56(3): 1(1.2) Development of the Magnetic Dipole Potential

The magnetic dipole potential is found from Jackson's eqn. (5.39) in the limit $r \gg a$, so:

$$A_\phi = \frac{\mu_0 I a^2}{4 r^2} \sin\theta \quad -(1)$$

in spherical polar coordinates. Transforming to Cartesian coordinates:

$$r^2 = x^2 + y^2 + z^2 \quad -(2)$$

and

$$\cos\theta = z/r \quad -(3)$$

so

$$\sin\theta = \left(1 - \frac{z^2}{r^2}\right)^{1/2} \quad -(4)$$

The unit vector transforms as:

$$\underline{e}_\phi = -\underline{i}\sin\phi + \underline{j}\cos\phi \quad -(5)$$

$$= -\frac{y}{(x^2+y^2)^{1/2}}\underline{i} + \frac{x}{(x^2+y^2)^{1/2}}\underline{j}$$

So:

$$A_\phi = \frac{\mu_0 I a^2}{4(x^2+y^2+z^2)}\left(1 - \frac{z^2}{r^2}\right)^{1/2}$$

$$= \mu_0 I a^2 \frac{(x^2+y^2)^{1/2}}{(x^2+y^2+z^2)^{3/2}} \quad -(6)$$

It follows that:

$$\underline{A} = \frac{\mu_0 I a^2}{4}\left(\frac{-y\,\underline{i}}{(x^2+y^2+z^2)^{3/2}} + \frac{x\,\underline{j}}{(x^2+y^2+z^2)^{3/2}}\right) \quad -(7)$$

) This potential is a particular case of the ECE2 Ampère Law:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad -(8)$$

where $\underline{J}$ is the current in the loop.

The ECE2 corresponding laws are:

$$\frac{\partial A_Z}{\partial Y} + \frac{\partial A_Y}{\partial Z} = \omega_Y A_Z + \omega_Z A_Y \quad -(9)$$

$$\frac{\partial A_X}{\partial Z} + \frac{\partial A_Z}{\partial X} = \omega_Z A_X + \omega_X A_Z \quad -(10)$$

$$\frac{\partial A_Y}{\partial X} + \frac{\partial A_X}{\partial Y} = \omega_X A_Y + \omega_Y A_X \quad -(11)$$

The spin connection vector:

$$\underline{\omega} = \omega_X \underline{i} + \omega_Y \underline{j} + \omega_Z \underline{k} \quad -(12)$$

can be found from eqs. (7) and (9) to (11).

Finally, the magnetic flux density is:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(13)$$

Note carefully that in the standard model:

$$\frac{\partial A_X}{\partial Y} \neq - \frac{\partial A_Y}{\partial X} \quad -(14)$$

from eq. (7), and:

$$\frac{\partial A_X}{\partial Z} \neq \frac{\partial A_Z}{\partial X} \quad -(15)$$

$$\frac{\partial A_Y}{\partial Z} \neq \frac{\partial A_Z}{\partial Y} \quad -(16)$$

The standard model violates conservation of anti-