86(b): Second Solution

This is:

$$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_z}{\partial x}\right) = \omega_z \left(A_y - A_x\right) \quad -(1)$$

$$\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_x}{\partial y}\right) = \omega_x \left(A_z - A_y\right) \quad -(2)$$

$$\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial z}\right) = \omega_y \left(A_x - A_z\right) \quad -(3)$$

These solutions can be considered to give:

$$\left(\frac{\partial A_z}{\partial y} + \frac{\partial A_z}{\partial x}\right)\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_z}{\partial x}\right) = \omega_z^2 \left(A_y + A_x\right)\left(A_y - A_x\right) \quad -(4)$$

$$\left(\frac{\partial A_x}{\partial z} + \frac{\partial A_x}{\partial y}\right)\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_x}{\partial y}\right) = \omega_x^2 \left(A_z + A_y\right)\left(A_z - A_y\right) \quad -(5)$$

$$\left(\frac{\partial A_y}{\partial x} + \frac{\partial A_y}{\partial z}\right)\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial z}\right) = \omega_y^2 \left(A_x + A_z\right)\left(A_x - A_z\right) \quad -(6)$$

Eqs. (4) to (6) can be considered to be the final and least restrictive solution. Given the assumption:

$$\underline{\nabla} \times \underline{A} = - \underline{\omega} \times \underline{A} \quad -(7)$$

any vector potential can be calculated from any current $\underline{J}(\underline{x}')$:

$$\underline{A} = \frac{\mu_0}{4\pi} \int \frac{\underline{J}(\underline{x}')}{|\underline{x} - \underline{x}'|} d^3x' \quad -(8)$$

In order to conserve antisymmetry, the spin connections can be calculated from Eqs (4) to (6). The first solution, given in note 386(5), is:

$$\left(\frac{\partial A_Z}{\partial Y} + \frac{\partial A_Z}{\partial X}\right) = \omega_Z \left(A_Y + A_X\right) \quad -(9)$$

$$\left(\frac{\partial A_X}{\partial Z} + \frac{\partial A_X}{\partial Y}\right) = \omega_X \left(A_Z + A_Y\right) \quad -(10)$$

$$\left(\frac{\partial A_Y}{\partial X} + \frac{\partial A_Y}{\partial Z}\right) = \omega_Y \left(A_X + A_Z\right) \quad -(11)$$

However, there are also relations such as:

$$\omega_X = \frac{1}{A_Y}\frac{\partial A_X}{\partial Y} = \frac{1}{A_Z}\frac{\partial A_X}{\partial Z} \quad -(12)$$

$$\omega_Y = \frac{1}{A_X}\frac{\partial A_Y}{\partial X} = \frac{1}{A_Z}\frac{\partial A_Y}{\partial Z} \quad -(13)$$

$$\omega_Z = \frac{1}{A_X}\frac{\partial A_Z}{\partial X} = \frac{1}{A_Y}\frac{\partial A_Z}{\partial Y} \quad -(14)$$

Eqs. (9) to (11), (1) to (3), and (12) to (14) are mathematically possible, but eqs. (12) to (14) are unphysical because they restrict the potential to unphysical solutions. For example, eqs. (12) to (14) are not obeyed by a circular current loop, but the other solutions are obeyed by a circular current loop. The most general solution is: $-(15)$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} = \underline{\nabla} \times \underline{A} + \underline{\nabla} \times \underline{\alpha},$$

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(16)$$

which will be considered in the next note.