```
(%i1) kill(all);
(%o0) done
```

1 Definitions

```
(%i1) cross(a,b) := [a[2]*b[3] - a[3]*b[2],
                     a[3]*b[1] - a[1]*b[3],
                     a[1]*b[2] - a[2]*b[1]];
```

(%o1) $\mathrm{cross}(a,b):=[a_2\,b_3-a_3\,b_2,\ a_3\,b_1-a_1\,b_3,\ a_1\,b_2-a_2\,b_1]$

```
(%i2) div(a) := diff(a[1],x) + diff(a[2],y) + diff(a[3],z);
```

(%o2) $\mathrm{div}(a):=\frac{d}{d\,x}\,a_1+\frac{d}{d\,y}\,a_2+\frac{d}{d\,z}\,a_3$

```
(%i3) curl(a) := [diff(a[3],y) - diff(a[2],z),
                  diff(a[1],z) - diff(a[3],x),
                  diff(a[2],x) - diff(a[1],y)];
```

(%o3) $\mathrm{curl}(a):=[\frac{d}{d\,y}\,a_3-\frac{d}{d\,z}\,a_2,\ \frac{d}{d\,z}\,a_1-\frac{d}{d\,x}\,a_3,\ \frac{d}{d\,x}\,a_2-\frac{d}{d\,y}\,a_1]$

```
(%i4) curl_s(a) := [1/(r*sin(theta))*(diff(sin(theta)*a[3],theta) - diff(a
                   1/(r*sin(theta))*diff(a[1],phi) - 1/r*diff(r*a[3],r),
                   1/r*(diff(r*a[2],r) - diff(a[1],theta))];
```

(%o4) $curl_s(a):=[\frac{1}{r\,\sin(\theta)}\left(\frac{d}{d\,\theta}\left(\sin(\theta)\,a_3\right)-\frac{d}{d\,\varphi}\,a_2\right),\ \frac{1}{r\,\sin(\theta)}\left(\frac{d}{d\,\varphi}\,a_1\right)-\frac{1}{r}\left(\frac{d}{d\,r}\left(r\,a_3\right)\right),\ \frac{1}{r}\left(\frac{d}{d\,r}\left(r\,a_2\right)-\frac{d}{d\,\theta}\,a_1\right)]$

```
(%i5) /* transform vector from spherical to cartesian coordinates */
      Transform_s_c(V) := block([S_s_cart,r,theta,phi,VT],
          S_s_cart: matrix([sin(theta)*cos(phi), cos(theta)*cos(phi), -sin
      [sin(theta)*sin(phi), cos(theta)*sin(phi), cos(phi)],
      [cos(theta), -sin(theta), 0]),
          r: sqrt(x^2+y^2+z^2),
          phi: atan2(y,x),
          theta: acos(z/sqrt(x^2+y^2+z^2)),
          VT: (factor(ev(S_s_cart.V))),
          [VT[1,1], VT[2,1], VT[3,1]]
      )$
```

2 Antisymm- Eqs

```
(%i6) depends([A_cart], [x,y,z]);
```

(%o6) $[A_{cart}(x,y,z)]$

```
(%i9)  E5: diff(A_cart[3],y)-omega_y*A_cart[3] = -(diff(A_cart[2],z)-omega_
       E6: diff(A_cart[1],z)-omega_z*A_cart[1] = -(diff(A_cart[3],x)-omega_
       E7: diff(A_cart[2],x)-omega_x*A_cart[2] = -(diff(A_cart[1],y)-omega_
```

(%o7) $\frac{d}{d\,y}\,A_{cart_3} - A_{cart_3}\,\omega_y = A_{cart_2}\,\omega_z - \frac{d}{d\,z}\,A_{cart_2}$

(%o8) $\frac{d}{d\,z}\,A_{cart_1} - A_{cart_1}\,\omega_z = A_{cart_3}\,\omega_x - \frac{d}{d\,x}\,A_{cart_3}$

(%o9) $\frac{d}{d\,x}\,A_{cart_2} - A_{cart_2}\,\omega_x = A_{cart_1}\,\omega_y - \frac{d}{d\,y}\,A_{cart_1}$

2.1 Step 1: start with A

Def.

```
(%i10) A_cart: -B_0/2*[-y, x, 2*z^3/(x*y)];
```

(%o10) $[\frac{B_0\,y}{2}, -\frac{B_0\,x}{2}, -\frac{B_0\,z^3}{x\,y}]$

```
(%i13) factor(diff(A_cart,x));
       factor(diff(A_cart,y));
       factor(diff(A_cart,z));
```

(%o11) $[0, -\frac{B_0}{2}, \frac{B_0\,z^3}{x^2\,y}]$

(%o12) $[\frac{B_0}{2}, 0, \frac{B_0\,z^3}{x\,y^2}]$

(%o13) $[0, 0, -\frac{3\,B_0\,z^2}{x\,y}]$

2.2 Step 2: Compute omega from antisymm. eqs.

```
(%i16) E5a: ev(E5),diff;
       E6a: ev(E6),diff;
       E7a: ev(E7),diff;
```

(%o14) $\frac{B_0\,\omega_y\,z^3}{x\,y} + \frac{B_0\,z^3}{x\,y^2} = -\frac{B_0\,\omega_z\,x}{2}$

(%o15) $-\frac{B_0\,\omega_z\,y}{2} = -\frac{B_0\,\omega_x\,z^3}{x\,y} - \frac{B_0\,z^3}{x^2\,y}$

(%o16) $\frac{B_0\,\omega_x\,x}{2} - \frac{B_0}{2} = \frac{B_0\,\omega_y\,y}{2} - \frac{B_0}{2}$

```
(%i17) E1: solve([E5a,E6a,E7a],[omega_x, omega_y, omega_z]);
```

(%o17) $[[\omega_x = -\frac{1}{x}, \omega_y = -\frac{1}{y}, \omega_z = 0]]$

(%i18)
```
omega: [rhs(first(first(E1))),
rhs(second(first(E1))),
rhs(third(first(E1)))];
```
(%o18) $[-\frac{1}{x}, -\frac{1}{y}, 0]$

(%i19)
```
curl(A_cart);
```
(%o19) $[\frac{B_0 z^3}{x y^2}, -\frac{B_0 z^3}{x^2 y}, -B_0]$

2.3 Step 3: Calculate B2 = omega x A

(%i20)
```
B2: cross(omega,A_cart);
```
(%o20) $[\frac{B_0 z^3}{x y^2}, -\frac{B_0 z^3}{x^2 y}, B_0]$

2.4 Step 4: Calculate B1 - B2

B given in spherical coordinates

(%i21)
```
B1: curl(A_cart);
```
(%o21) $[\frac{B_0 z^3}{x y^2}, -\frac{B_0 z^3}{x^2 y}, -B_0]$

Check from B=curl A

(%i22)
```
B1-B2;
```
(%o22) $[0, 0, -2 B_0]$

div checks

(%i23)
```
div(B);
```
(%o23) 0

(%i24)
```
factor(div(B1));
```
(%o24) 0

(%i25)
```
div(A_cart);
```
(%o25) $-\frac{3 B_0 z^2}{x y}$

(%i26)
```
factor(div(omega));
```
(%o26) $\frac{y^2+x^2}{x^2 y^2}$