386(8): Commutator Proof of U(1) Antisymmetry

As in UFT 131, the U(1) gauge theory of the standard model defines the U(1) electromagnetic field as:

$$F_{\mu\nu} = [D_\mu, D_\nu]\phi = [\partial_\mu - igA_\mu, \partial_\nu - igA_\nu]\phi \quad -(1)$$

where ϕ is the gauge field. The standard U(1) covariant derivative is:

$$D_\mu = \partial_\mu - igA_\mu \quad -(2)$$

As is well known. The commutator of covariant derivatives is defined by:

$$[D_\mu, D_\nu]\phi = -[D_\nu, D_\mu]\phi \quad -(3)$$

and is antisymmetric by definition.

From eq. (1):

$$[D_\mu, D_\nu]\phi = [\partial_\mu, \partial_\nu]\phi - ig[A_\mu, \partial_\nu - igA_\nu]\phi$$

$$= -ig[A_\mu, \partial_\nu - igA_\nu]\phi$$

$$= -ig[A_\mu, \partial_\nu]\phi - g^2[A_\mu, A_\nu]\phi$$

$$= -ig[A_\mu, \partial_\nu]\phi \quad -(4)$$

because

$$[\partial_\mu, \partial_\nu]\phi = 0 \quad -(5)$$

$$[A_\mu, A_\nu]\phi = 0 \quad -(6)$$

Consider:

$$[A_\mu, \partial_\nu]\phi = A_\mu\partial_\nu\phi - \partial_\nu(A_\mu\phi)$$

2)
$$= A_\mu \partial_\nu \phi - A_\mu \partial_\nu \phi - \partial_\nu A_\mu \phi \quad -(7)$$

It follows that:

$$\boxed{[D_\mu, D_\nu]\phi = ig\, \partial_\nu A_\mu \phi} \quad -(8)$$

From eq. (3) it follows that:

$$\boxed{\partial_\mu A_\nu = -\partial_\nu A_\mu} \quad -(9)$$

Q.E.D.

However, in U(1) electrodynamics:

$$\partial_\mu A_\nu \neq -\partial_\nu A_\mu \quad -(10)$$

in general. So standard electrodynamics is entirely refuted. On the ECE2 level the commutator (3) is replaced by:

$$[D_\mu, D_\nu] V^\rho$$

which generates the torsion tensor and curvature tensor simultaneously. ECE2 conserves antisymmetry through suitable choice of spin connection.