2}}-\frac{3\,I\,a^2\,\mu_0\,y^2}{4\left(z^2+y^2+x^2\right)^{5/2}}$$

```
(%i22)  E1: factor(solve([E5a,E6a,E7a], [omega_x, omega_y, omega_z]));
solve: dependent equations eliminated: (2)
```

$$(\%o22)\quad \left[\left[\omega_x=\frac{\%r1\,y\,z^2+\%r1\,y^3+3\,y^2+\%r1\,x^2\,y-3\,x^2}{x\left(z^2+y^2+x^2\right)},\omega_y=\%r1,\omega_z=-\frac{3\,z}{z^2+y^2+x^2}\right]\right]$$

```
(%i23)  E2: ev(E1, [%r1=0]);
```

$$(\%o23)\quad \left[\left[\omega_x=\frac{3\,y^2-3\,x^2}{x\left(z^2+y^2+x^2\right)},\omega_y=0,\omega_z=-\frac{3\,z}{z^2+y^2+x^2}\right]\right]$$

2.3 Step 3: Calculate B1 = omega x A

```
(%i24)  omega: [rhs(first(first(E2))),
                rhs(second(first(E2))),
                rhs(third(first(E2)))];
```

$$(\%o24)\quad \left[\frac{3\,y^2-3\,x^2}{x\left(z^2+y^2+x^2\right)},0,-\frac{3\,z}{z^2+y^2+x^2}\right]$$

(%i25) `B1: factor(cross(omega,A_cart));`

(%o25) $$\left[\frac{3\,I\,a^2\,\mu_0\,x\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},\frac{3\,I\,a^2\,\mu_0\,y\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},\frac{3\,I\,a^2\,\mu_0\left(y-x\right)\left(y+x\right)}{4\left(z^2+y^2+x^2\right)^{5/2}}\right]$$

2.4 Step 4: Calculate B - B1

B given in spherical coordinates

(%i26) `B: mu_0*I*a^2* [1/2*cos(theta)/r^3, 1/4*sin(theta)/r^3,0];`

(%o26) $$\left[\frac{I\,a^2\,\mu_0\cos\left(\theta\right)}{2\,r^3},\frac{I\,a^2\,\mu_0\sin\left(\theta\right)}{4\,r^3},0\right]$$

(%i27) `B_cart: Transform_s_c(B);`

(%o27) $$\left[\frac{3\,I\,a^2\,\mu_0\,x\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},\frac{3\,I\,a^2\,\mu_0\,y\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},\frac{I\,a^2\,\mu_0\left(2\,z^2-y^2-x^2\right)}{4\left(z^2+y^2+x^2\right)^{5/2}}\right]$$

Check from B=curl A

(%i28) `B_check: factor(curl(A_cart));`

(%o28) $$\left[\frac{3\,I\,a^2\,\mu_0\,x\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},\frac{3\,I\,a^2\,\mu_0\,y\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},\frac{I\,a^2\,\mu_0\left(2\,z^2-y^2-x^2\right)}{4\left(z^2+y^2+x^2\right)^{5/2}}\right]$$

(%i29) `B_cart-B_check;`

(%o29) $$[0,0,0]$$

(%i30) `B1;`

(%o30) $$\left[\frac{3\,I\,a^2\,\mu_0\,x\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},\frac{3\,I\,a^2\,\mu_0\,y\,z}{4\left(z^2+y^2+x^2\right)^{5/2}},\frac{3\,I\,a^2\,\mu_0\left(y-x\right)\left(y+x\right)}{4\left(z^2+y^2+x^2\right)^{5/2}}\right]$$

(%i31) `factor(B_cart-B1);`

(%o31) $$\left[0,0,\frac{I\,a^2\,\mu_0\left(z^2-2\,y^2+x^2\right)}{2\left(z^2+y^2+x^2\right)^{5/2}}\right]$$