889(3): Newtonian Limit of ECE2 Gravitation.

1) This is defined by:

$$\underline{g} = -mG\frac{\underline{r}}{r^3} = -\underline{\nabla}\Phi + \underline{\omega}\Phi = -\underline{\nabla}\Phi - \frac{\partial \underline{Q}}{\partial t}(\text{total}) \quad -(1)$$

2) One possible solution is:

$$\Phi = -\frac{mG}{2r}, \quad \underline{\omega} = \frac{\underline{r}}{r^2} \quad -(2)$$

3) Now find $\underline{Q}$ from the antisymmetry equations:

$$\frac{\partial Q_Z}{\partial Y} + \frac{\partial Q_Y}{\partial Z} = \omega_Y Q_Z + \omega_Z Q_Y \quad -(3)$$

$$\frac{\partial Q_X}{\partial Z} + \frac{\partial Q_Z}{\partial X} = \omega_Z Q_X + \omega_X Q_Z \quad -(4)$$

$$\frac{\partial Q_Y}{\partial X} + \frac{\partial Q_X}{\partial Y} = \omega_X Q_Y + \omega_Y Q_X \quad -(5)$$

4) Find ω_0 from the Lindstrom constraint:

$$\omega_0 = \frac{1}{\Phi}\left(c^2(\underline{\nabla} - \underline{\omega})\cdot\underline{Q} - \frac{\partial \Phi}{\partial t}\right) \quad -(6)$$

5) Using Eq. (6) find:

$$\frac{\partial \underline{Q}}{\partial t}(\text{total}) = -\underline{\omega}\Phi \quad -(7)$$

6) Find

$$\underline{Q}(\text{total}) = -\int \underline{\omega}\Phi \, dt + \underline{Q}_2 \quad -(8)$$

where $\underline{Q}_2$ is a constant of integration:

$$\underline{\mathcal{Q}}_2 = \underline{\mathcal{Q}}(\text{total}) + \underline{\omega}\,\underline{\Phi} \quad - (9)$$

where

$$\underline{\mathcal{Q}}(\text{total}) = \underline{\mathcal{Q}} + \underline{\mathcal{Q}}_1 \quad - (10)$$

and

$$\underline{\nabla} \times \underline{\mathcal{Q}}_1 := -\underline{\omega} \times \underline{\mathcal{Q}} \quad - (11)$$

So

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{\mathcal{Q}}_1 = -\underline{\nabla} \cdot (\underline{\omega} \times \underline{\mathcal{Q}}) \quad - (12)$$
$$= 0$$

Non Newtonian effects are described by using a spin connection different from eq. (2). For example orbital precession, electromagnetic deflection due to gravitation, velocity curve of a whirlpool galaxy, Lense Thirring precession, de Sitter precession, and so on.

In addition to the above set of equations:

$$\underline{\nabla} \cdot \underline{g} = \Box \phi = 4\pi G \rho_m \quad - (13)$$

and

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2}\frac{\partial \underline{g}}{\partial t} = \Box \underline{\mathcal{Q}} = \frac{4\pi G}{c^2} \underline{J}_m \quad - (14)$$

Therefore the current of mass density $\underline{J}_m$ is defined by eq. (14).