

```
(%i1) kill(all);
(%o0) done
```

## 1 Definitions

```
(%i1) cross(a,b) := [a[2]*b[3] - a[3]*b[2],
                    a[3]*b[1] - a[1]*b[3],
                    a[1]*b[2] - a[2]*b[1]];
(%o1) cross(a,b) := [a2 b3 - a3 b2, a3 b1 - a1 b3, a1 b2 - a2 b1]
```

```
(%i2) div(a) := diff(a[1],x) + diff(a[2],y) + diff(a[3],z);
(%o2) div(a) :=  $\frac{d}{dx} a_1 + \frac{d}{dy} a_2 + \frac{d}{dz} a_3$ 
```

```
(%i3) curl(a) := [diff(a[3],y) - diff(a[2],z),
                  diff(a[1],z) - diff(a[3],x),
                  diff(a[2],x) - diff(a[1],y)];
(%o3) curl(a) := [ $\frac{d}{dy} a_3 - \frac{d}{dz} a_2$ ,  $\frac{d}{dz} a_1 - \frac{d}{dx} a_3$ ,  $\frac{d}{dx} a_2 - \frac{d}{dy} a_1$ ]
```

## 2 Antisymm-Eqs. (6-8)

```
(%i4) depends([A], [x,y,z]);
(%o4) [A(x,y,z)]
```

```
(%i7) E6: diff(A[3],y) - omega[2]*A[3] = -(diff(A[2],z) - omega[3]*A[2]);
      E7: diff(A[1],z) - omega[3]*A[1] = -(diff(A[3],x) - omega[1]*A[3]);
      E8: diff(A[2],x) - omega[1]*A[2] = -(diff(A[1],y) - omega[2]*A[1]);
(%o5)  $\frac{d}{dy} A_3 - \omega_2 A_3 = A_2 \omega_3 - \frac{d}{dz} A_2$ 
(%o6)  $\frac{d}{dz} A_1 - A_1 \omega_3 = \omega_1 A_3 - \frac{d}{dx} A_3$ 
(%o7)  $\frac{d}{dx} A_2 - \omega_1 A_2 = A_1 \omega_2 - \frac{d}{dy} A_1$ 
```

## 3 Eq. (1) --> (9-11)

```
(%i8) E1: curl(A) = -cross(omega,A);
(%o8) [ $\frac{d}{dy} A_3 - \frac{d}{dz} A_2$ ,  $\frac{d}{dz} A_1 - \frac{d}{dx} A_3$ ,  $\frac{d}{dx} A_2 - \frac{d}{dy} A_1$ ] = [A2  $\omega_3 - \omega_2 A_3$ ,  $\omega_1 A_3$ ,
-A1  $\omega_3$ , A1  $\omega_2 - \omega_1 A_2$ ]
```

```
(%i11) E9: first(lhs(E1)) = first(rhs(E1));
E10: second(lhs(E1)) = second(rhs(E1));
E11: third(lhs(E1)) = third(rhs(E1));

(%o9)  $\frac{d}{d y} A_3 - \frac{d}{d z} A_2 = A_2 \omega_3 - \omega_2 A_3$ 

(%o10)  $\frac{d}{d z} A_1 - \frac{d}{d x} A_3 = \omega_1 A_3 - A_1 \omega_3$ 

(%o11)  $\frac{d}{d x} A_2 - \frac{d}{d y} A_1 = A_1 \omega_2 - \omega_1 A_2$ 
```

#### 4 Solve antisymm. eqs. (6-8)

```
(%i12) ESa: solve([E6,E7,E8],[omega[1], omega[2], omega[3]]);
(%o12) [[omega_1 = -

$$\frac{A_1 \left( \frac{d}{d y} A_3 \right) + A_2 \left( -\frac{d}{d x} A_3 - \frac{d}{d z} A_1 \right) + A_1 \left( \frac{d}{d z} A_2 \right) + A_3 \left( -\frac{d}{d x} A_2 - \frac{d}{d y} A_1 \right)}{2 A_2 A_3}, \omega_2 =$$


$$\frac{A_1 \left( \frac{d}{d y} A_3 \right) + A_2 \left( -\frac{d}{d x} A_3 - \frac{d}{d z} A_1 \right) + A_1 \left( \frac{d}{d z} A_2 \right) + A_3 \left( \frac{d}{d x} A_2 + \frac{d}{d y} A_1 \right)}{2 A_1 A_3}, \omega_3 =$$


$$\frac{A_1 \left( \frac{d}{d y} A_3 \right) + A_2 \left( \frac{d}{d x} A_3 + \frac{d}{d z} A_1 \right) + A_1 \left( \frac{d}{d z} A_2 \right) + A_3 \left( -\frac{d}{d x} A_2 - \frac{d}{d y} A_1 \right)}{2 A_1 A_2} ] ]$$

```

#### 5 Solve eqs. (9-11)

```
(%i13) ESb: solve([E9,E10,E11],[omega[1], omega[2], omega[3]]);
(%o13) [ ]
```

#### 6 Check eqs. (18-20)

```
(%i16) E18: diff(A[3],y)+diff(A[3],x) = omega[3]*(A[2]+A[1]);
E19: diff(A[1],z)+diff(A[1],y) = omega[1]*(A[3]+A[2]);
E20: diff(A[2],x)+diff(A[2],z) = omega[2]*(A[1]+A[3]);

(%o14)  $\frac{d}{d y} A_3 + \frac{d}{d x} A_3 = (A_2 + A_1) \omega_3$ 

(%o15)  $\frac{d}{d z} A_1 + \frac{d}{d y} A_1 = \omega_1 (A_3 + A_2)$ 

(%o16)  $\frac{d}{d z} A_2 + \frac{d}{d x} A_2 = \omega_2 (A_3 + A_1)$ 
```

```
(%i17) ESd: solve([E18,E19,E20],[omega[1], omega[2], omega[3]]);
(%o17) [[omega_1 =  $\frac{\frac{d}{d z} A_1 + \frac{d}{d y} A_1}{A_3 + A_2}$ , omega_2 =  $\frac{\frac{d}{d z} A_2 + \frac{d}{d x} A_2}{A_3 + A_1}$ , omega_3 =  $\frac{\frac{d}{d y} A_3 + \frac{d}{d x} A_3}{A_2 + A_1}$  ] ]
```

## 7 Choice of A

### 7.1 Define A

```
(%i18) A: mu_0*I*a^2/(4*(x^2+y^2+z^2)^(1/2))*[-y, x, 0];
```

$$(\%o18) \left[ -\frac{I a^2 \mu_0 y}{4 \sqrt{z^2+y^2+x^2}}, \frac{I a^2 \mu_0 x}{4 \sqrt{z^2+y^2+x^2}}, 0 \right]$$

```
(%i19) A: mu_0*I*a^2/(4*(x^2+y^2+z^2)^(3/2))*[-y, x, 0];
```

$$(\%o19) \left[ -\frac{I a^2 \mu_0 y}{4 (z^2+y^2+x^2)^{3/2}}, \frac{I a^2 \mu_0 x}{4 (z^2+y^2+x^2)^{3/2}}, 0 \right]$$

Eqs. (6-8)

```
(%i22) factor(ev(E6,diff)); factor(ev(E7,diff)); factor(ev(E8,diff));
```

$$(\%o20) 0 = \frac{I a^2 \mu_0 x (\omega_3 z^2 + 3 z + \omega_3 y^2 + \omega_3 x^2)}{4 (z^2 + y^2 + x^2)^{5/2}}$$

$$(\%o21) \frac{I a^2 \mu_0 y (\omega_3 z^2 + 3 z + \omega_3 y^2 + \omega_3 x^2)}{4 (z^2 + y^2 + x^2)^{5/2}} = 0$$

$$(\%o22) -\frac{I a^2 \mu_0 (\omega_1 x z^2 - z^2 + \omega_1 x y^2 - y^2 + \omega_1 x^3 + 2 x^2)}{4 (z^2 + y^2 + x^2)^{5/2}} = -$$

$$\frac{I a^2 \mu_0 (\omega_2 y z^2 - z^2 + \omega_2 y^3 + 2 y^2 + \omega_2 x^2 y - x^2)}{4 (z^2 + y^2 + x^2)^{5/2}}$$

Eqs. (9-11)

```
(%i25) factor(ev(E9,diff)); factor(ev(E10,diff)); factor(ev(E11,diff));
```

$$(\%o23) \frac{3 I a^2 \mu_0 x z}{4 (z^2 + y^2 + x^2)^{5/2}} = \frac{\omega_3 I a^2 \mu_0 x}{4 (z^2 + y^2 + x^2)^{3/2}}$$

$$(\%o24) \frac{3 I a^2 \mu_0 y z}{4 (z^2 + y^2 + x^2)^{5/2}} = \frac{\omega_3 I a^2 \mu_0 y}{4 (z^2 + y^2 + x^2)^{3/2}}$$

$$(\%o25) \frac{I a^2 \mu_0 (2 z^2 - y^2 - x^2)}{4 (z^2 + y^2 + x^2)^{5/2}} = -\frac{I a^2 \mu_0 (\omega_2 y + \omega_1 x)}{4 (z^2 + y^2 + x^2)^{3/2}}$$

```
(%i26) ESa: solve([ev(E6,diff),ev(E7,diff),ev(E8,diff)], [omega[1], omega[2]]
solve: dependent equations eliminated: (2)
```

$$(\%o26) \left[ \left[ \omega_1 = \frac{\%r1 y z^2 + \%r1 y^3 + 3 y^2 + x^2 (\%r1 y - 3)}{x z^2 + x y^2 + x^3}, \omega_2 = \%r1, \omega_3 = -\frac{3 z}{z^2 + y^2 + x^2} \right] \right]$$

```
(%i27) ESb: solve([ev(E9,diff),ev(E10,diff),ev(E11,diff)], [omega[1], omega[2], omega[3]
solve: dependent equations eliminated: (2)
(%o27) [[omega_1 = - ((%r2 y+2) z^2+%r2 y^3-y^2+x^2 (%r2 y-1)) / (x z^2+x y^2+x^3), omega_2 = %r2, omega_3 =
3 z / (z^2+y^2+x^2) ] ]
```

```
(%i28) ESc: solve([ev(E8,diff),ev(E11,diff),ev(E9,diff)], [omega[1], omega[2], omega[3]
(%o28) [[omega_1 = (-z^2+2 y^2-x^2) / (x z^2+x y^2+x^3), omega_2 = -(z^2+y^2-2 x^2) / (y (z^2+x^2)+y^3), omega_3 = 3 z / (z^2+y^2+x^2) ] ]
```

```
--> /*ESc: solve([ev(E18,diff),ev(E19,diff),ev(E20,diff)], [omega[1], omega[2], omega[3]

```

## 7.2 Solution omega for A

```
(%i29) omega: factor([rhs(first(ESc)[1]),rhs(first(ESc)[2]),
rhs(first(ESc)[3])]);
(%o29) [ - (z^2-2 y^2+x^2) / (x (z^2+y^2+x^2)), - (z^2+y^2-2 x^2) / (y (z^2+y^2+x^2)), 3 z / (z^2+y^2+x^2) ]
```

## 7.3 Other terms

```
(%i30) oxA: factor(cross(omega,A));
(%o30) [ - (3 I a^2 mu_0 x z) / (4 (z^2+y^2+x^2)^{5/2}), - (3 I a^2 mu_0 y z) / (4 (z^2+y^2+x^2)^{5/2}), - (I a^2 mu_0 (2 z^2-y^2-x^2)) / (4 (z^2+y^2+x^2)^{5/2}) ]
```

```
(%i31) cA: factor(curl(A));
(%o31) [ (3 I a^2 mu_0 x z) / (4 (z^2+y^2+x^2)^{5/2}), (3 I a^2 mu_0 y z) / (4 (z^2+y^2+x^2)^{5/2}), (I a^2 mu_0 (2 z^2-y^2-x^2)) / (4 (z^2+y^2+x^2)^{5/2}) ]
```

```
(%i32) B: cA-oxA;
(%o32) [ (3 I a^2 mu_0 x z) / (2 (z^2+y^2+x^2)^{5/2}), (3 I a^2 mu_0 y z) / (2 (z^2+y^2+x^2)^{5/2}), (I a^2 mu_0 (2 z^2-y^2-x^2)) / (2 (z^2+y^2+x^2)^{5/2}) ]
```

## 7.4 Tests

```
(%i33) cA+oxA;
(%o33) [0, 0, 0]
```

```
(%i34) factor(div(oxA));
(%o34) 0
```

```
(%i35) factor(div(B));
(%o35) 0
```

```
(%i36) factor(curl(B));  
(%o36) [0, 0, 0]
```